On Counter Functions and Operators for Modeling Discrete-Event Dynamic Systems

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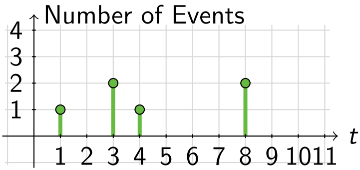
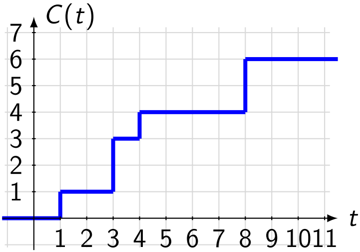
Counter of events





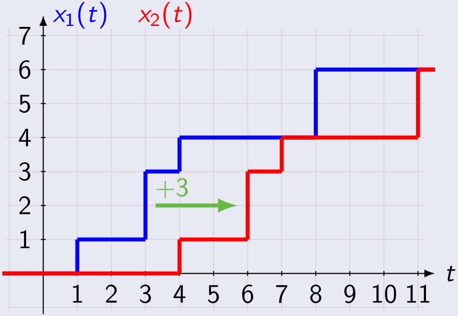
Counters of events (cumulative value)

Function $C(t)$ gives the number of event's occurrence up to time t, e.g., $C(6) = 4$ means that 4 events occurred up to time 6.
It is a non-decreasing and non-continuous function.
The set of Counters is denoted $\mathcal{C}$ in the sequel.

Outline

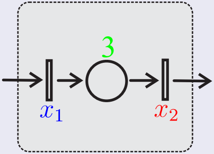
1. Transformations in $\mathcal{C}$: δ-operator

Time Shifting of Counter



Delay in timed Petri net

$x_2(t) = x_1(t - 3)$

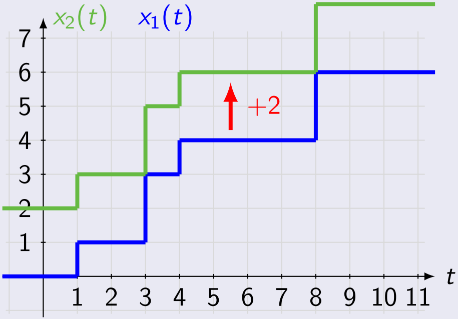


Operator δ^τ, shifting of τ time unit :

$\delta^\tau : \mathcal{C} \to \mathcal{C},\ C(t) \to C(t - \tau)$, e.g.,
$\delta^3 : \mathcal{C} \to \mathcal{C},\ C(t) \to C(t - 3)$, is the mapping delaying a signal of 3 times unit, i.e., $x2(t) = \delta^3(x_1(t))$.

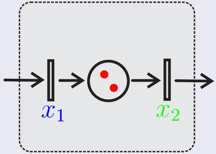
1. Transformations in $\mathcal{C}$: γ-operator

Event Shifting



Initial marking

$x_2(t) = x_1(t) + 2$



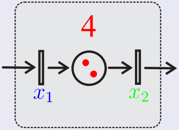
Operator γ^n, shifting of n events :

$\gamma^n : \mathcal{C} \to \mathcal{C}, C(t) \to n + C(t)$, e.g.,
$\gamma^2 : \mathcal{C} \to \mathcal{C}, C(t) \to 2 + C(t)$, is the mapping shifting a signal of 2 events, i.e., $x2(t) = \gamma^2(x_1(t))$.

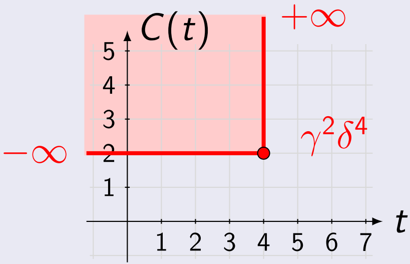
$$\delta^\tau \quad : \quad \mathcal{C} \to \mathcal{C}, C(t) \to C(t-\tau)$$
$$\gamma^n \quad : \quad \mathcal{C} \to \mathcal{C}, C(t) \to n + C(t)$$
$$\gamma^n \circ \delta^\tau \quad = \quad \delta^\tau \circ \gamma^n$$

Composition of operators

$$x_2 = \gamma^2(\delta^4(x_1)) = \delta^4(\gamma^2(x_1))$$

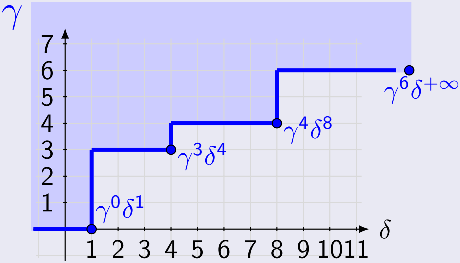


$\gamma^n\delta^\tau$ to code a counter function



Counters $C(t)$ can be coded as a union of monomial $\gamma^n\delta^\tau$

$C(\gamma,\delta) = \bigcup \gamma^{C(t)}\delta^t$ where γ is the event shift operator and δ is the time shift operator.



2. Idempotent semi-ring

Idempotent Semiring $\mathcal{S}$ (Tropical Algebra (I. Simon))

- Sum $\oplus$, associative,commutative, zero element denoted ε ,
- Product $\otimes$, associative, identity element denoted e,
- Product $\otimes$ distributes with respect of sum,
  $(a \oplus b) \otimes c = a \otimes c \oplus b \otimes c$,
- Zero element ε is absorbing, $a \otimes \varepsilon = \varepsilon$
- The sum is idempotent, $a \oplus a = a$.
- $b \preceq a \Leftrightarrow a \oplus b = a = a \vee b \Leftrightarrow a \wedge b = b$
  hence an idempotent semiring has a complete lattice structure, with (ε) as bottom element and ($T = \bigoplus_{x \in \mathcal{S}} x$) as top element.

(min,+) algebra, $\overline{\mathbb{Z}}_{\min}$

▸ More

Sum $\oplus$ is the operator *min*, product $\otimes$ is classical sum $+$, $\varepsilon = +\infty$, $T = -\infty$ and $e = 0$, then :

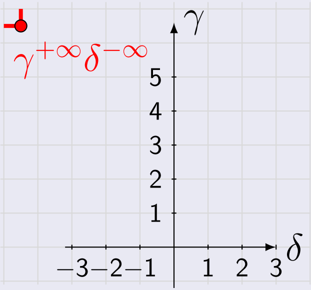
$$1 \oplus 2 = 1 = min(1, 2), \text{ (warning } 2 \preceq 1)$$
$$2 \otimes 1 = 3 = 2 + 1.$$

2. Semiring $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$ (Cohen et al.)

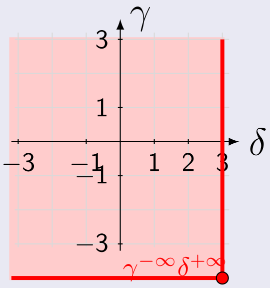
Semiring of formal power series coded with γ^n and δ^τ

Sum $\oplus$ is the $\cup$, product $\otimes$ is the composition $\circ$, $\varepsilon = \gamma^{+\infty}\delta^{-\infty}$, $T = \gamma^{-\infty}\delta^{+\infty}$ and $e = \gamma^0\delta^0$.

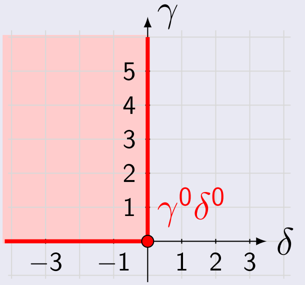
$\varepsilon = \gamma^{+\infty}\delta^{-\infty}$



$T = \gamma^{-\infty}\delta^{+\infty}$



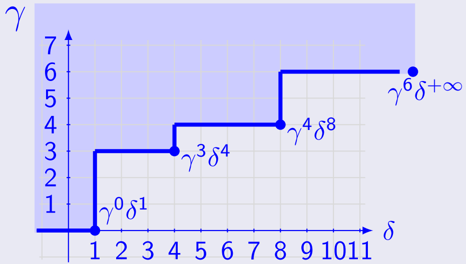
$e = \gamma^0\delta^0$



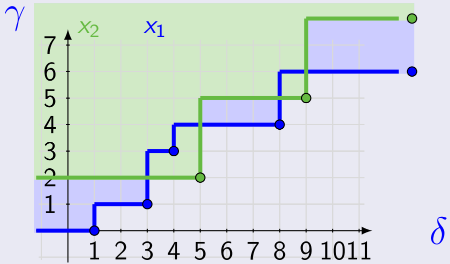
Counters $C(t)$ can be coded as a non decreasing series in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$

$C(\gamma,\delta) = \bigoplus \gamma^{C(t)}\delta^{t}$ where γ is a event shift operator and δ is a time shift operator, a series admits a minimal representation, e.g.,

$$C(\gamma,\delta) = \gamma^0\delta^1 \oplus \gamma^3\delta^4 \oplus \gamma^4\delta^8 \oplus \gamma^6\delta^{+\infty}$$

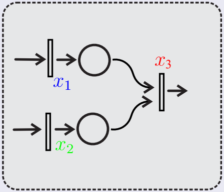


Synchronization of signals

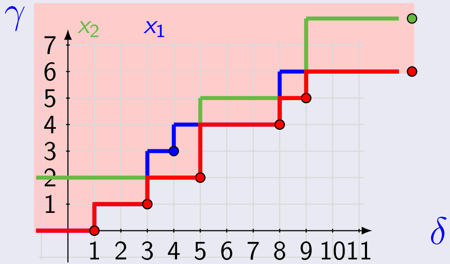


Synchronization phenomena

$x_3 = x_1 \oplus x_2$



Synchronization



Synchronization phenomena

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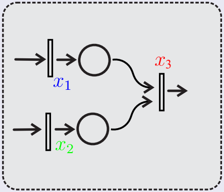
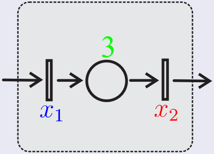
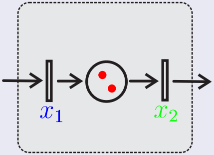


2. Elementary Operations

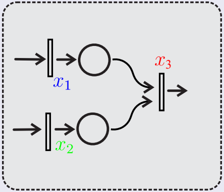
δ^t : time shifting, $x_2 = \delta^3 x_1$

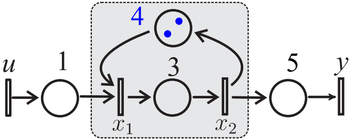


γ^n : event shifting , $x_2 = \gamma^2 x_1$



$\oplus$: synchronization phenomena $x_3 = x_1 \oplus x_2$



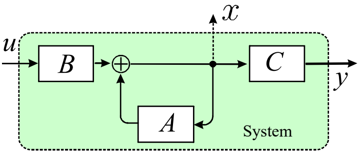


Timed Event Graphs (TEG)

are perfectly described by composition of operators γ and δ.
Internal transitions are denoted x_i (inputs transitions u_i, outputs transitions y_i).

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \varepsilon & \gamma^2\delta^4 \\ \delta^3 & \varepsilon \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus \begin{pmatrix} \delta^1 \\ \varepsilon \end{pmatrix} u$$

$$y = \begin{pmatrix} \varepsilon & \delta^5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$



Standard representation in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$

with vectors of inputs trajectories $u \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^p$, of internal states trajectories $x \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^n$ and outputs trajectories $y \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^q$), hence $A \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times n}$, $B \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{n\times p}$, $C \in \mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]^{q\times n}$:

$$\begin{aligned} x &= Ax \oplus Bu \\ y &= Cx \end{aligned}$$

3. Transfer Relation of TEG

Fixed point equations

For non-decreasing function, ($x \preceq y \Rightarrow f(x) \preceq f(y)$), it is possible to compute fixed points $f(x) = x$.

Application : $x = ax \oplus b = f(x)$

Theorem : Over a complete idempotent semiring $\mathcal{S}$, the least solution to $x = ax \oplus b$ is $x = a^*b$ with $a^* = \bigoplus_{i \in \mathbb{N}_0} a^i = e \oplus a \oplus a^2 \oplus \ldots$

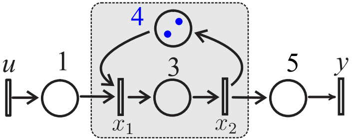
$*$ is called Kleene star operator.

State Equation :

$$\begin{aligned} x &= Ax \oplus Bu \\ y &= Cx \end{aligned}$$

Transfer Relation :

$$\begin{aligned} x &= A^*Bu \\ y &= CA^*Bu \end{aligned}$$



Transfer relation

$$A^* = \begin{pmatrix} (\gamma^2\delta^7)^* & \gamma^2\delta^4(\gamma^2\delta^7)^* \\ \delta^3(\gamma^2\delta^7)^* & (\gamma^2\delta^7)^* \end{pmatrix}$$

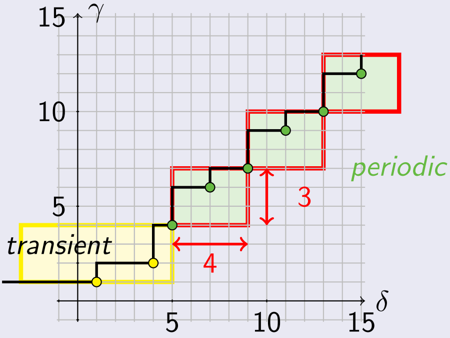
$$x = A^*Bu = \begin{pmatrix} \gamma^0\delta^1(\gamma^2\delta^7)^* \\ \gamma^0\delta^4(\gamma^2\delta^7)^* \end{pmatrix} u$$

$$y = CA^*Bu = \delta^9(\gamma^2\delta^7)^* u$$

A periodic series in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

$s = p \oplus q(\gamma^{\nu}\delta^{\tau})^*$ where $p = \bigoplus_i^n \gamma^{n_i}\delta^{t_i}$ and $q = \bigoplus_j^m \gamma^{n_j}\delta^{t_j}$ are polynomials and $\mathcal{C}_\infty(s) = \nu/\tau$ is the asymptotic slope (the throughput). Canonical form exists.

$s = (\delta^1\gamma^1 \oplus \delta^4\gamma^2) \oplus (\delta^5\gamma^4 \oplus \delta^7\gamma^6)(\delta^4\gamma^3)^*$ and $\mathcal{C}_\infty(s) = 3/4$



3. Idempotent semi-ring and pseudo-inverse

Residuation Theory (Galois Connection) (Croisot et al., Blyth)

A pseudo inverse exists for non-decreasing function defined over ordered sets.

Inequality $a \otimes x \preceq b$ (Baccelli et al.)

Over a complete idempotent semi-ring, inequality $a \otimes x \preceq b$ admits a greatest solution , denoted, $x = a \diamond\!\!\!\backslash b$,
($i.e.$ $a(a \diamond\!\!\!\backslash b) \preceq b$ and equality is achieved, if possible).

Example : (min,+) algebra $\overline{\mathbb{Z}}_{\min}$

Inequality $3 \otimes x \preceq 5$ admits a greatest solution $x = 3 \diamond\!\!\!\backslash 5 = 5 - 3 = 2$. It achieves equality in the scalar case.
(warning : $\preceq$ is the inverse order in this semi-ring)

3. Semiring of periodic series (Cohen et al.)

Operations over semi-ring of periodic series over $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$

- $s = s_1 \oplus s_2$ is a periodic series, asymptotic slope $\sigma_\infty(s) = min(\sigma_\infty(s_1), \sigma_\infty(s_2))$
- $s = s_1 \otimes s_2$ is a periodic series, asymptotic slope $\sigma_\infty(s) = min(\sigma_\infty(s_1), \sigma_\infty(s_2))$
- $s = s_1 \wedge s_2$ is a periodic series, asymptotic slope $\sigma_\infty(s) = max(\sigma_\infty(s_1), \sigma_\infty(s_2))$
- $s = s_1 \backslash\!\!\!\circ s_2$ is a periodic series, $\sigma_\infty(s) = \sigma_\infty(s_2)$ if $\sigma_\infty(s_2) \leq \sigma_\infty(s_1)$ else $s = \varepsilon$.

Software Tools (MinmaxGD)

Software to handle periodic series is available on :
http://perso-laris.univ-angers.fr/~hardouin/outils.html
http://perso-laris.univ-angers.fr/~lhommeau/

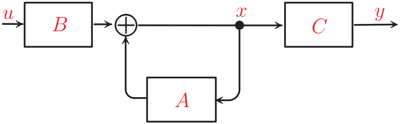
Matrix

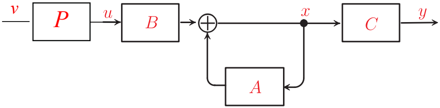
Let A, B, C three matrices in $\mathcal{S}^{n \times n}$

- $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$
- $(A \otimes B)_{ik} = \bigoplus_{j=1 \ldots n} (A_{ij} \otimes B_{jk})$
- $(A \circ\!\!\!\backslash B)_{ik} = \bigwedge_{j=1 \ldots n} (A_{ji} \circ\!\!\!\backslash B_{jk})$, where $A \circ\!\!\!\backslash B$ is the greatest matrix s.t. $AX \preceq B$
- $(B \circ\!\!\!/ A)_{ik} = \bigwedge_{j=1 \ldots n} (A_{ij} \circ\!\!\!/ B_{kj})$, where $A \circ\!\!\!/ B$ is the greatest such $XA \preceq B$
- $(X)_{ij} = A^*_{ij}$ is the greatest matrix s.t. $X \preceq A^*$

More

See : http://perso-laris.univ-angers.fr/~hardouin/GET_BO.html





System Equation :

$$\begin{cases} x &= Ax \oplus Bu \\ y &= Cx = CA^*Bu \end{cases}$$

Open-loop control

$$y = CA^*BPv$$

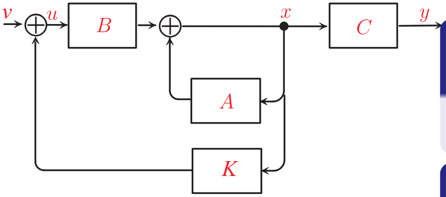
such that

$$CA^*BP = CA^*B$$

Optimal solution

$$P_{opt} = Pr_+((CA^*B) \circ\!\!\!\!\backslash (CA^*B))$$

3. Controller synthesis (Maia et al.)



Controlled system :

$$\begin{cases} x = Ax \oplus B(v \oplus Kx) \\ y = Cx \end{cases}$$

Closed-loop transfer function :

$$y = C(A \oplus BK)^* Bv$$

Objective :

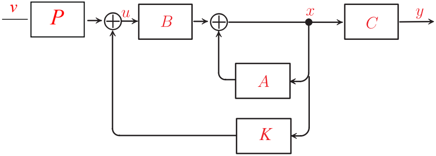
Compute the greatest K s.t. :

$$C(A \oplus BK)^* B = CA^* B$$

Optimal solution

$$K_{opt} = Pr_+((A^*B) \circ\!\!\!\!\backslash (CA^*B) \circ\!\!\!\!/ (CA^*B)$$

3. Controller synthesis (Maia et al.)



Controlled system :

$$\begin{cases} x = Ax \oplus B(v \oplus Kx) \oplus Pv \\ y = Cx \end{cases}$$

Controlled transfer function :

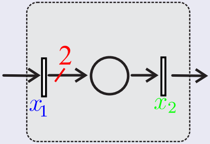
$$y = C(A \oplus BK)^* BPv$$

Controller P_{opt} and K_{opt} :

$$P_{opt} = Pr_+((CA^*B) \circ\!\!\!\backslash (CA^*B))$$

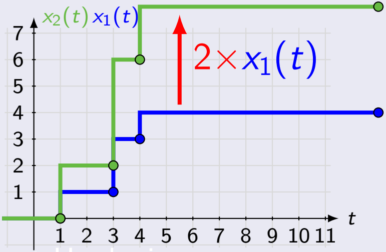
$$K_{opt} = Pr_+((A^*B) \circ\!\!\!\backslash (CA^*B) \circ\!\!\!/ (CA^*B))$$

Split

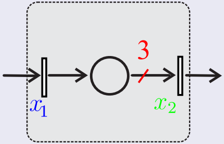


$$x_2(t) = 2\times x_1(t)$$

Event counter is multiplied by 2 (input weight).

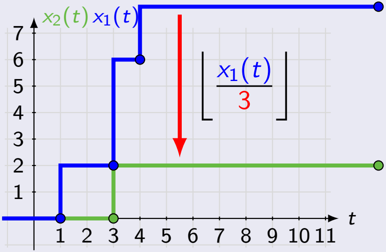


Batch



$$x_2(t) = \left\lfloor \frac{x_1(t)}{3} \right\rfloor$$

Event counter is divided by 3 (output weight).



4. Operators to Model Weights (Cottenceau et al.)

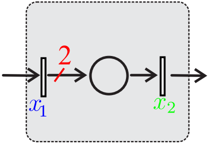
Additive operators μ_m and β_b

$m \in \mathbb{N}^+, \ \mu_m : \mu_m(C(t)) = m \times C(t)$

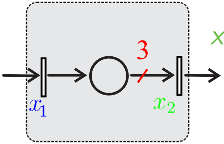
$b \in \mathbb{N}^+, \ \beta_b : \beta_b(C(t)) = \lfloor C(t)/b \rfloor$

Commutation

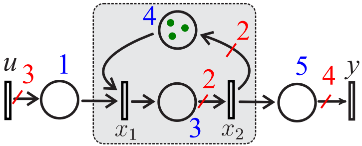
- μ_m, β_b and γ^ν do not commute
- μ_m, β_b and γ^ν commute with δ^τ
- $\mu_m \gamma^1 = \gamma^m \mu_m$ and $\gamma^1 \beta_b = \beta_b \gamma^b$



$x_2(t) = \mu_2(x_1(t))$



$x_2(t) = \beta_3(x_1(t))$



WB-TEG model

Delays are in blue, weights in red, tokens in green.

$$x = \begin{pmatrix} \varepsilon & \delta^4\gamma^3\mu_2 \\ \delta^3\beta_2 & \varepsilon \end{pmatrix} x \oplus \begin{pmatrix} \delta^1\mu_3 \\ \varepsilon \end{pmatrix} u$$

$$y = \begin{pmatrix} \varepsilon & \delta^5\beta_4 \end{pmatrix} x$$

Gain of a Path

Gain of a path is the product of each weight, e.g. $u \to y$ the ratio of input weight and output weight is $3/1 \times 1/2 \times 1/4 = 3/8$.

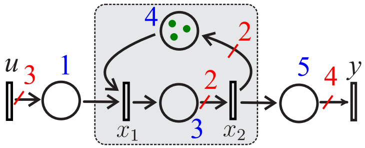
Weights Balanced TEG (WB-TEG)

Each parallel path is with the same gain, this implies that each circuit is with a gain equal to 1.

4. Weights Balanced Timed Event Graph (WB-TEG)

Semi-ring $\mathcal{E}^*[\![\delta]\!]$

Transfer behavior of WB-TEGs are described by rational expression over $\{\gamma^\nu, \delta^\tau, \mu_m, \beta_b\}$ in a specific semi-ring.



$$y = \beta_4\delta^5(\beta_2\delta^3\delta^4\gamma^3\mu_2)^*\beta_2\delta^3\delta\mu_3 u$$
$$y = \beta_4\delta^5(\beta_2\delta^7\gamma^3\mu_2)^*\beta_2\delta^4\mu_3 u$$
$$y = \delta^9\beta_4(\delta^7\mu_2\gamma^3\beta_2)^*\beta_2\mu_3 u$$

WB-TEG transfer in $\mathcal{E}^*[\![\delta]\!]$

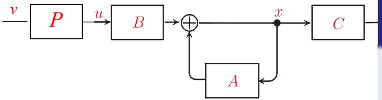
$$x = Ax \oplus Bu$$
$$y = Cx$$
$$\Rightarrow$$
$$y = CA^*Bu$$

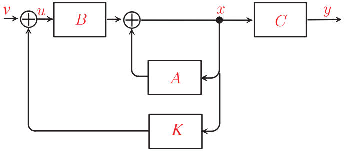
Software tools, *ETVO*, Canonical form exists.

`http://perso-laris.univ-angers.fr/~cottenceau/etvo.html`

Residuation $a \otimes x \preceq b$

Semi-ring $\mathcal{E}^*[\![\delta]\!]$ is complete, hence $a \otimes x \preceq b$ admits a greatest solution $x \preceq a \circ\!\!\!\!\backslash\, b$.





Optimal Neutral Controller for WB-TEG in $\mathcal{E}^*[\![\delta]\!]$

Open Loop Controller

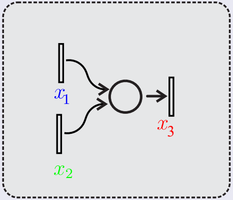
$$P_{opt} = Pr_+((CA^*B) \circ\!\!\!\!\backslash\, (CA^*B))$$

Closed Loop Controller

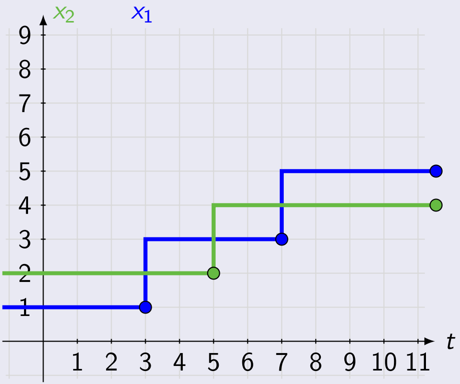
$$K_{opt} = Pr_+((A^*B) \circ\!\!\!\!\backslash\, (CA^*B) \circ\!\!\!\!/\, (CA^*B))$$

Convergence of events

$$x_3(t) = x_1(t) + x_2(t))$$

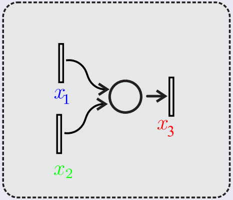


Hadamard Product

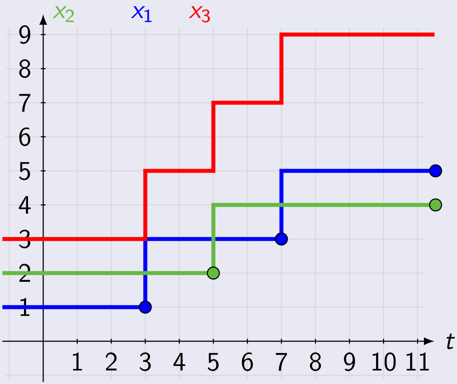


Convergence of events

$x_3(t)=x_1(t)+x_2(t))$



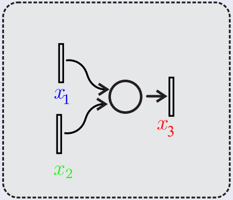
Hadamard Product



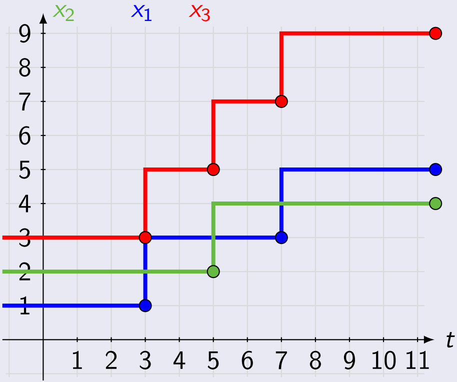
5. Hadamard Product (Hardouin et al.)

Petri Net

$x_3 = x_1 \odot x_2$



Hadamard Product



Computation in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$, $x3 = x1 \odot x2$

$x1 = \gamma^1\delta^3 \oplus \gamma^3\delta^7 \oplus \gamma^5\delta^{+\infty}$ and $x2 = \gamma^5\delta^2 \oplus \gamma^4\delta^{+\infty}$

$x_3 = x_1 \odot x_2 = \gamma^3\delta^3 \oplus \gamma^5\delta^5 \oplus \gamma^7\delta^7 \oplus \gamma^9\delta^{+\infty}$

5. Hadamard Product

Hadamard product $\odot$

Let $s_1 = \bigoplus_{t\in\mathbb{Z}} \gamma^{s_1(t)}\delta^t$ and $s_2 = \bigoplus_{t\in\mathbb{Z}} \gamma^{s_2(t)}\delta^t$ be two series,

$$s_1 \odot s_2 = \bigoplus_{t\in\mathbb{Z}} \gamma^{s_1(t)+s_2(t)}\delta^t$$

Properties of Law $\odot$

- Associative, commutative, neutral element $e_\odot = \gamma^0\delta^{+\infty}$
- Zero element $\varepsilon = \gamma^{-\infty}\delta^{+\infty}$ is absorbing $a \odot \varepsilon = \varepsilon$
- Distributes with respect of sum $\oplus$, i.e., $a \odot (b \oplus c) = a \odot b \oplus a \odot c$
- Distributes with respect of $\wedge$, i.e., $a \odot (b \wedge c) = (a \odot b) \wedge (a \odot c)$

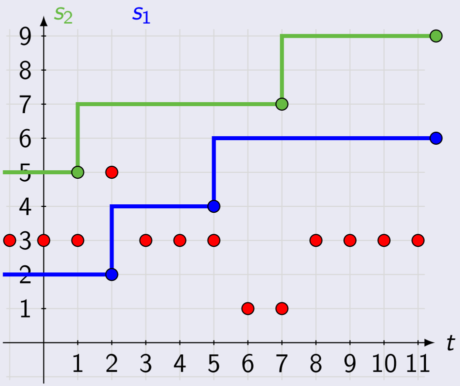
Residuation of Hadamard product $\odot^\sharp$

$s_1 \odot x \preceq s_2$ admits a greatest solution $x^\sharp = s_2 \odot^\sharp s_1$

Substraction of Counters

$s_2(t) - s_1(t)$ is not a counter (red bullet).

Residuation of Hadamard product



5. Hadamard Product and Residuation

Residuation of Hadamard product $\odot^\sharp$

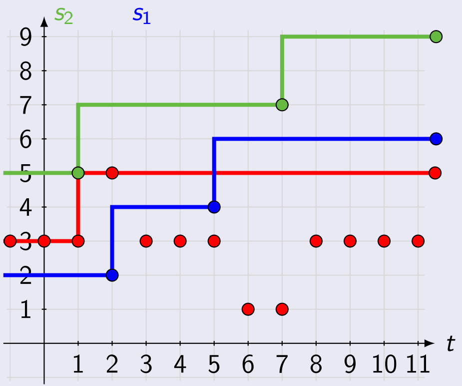
$s_1 \odot x \preceq s_2$ admits a greatest solution $x^\sharp = s_2 \odot^\sharp s_1$

Non decreasing trajectory

the upper hull is a counter function.

$x^\sharp(t) = \max(s_2(t) - s_1(t), x^\sharp(t-1))$

Residuation of Hadamard Product

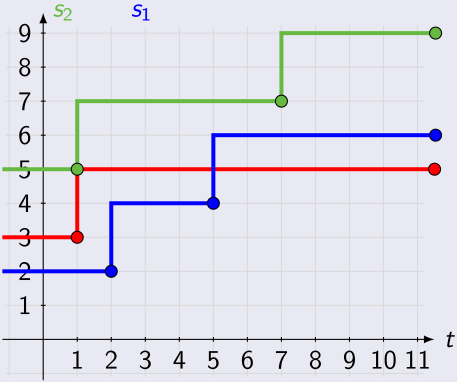


5. Hadamard Product and Residuation

Residuation of Hadamard product $\odot^\sharp$

$s_1 \odot x \preceq s_2$ admits a greatest solution $x^\sharp = s_2 \odot^\sharp s_1$

Residuation of Hadamard product



Computation in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, $x^\sharp = s2 \odot^\sharp s1$

$s1 = \gamma^1\delta^2 \oplus \gamma^4\delta^5 \oplus \gamma^6\delta^{+\infty}$ and $s2 = \gamma^5\delta^1 \oplus \gamma^7\delta^7 \oplus \gamma^9\delta^{+\infty}$

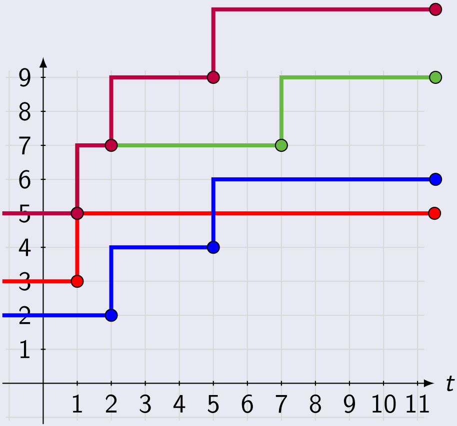
$x^\sharp = s_2 \odot^\sharp s_1 = \gamma^3\delta^1 \oplus \gamma^5\delta^{+\infty}$

5. Hadamard Product and Residuation

Residuation of Hadamard product

$s_1 \odot x \preceq s_2$ admits a greatest solution $x^\sharp = s_2 \odot^\sharp s_1$ then $s_1 \odot x^\sharp = s_2 \preceq s_2$ is the (purple) trajectory as close as possible from above to the trajectory s_2

Residuation of Hadamard product



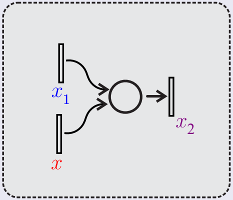
Computation in $\mathcal{M}_{in}^{ax}[\![\gamma, \delta]\!]$, $s1 \odot x^\sharp$

$$(\gamma^1\delta^2 \oplus \gamma^4\delta^5 \oplus \gamma^6\delta^{+\infty}) \odot (\gamma^3\delta^1 \oplus \gamma^5\delta^{+\infty}) = \gamma^5\delta^1 \oplus \gamma^7\delta^2 \oplus \gamma^5\delta^9 \oplus \gamma^1 1\delta^{+\infty}$$

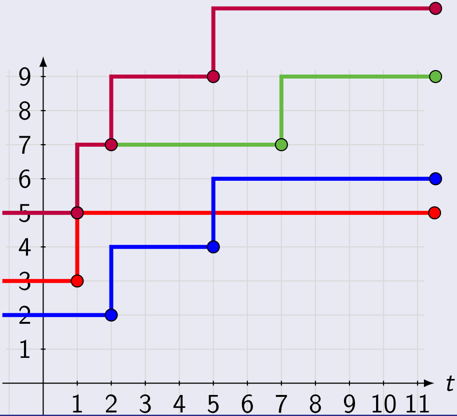
5. Hadamard Product and Residuation

Petri Net

$$x_2 = x_1 \odot x^\sharp \preceq x_2$$
$$x^\sharp = x_2 \odot^\sharp x_1$$



Residuation of Hadamard product



Interpretation of $x^\sharp$

x_2 and x_1 being given, $x^\sharp$ is the minimum number of token to add s.t. x_2 serves more token than the desired quantity depicted by x_2.

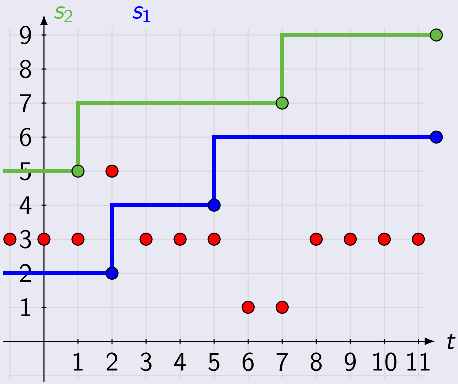
Dual Residuation of Hadamard product $\odot^\flat$

$s_1 \odot x \succeq s_2$ admits a smallest solution $x^\flat = s_2 \odot^\flat s_1$

Substraction of Counters

$s_2(t) - s_1(t)$ is not a counter (red bullet).

Dual Residuation of Hadamard product



5. Hadamard Product and Resource Sharing

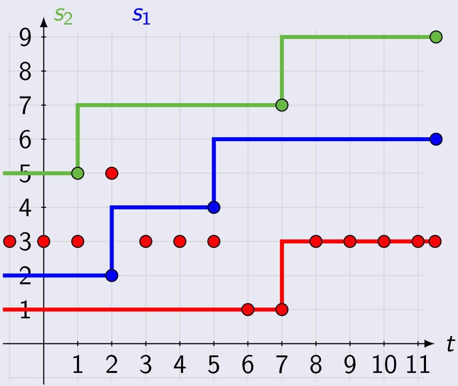
Dual Residuation of Hadamard product $\odot^\flat$

$s_1 \odot x \succeq s_2$ admits a smallest solution $x^\flat = s_2 \odot^\flat s_1$

Non decreasing trajectory

the lower hull is a counter
$x^\flat(t) =$
$\min(s_2(t) - s_1(t), x^\flat(t+1))$

Dual Residuation of Hadamard Product

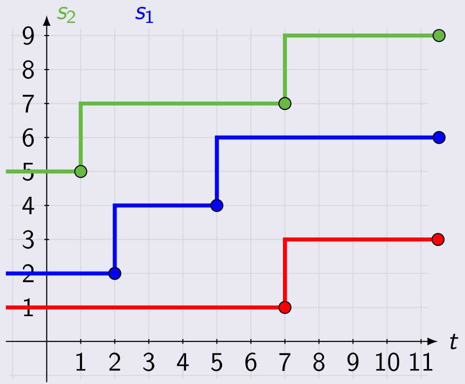


5. Hadamard Product and Residuation

Dual Residuation of Hadamard product $\odot^\flat$

$s_1 \odot x \succeq s_2$ admits a smallest solution $x^\flat = s_2 \odot^\flat s_1$

Residuation of Hadamard product



Computation in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$, $x^\flat = s2 \odot^\flat s1$

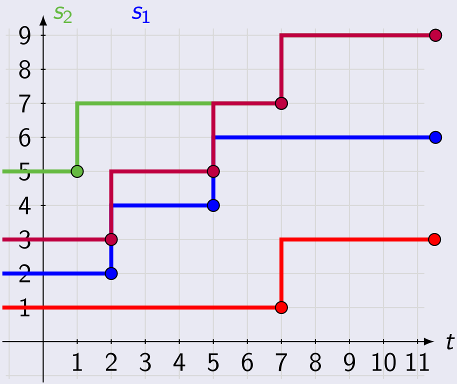
$s2 = \gamma^5\delta^1 \oplus \gamma^7\delta^7 \oplus \gamma^9\delta^{+\infty}$ and $s1 = \gamma^1\delta^2 \oplus \gamma^4\delta^5 \oplus \gamma^6\delta^{+\infty}$

$x^\flat = s_2 \odot^\flat s_1 = \gamma^7\delta^1 \oplus \gamma^3\delta^{+\infty}$

Dual Residuation of Hadamard product

$s_1 \odot x \succeq s_2$ admits a smallest solution $x^\flat = s_2 \odot^\flat s_1$ then $s_1 \odot x^\flat = s_2 \succeq s_2$ is the (purple) trajectory as close as possible from below to the trajectory s_2

Dual Residuation of Hadamard product



Computation in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$, $s1 \odot x^\flat$

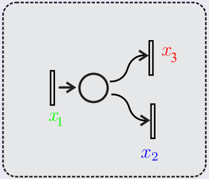
$$(\gamma^1\delta^2 \oplus \gamma^4\delta^5 \oplus \gamma^6\delta^{+\infty}) \odot (\gamma^7\delta^1 \oplus \gamma^3\delta^{+\infty}) = \gamma^3\delta^2 \oplus \gamma^5\delta^5 \oplus \gamma^7\delta^7 \oplus \gamma^9\delta^{+\infty}$$

5. Hadamard Product and Residuation

Petri Net

$$x_1(t) = x_3(t) + x_2(t)$$

$$x_1 = x_3 \odot x_2$$

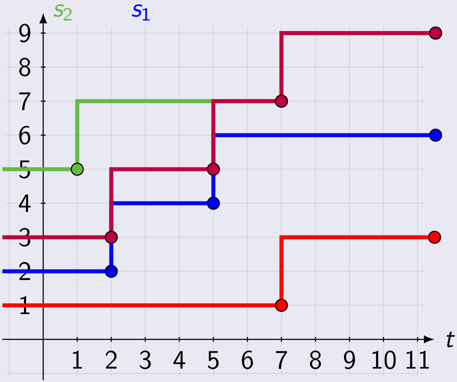


$$x_1 \preceq x_3 \odot x_2$$

$$x_3^\flat = x_1 \odot^\flat x_3$$

$$x_2 \odot x_3^\flat = x_1 \succeq x_1$$

Dual Residuation of Hadamard Product



Interpretation of $x^\flat$

x_1 and x_2 being given, $x_3^\flat$ is the maximum number of token you can consume by ensuring that x_2 is still satisfied.

MinmaxGD

Thanks to D. Zorzenon (Wodes '22), this product and residuation is included in software MinmaxGD.

Resource Sharing Problem (Moradi et al.)

$$x_1 \odot x_3 \succeq (\gamma^3\delta^2) \otimes x_2 \odot x_4$$

$$x \preceq Ax \oplus Bu \preceq A^*Bu \text{ and } y = Cx$$

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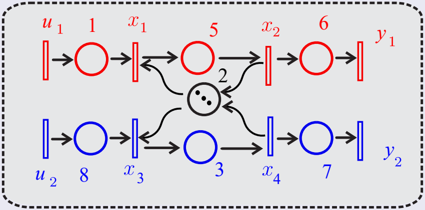

5. Resource Sharing (Moradi et al.)

Resource Sharing

$$(\gamma^3\delta^2)\otimes x_2 \odot x_4 \preceq x_1 \odot x_3$$

$$x \preceq Ax \oplus Bu \preceq A^*Bu \text{ and } y = Cx$$

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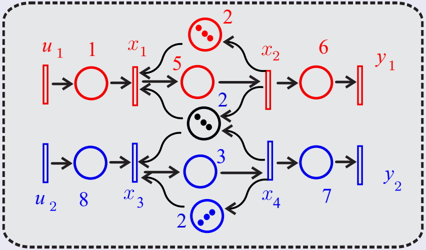

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \preceq \begin{pmatrix} \varepsilon & \gamma^3\delta^2 \\ \delta^5 & \varepsilon \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus \begin{pmatrix} \delta^1 \\ \varepsilon \end{pmatrix} u_1 \text{ and } y_1 = \begin{pmatrix} \varepsilon & \delta^6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Resource Sharing

$$(\gamma^3\delta^2)\otimes x_2 \odot x_4 \preceq x_1 \odot x_3$$

$$x \preceq Ax \oplus Bu \preceq A^* Bu \text{ and } y = Cx$$

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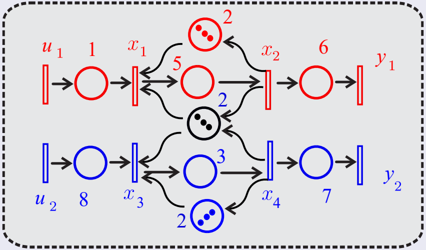


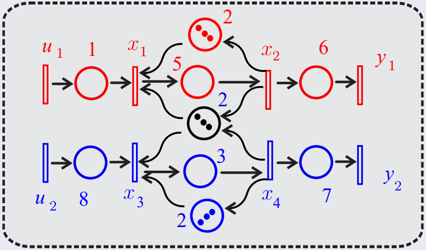
$$\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \preceq \begin{pmatrix} \varepsilon & \gamma^3\delta^2 \\ \delta^3 & \varepsilon \end{pmatrix} \begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \oplus \begin{pmatrix} \delta^8 \\ \varepsilon \end{pmatrix} u_2 \text{ and } y_2 = \begin{pmatrix} \varepsilon & \delta^7 \end{pmatrix} \begin{pmatrix} x_3 \\ x_4 \end{pmatrix}$$

Resource Sharing

$$(\gamma^3\delta^2)\otimes x_2 \odot x_4 \preceq x_1 \odot x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \preceq \begin{pmatrix} \delta^1(\gamma^3\delta^7)^* \\ \delta^6(\gamma^3\delta^7)^* \end{pmatrix} u_1 \text{ and } y_1 = \delta^{12}(\gamma^3\delta^7)^* u_1$$

$$\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} \preceq \begin{pmatrix} \delta^8(\gamma^3\delta^5)^* \\ \delta^{11}(\gamma^3\delta^5)^* \end{pmatrix} u_2 \text{ and } y_2 = \delta^{18}(\gamma^3\delta^5)^* u_2$$



5. Resource Sharing (Moradi et al.)

$$(\gamma^3\delta^2)\otimes x_2 \odot x_4 \preceq x_1 \odot x_3$$

$$x \preceq Ax \oplus Bu \preceq A^*Bu \text{ and } y = Cx$$

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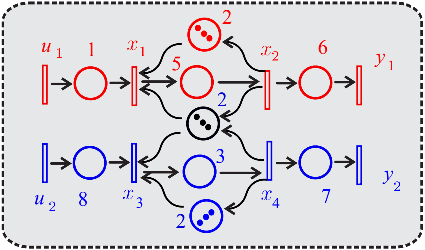
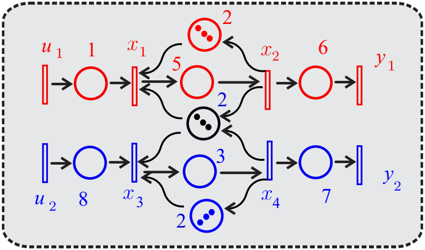


Optimal Control for Red Line (Highest Priority)

A desired output z_1 is supposed known, i.e., the objective is $y_1 \preceq z_1$:

$$\begin{aligned} u_1^{opt} &= (CA^*B)\diamond\!\!\backslash z_1 \\ CA^*Bu_1^{opt} &\preceq z_1 \\ \begin{pmatrix} x_1^{opt} & x_2^{opt} \end{pmatrix}^T = A^*Bu_1^{opt} &= \begin{pmatrix} (A^*Bu_1^{opt})_1 & (A^*Bu_1^{opt})_2 \end{pmatrix}^T \end{aligned}$$

5. Resource Sharing (Moradi et al.)



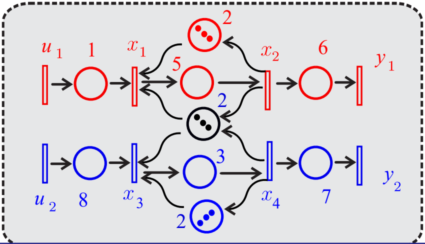
Constraint for Blue Line, (Lowest Priority)

$$(\gamma^3\delta^2) \otimes x_2^{opt} \odot x_4 \preceq x_1^{opt} \odot x_3$$

$$(\gamma^3\delta^2) \otimes x_2^{opt} \odot (A^*B)_2 \otimes u_2 \preceq x_1^{opt} \odot (A^*B)_1 \otimes u_2$$

Constraint on u_2

$$u_2 \preceq (A^*B)_2 \circ\!\!\!\backslash ((x_1^{opt} \odot (A^*B)_1 \otimes u_2) \odot^{\sharp} ((\gamma^3\delta^2) \otimes x_2^{opt}))$$



Optimal Control for Blue Line, (Lowest Priority)

A desired output z_2 is supposed known, i.e., $y_2 \preceq z_2$:

$$u_2 \preceq (CA^*B) \circ\!\!\!\backslash z_2$$

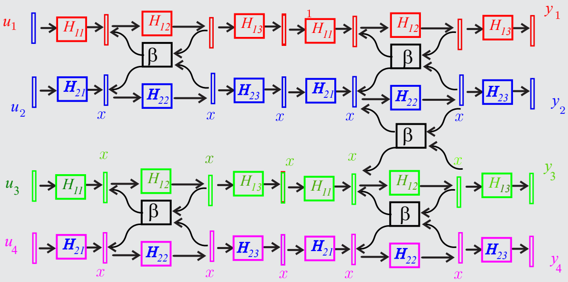
$$u_2 \preceq ((CA^*B) \circ\!\!\!\backslash z_2) \wedge ((A^*B)_2 \circ\!\!\!\backslash ((x_1^{opt} \odot (A^*B)_1 \otimes u_2) \odot^\sharp ((\gamma^3\delta^2) \otimes x_2^{opt})) \wedge u_2$$

$$u_2 \preceq \Phi(u_2)$$

Optimal Control

A greatest fixed point of $u_2 \preceq \Phi(u_2)$ exists and it is the optimal control u_2^{opt}, i.e., the greatest control respecting the constraints.

5. Resource Sharing (Schafaschek et al.)



With updating of the reference output z_i

- Many resource and many priorities
- Updating reference input z_i (receding horizon, MPC approach)

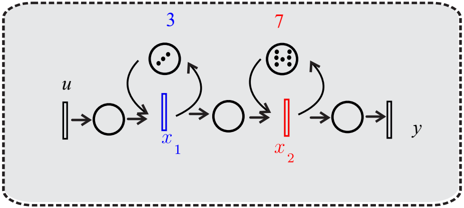
Conclusion

- Model TEG (γ and δ operators)
- Model WBTEG (γ, μ, β and δ operators)
- Open and closed loop controllers synthesis of the both (off line computation)
- Optimal control when resources are shared, on receding horizon (on line computation)

Open Problems

Complexity

Algorithms to manipulate periodic series are polynomial according to the size of the series not to the number of states.



2

The canonical representation can be large

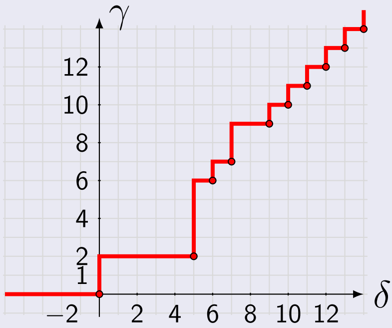
- Two internal transitions
- A transient pattern with 6 monomials $(\gamma^3\delta^3)^*(\gamma^7\delta^7)^* =$ $(\gamma^0\delta^0 \oplus \gamma^3\delta^3 \oplus \gamma^6\delta^6 \oplus \gamma^7\delta^7 \oplus \gamma^9\delta^9 \oplus \gamma^{10}\delta^{10}) \oplus (\gamma^{12}\delta^{12})(\gamma^1\delta^1)^*$

Open Problems

Alternative : Legendre-Fenchel Transform

$$\mathcal{L}(C) = \bigoplus_{t \in \mathbb{R}} (t.s - C(t))$$

Series $(\gamma^0\delta^0 \oplus \gamma^2\delta^5) \oplus (\gamma^6\delta^6)(\gamma^1\delta^1)^*$ and its approximation

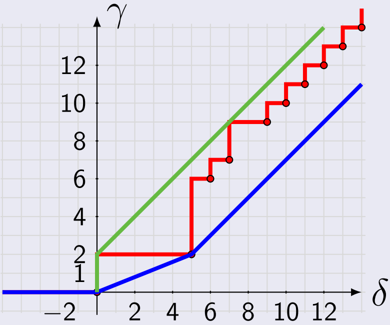


Open Problems

Alternative : Legendre-Fenchel Transform

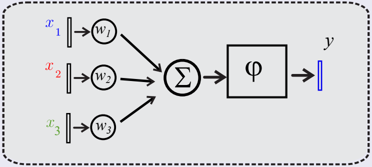
$$\mathcal{L}(C) = \bigoplus_{t \in \mathbb{R}} (t.s - C(t))$$

Series $(\gamma^0\delta^0 \oplus \gamma^2\delta^5) \oplus (\gamma^6\delta^6)(\gamma^1\delta^1)^*$ and its approximation (E. Le Corronc)



Open Problems

An artificial neuron (P. Maragos et al.)



$$y = \varphi(\Sigma_i(w_i x_i))$$

where φ is the activation function. Rectifier (ReLu) is an activation function such as : $\max(0, x)$, hence $y = max(0, \sum_i(w_i x_i))$

Equation of an artificial neuron in idempotent semi-ring

$$y = e \oplus \bigodot_i \mu_{w_i}(x_i)$$

The reachable set is a max-plus polyhedron.

Open Problems

Log semi-ring (Maslov et al.)

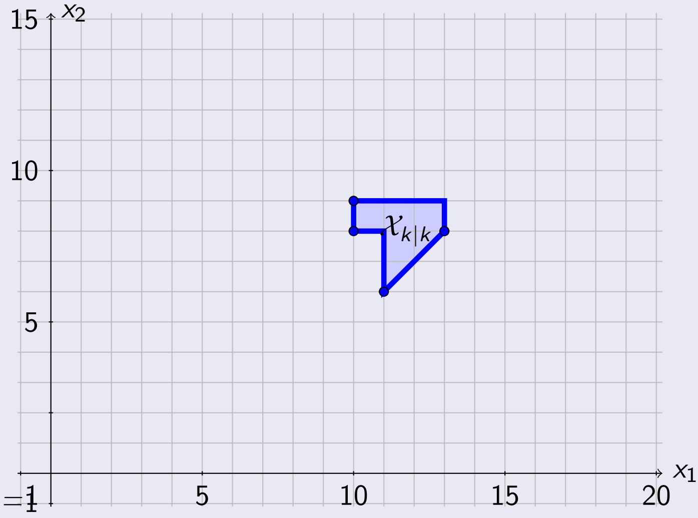
$$a \oplus b = log(e^a + e^b)$$

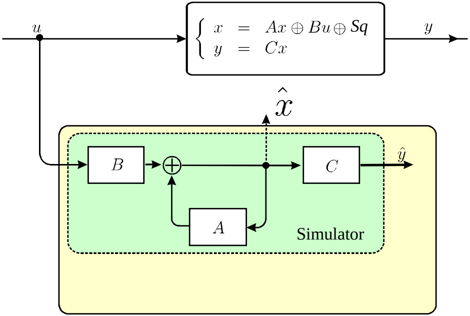
$$a \otimes b = a + b$$

Useful to address some problems of filtering in stochastic context (Analogous to Kalman filter, see G. Winck et al.).

Thank you for your attention.

A max-plus polyhedron





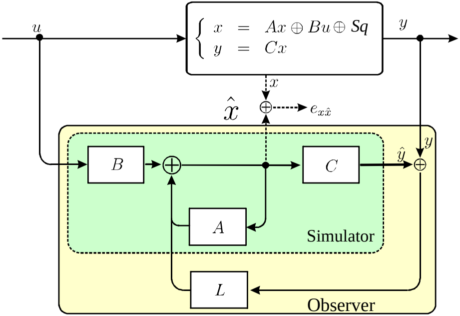
Prediction computation :

$$\hat{x}(\gamma) = Ax(\gamma) \oplus Bu(\gamma).$$

or

$$\hat{x}(k) = Ax(k-1) \oplus Bu(k).$$

State Estimation : Observer Synthesis

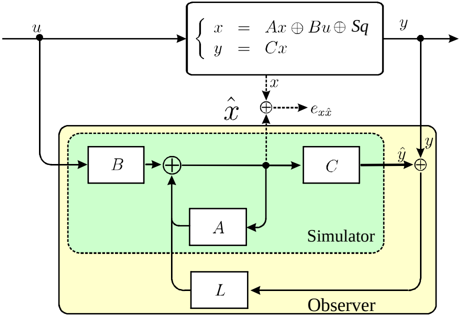


Objective :

Compute the greatest observer matrix L such that

$$\hat{x} \preceq x.$$

State Estimation : Observer Synthesis



System Equations :

Matrix S

$$
\begin{aligned}
x &= Ax \oplus Bu \oplus Sq = A^*Bu \oplus A^*Sq \\
y &= Cx = CA^*Bu \oplus CA^*Sq.
\end{aligned}
$$

Estimated State Equations :

$$
\begin{aligned}
\hat{x} &= A\hat{x} \oplus Bu \oplus L(\hat{y} \oplus y) \\
\hat{y} &= C\hat{x}.
\end{aligned}
$$

State Estimation : Observer Synthesis

Constraints Satisfaction :

Compute the greatest observer matrix L such that

$$
\begin{array}{lcll}
(A \oplus LC)^* Bu & \preceq & A^* Bu & \forall u \\
(A \oplus LC)^* LCA^* Sq & \preceq & A^* Sq & \forall q,
\end{array}
$$

Constraints Satisfaction :

Compute the greatest matrix L such that

$$
\begin{array}{lcl}
(A \oplus LC)^* B & \preceq & A^* B \Leftrightarrow L \preceq (A^* B) \phi (CA^* B) \\
(A \oplus LC)^* LCA^* S & \preceq & A^* S \Leftrightarrow L \preceq (A^* S) \phi (CA^* S).
\end{array}
$$

Optimal Matrix : (Hardouin et al. IEEE TAC 2010, Hardouin et al. 2019)

$$L_{opt} = ((A^*B) \phi\!\!\!/ (CA^*B)) \wedge ((A^*S) \phi\!\!\!/ (CA^*S))$$

is the greatest such that

$$\hat{x} \preceq x.$$

State Estimation : Observer Synthesis : Performance Analysis

Equality of the asymptotic slope (Hardouin et al. IEEE TAC 2010)

If matrix C linking state vector to the output is connected to all connected components of the graph then

$$\sigma_\infty(\hat{x}_i) = \sigma_\infty(x_i) \ \forall i$$

Corollary :

If state x_i belongs to a connected component whose at least one transition is measured then the error $\hat{x}_i - x_i$ is bounded.

State Estimation : Set-membership approach

Uncertain system
$A(k) \in [\underline{A}, \overline{A}] = [A], B(k) \in [\underline{B}, \overline{B}] = [B], C(k) \in [\underline{C}, \overline{C}] = [C]$

Each matrices entries is supposed bounded and $A(k), B(k), C(k)$ is a realization at step k

$$\begin{aligned} x(k) &= A(k)x(k-1) \oplus B(k)u(k) \\ y(k) &= C(k)x(k) \end{aligned}$$

In the sequel, to enlighten the notation, we assume (without lost of generality) autonomous systems, i.e., $x(k) = A(k)x(k-1)$.

Indeed we can consider $\tilde{x} = (x^t u^t)^t$ and $\tilde{A} = \begin{pmatrix} A & \varepsilon \\ \varepsilon & B \end{pmatrix}$

Uncertain system $A(k) \in [\underline{A}, \overline{A}] = [A]$, $C(k) \in [\underline{C}, \overline{C}] = [C]$

$$\begin{array}{rcl} x(k) & = & A(k)x(k-1) \\ y(k) & = & C(k)x(k) \end{array}$$

Q1 : Assuming $x(k-1) \in \mathcal{X}_{k-1|k-1}$ a known set, is it possible to compute the set $\mathcal{X}_{k|k-1} = \left\{Ax \mid A \in [\underline{A}, \overline{A}], x \in \mathcal{X}_{k-1|k-1}\right\}$? (prediction)
Q2 : Assuming $y(k)$ available, is it possible to compute the inverse image set $[C]^{-1}(y(k)) = \left\{x \mid y(k) = Cx, C \in [\underline{C}, \overline{C}]\right\}$? (likelihood)
Q3 : Is it possible to compute the intersection of the two previous sets to obtain the set $\mathcal{X}_{k|k} = \mathcal{X}_{k|k-1} \cap [C]^{-1}(y(k))$? (estimation)

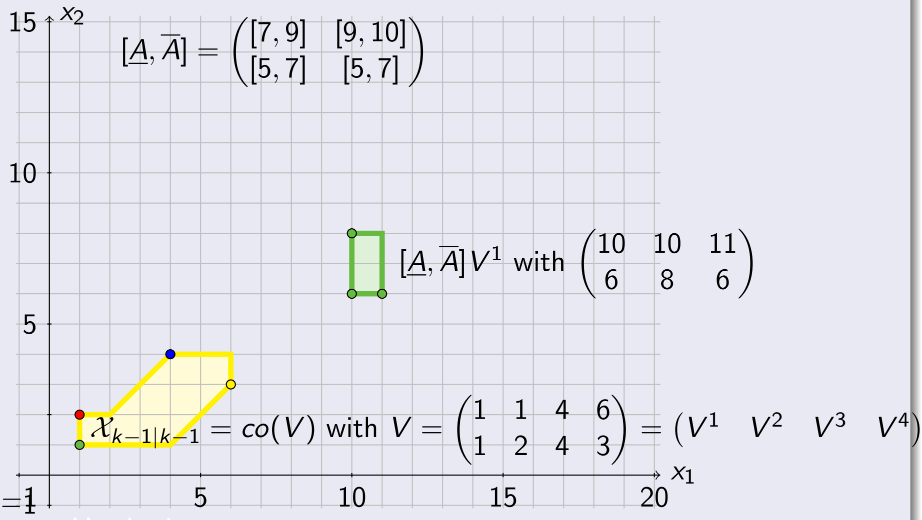
Uncertain system $A \in [\underline{A}, \overline{A}]$, $C \in [\underline{C}, \overline{C}]$

$$\begin{array}{rcl} x(k) & = & A(k)x(k-1) \\ y(k) & = & C(k)x(k) \end{array}$$

Q1 : Assuming $x(k-1) \in \mathcal{X}_{k-1|k-1}$ a known set, is it possible to compute the set $\mathcal{X}_{k|k-1} = \left\{Ax \mid A \in [\underline{A}, \overline{A}], x \in \mathcal{X}_{k-1|k-1}\right\}$?
Assumption : $\mathcal{X}_{k-1|k-1}$ is depicted as a tropical polytope.
(Lemma 2.1 PhD Guilherme Winck (University of Angers)).

State Estimation : Set-membership approach

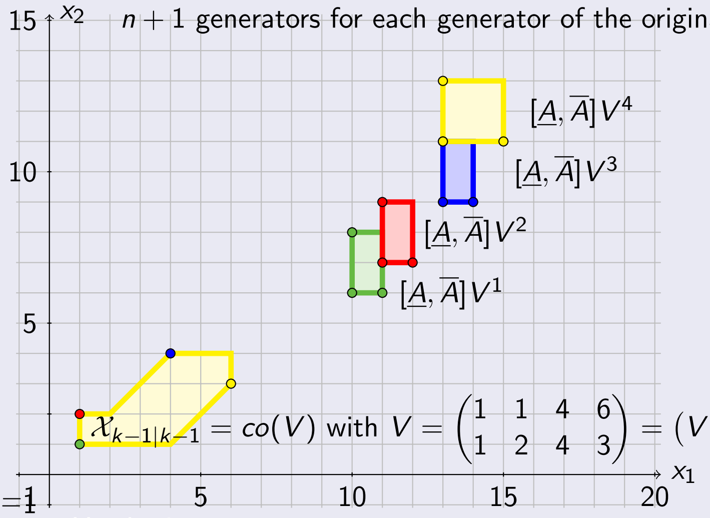
Q1 : Assuming $x(k-1) \in \mathcal{X}_{k-1|k-1}$ a known set, is it possible to compute the set $\mathcal{X}_{k|k-1} = \left\{ Ax \mid A \in [\underline{A}, \overline{A}], x \in \mathcal{X}_{k-1|k-1} \right\}$?



State Estimation : Set-membership approach

Q1 : Assuming $x(k-1) \in \mathcal{X}_{k-1|k-1}$ a known set, is it possible to compute the set $\mathcal{X}_{k|k-1} = \{Ax \mid A \in [\underline{A}, \overline{A}], x \in \mathcal{X}_{k-1|k-1}\}$?

$n+1$ generators for each generator of the original set

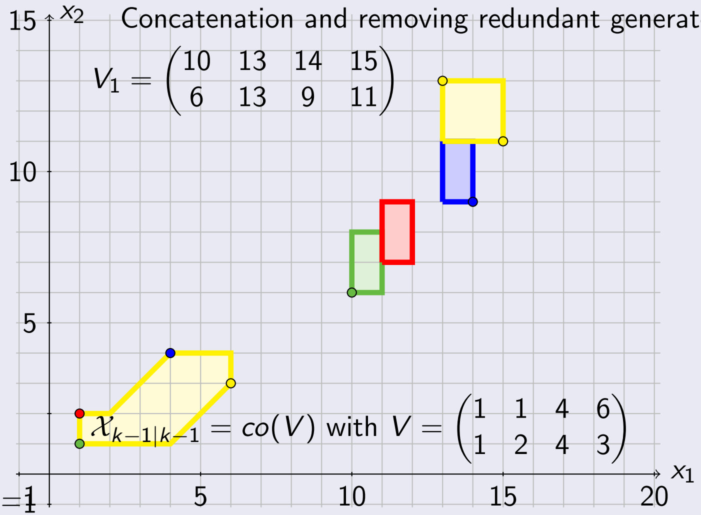


$$\mathcal{X}_{k-1|k-1} = co(V) \text{ with } V = \begin{pmatrix} 1 & 1 & 4 & 6 \\ 1 & 2 & 4 & 3 \end{pmatrix} = \begin{pmatrix} V^1 & V^2 & V^3 & V^4 \end{pmatrix}$$

Q1 : Assuming $x(k-1) \in \mathcal{X}_{k-1|k-1}$ a known set, is it possible to compute the set $\mathcal{X}_{k|k-1} = \{Ax \mid A \in [\underline{A}, \overline{A}], x \in \mathcal{X}_{k-1|k-1}\}$?

Concatenation and removing redundant generators

$$V_1 = \begin{pmatrix} 10 & 13 & 14 & 15 \\ 6 & 13 & 9 & 11 \end{pmatrix}$$

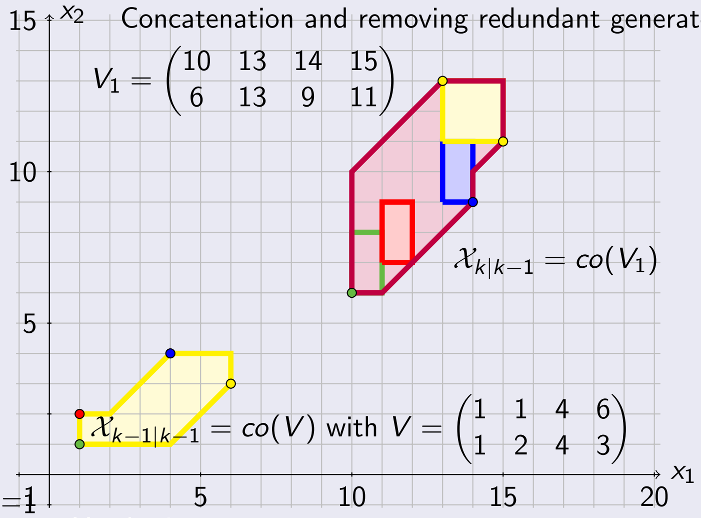


$$\mathcal{X}_{k-1|k-1} = co(V) \text{ with } V = \begin{pmatrix} 1 & 1 & 4 & 6 \\ 1 & 2 & 4 & 3 \end{pmatrix}$$

Q1 : Assuming $x(k-1) \in \mathcal{X}_{k-1|k-1}$ a known set, is it possible to compute the set $\mathcal{X}_{k|k-1} = \left\{Ax \mid A \in [\underline{A}, \overline{A}], x \in \mathcal{X}_{k-1|k-1}\right\}$?

Concatenation and removing redundant generators

$$V_1 = \begin{pmatrix} 10 & 13 & 14 & 15 \\ 6 & 13 & 9 & 11 \end{pmatrix}$$



$\mathcal{X}_{k|k-1} = co(V_1)$

$\mathcal{X}_{k-1|k-1} = co(V)$ with $V = \begin{pmatrix} 1 & 1 & 4 & 6 \\ 1 & 2 & 4 & 3 \end{pmatrix}$

Uncertain system $A \in [\underline{A}, \overline{A}]$, $C \in [\underline{C}, \overline{C}]$

$$x(k) = A(k)x(k-1)$$
$$y(k) = C(k)x(k)$$

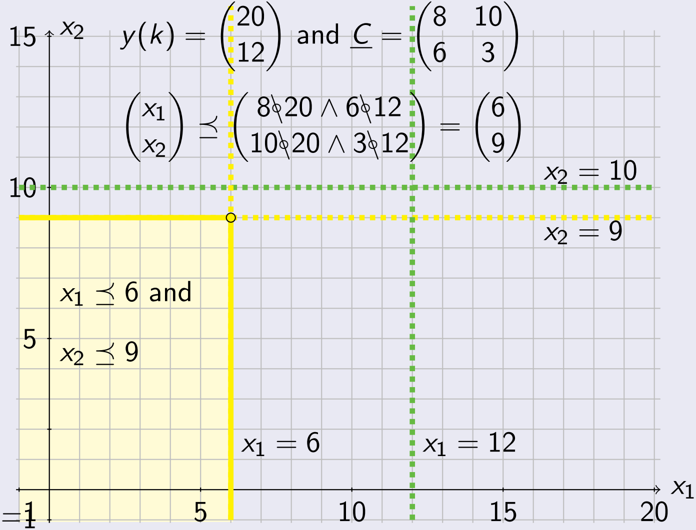
Q2 : Assuming $y(k)$ available, is it possible to compute the inverse image set $[C]^{-1}(y(k)) = \{x \mid y(k) = Cx, C \in [\underline{C}, \overline{C}]\}$?
The set can be written as $[C]^{-1}(y(k)) = \{x | \underline{C}x \preceq y(k) \preceq \overline{C}x\}$, which can be decomposed in two sets :

$$\mathcal{X} = \overline{\mathcal{X}} \cap \underline{\mathcal{X}}$$

where $\overline{\mathcal{X}} = \{x | \underline{C}x \preceq y(k)\}$ and $\underline{\mathcal{X}} = \{x | y(k) \preceq \overline{C}x\}$
Renato Cândido et al., "An Algorithm to Compute the Inverse Image of a Point with Respect to a Nondeterministic Max Plus Linear System", in IEEE TAC, 2021.

State Estimation : Set-membership approach

Computation $\overline{\mathcal{X}} = \{x | \underline{C}x \preceq y(k)\} \Leftrightarrow \overline{\mathcal{X}} = \{x | x \preceq \underline{C} \circ\!\!\!\backslash\, y(k)\}$

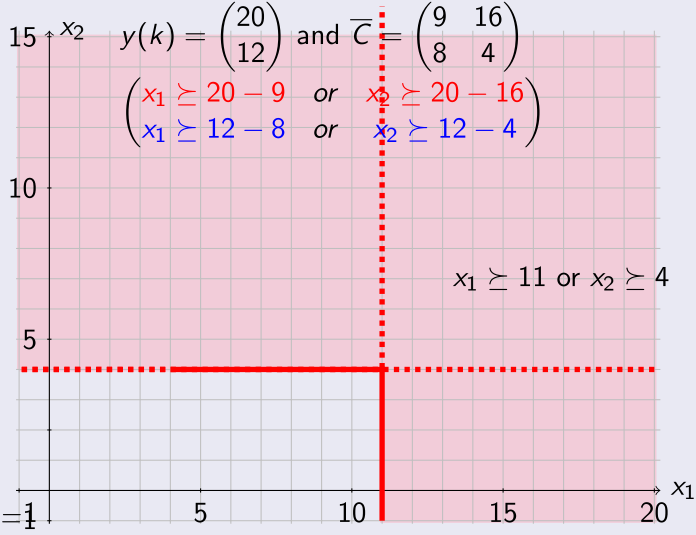


State Estimation : Set-membership approach

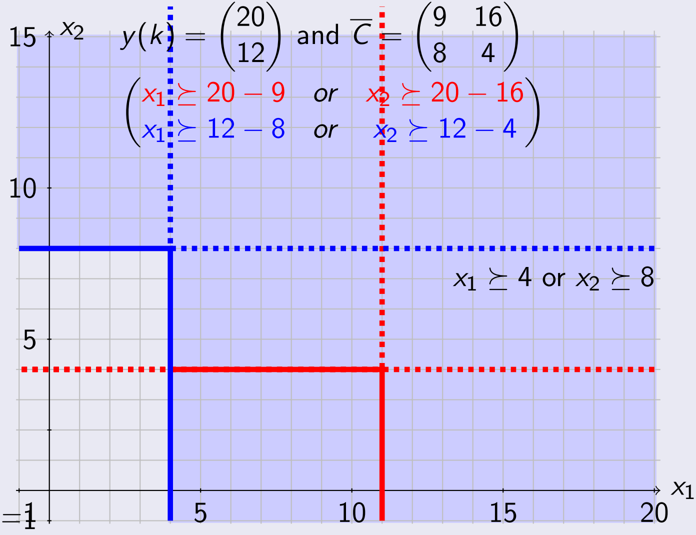
Computation $\underline{\mathcal{X}} = \{x | y(k) \preceq \overline{C} x\}$

$$y(k) = \begin{pmatrix} 20 \\ 12 \end{pmatrix} \text{ and } \overline{C} = \begin{pmatrix} 9 & 16 \\ 8 & 4 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \succeq 20 - 9 & or & x_2 \succeq 20 - 16 \\ x_1 \succeq 12 - 8 & or & x_2 \succeq 12 - 4 \end{pmatrix}$$



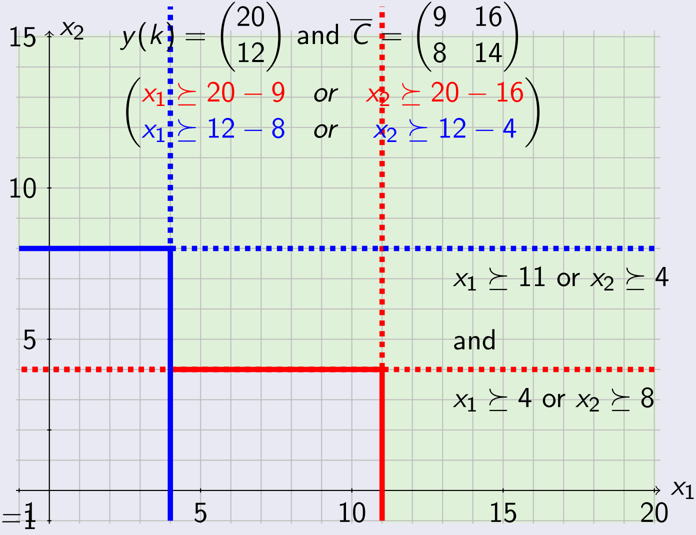
Computation $\underline{\mathcal{X}} = \{x | y(k) \preceq \overline{C}x\}$



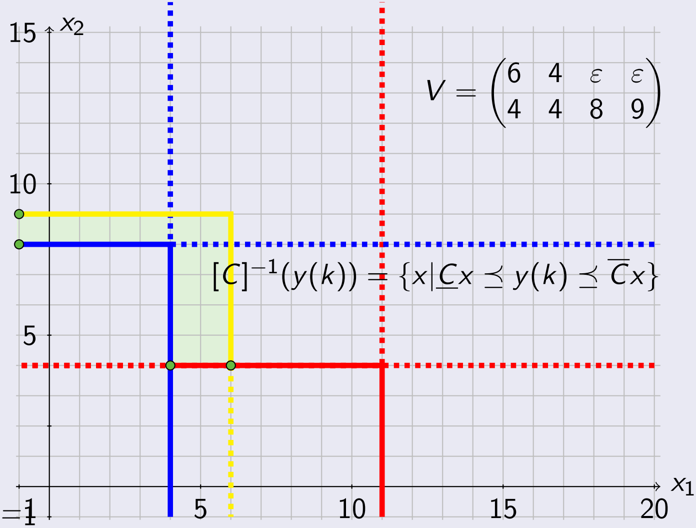
Computation $\underline{\mathcal{X}} = \{x | y(k) \preceq \overline{C}x\}$

$y(k) = \begin{pmatrix} 20 \\ 12 \end{pmatrix}$ and $\overline{C} = \begin{pmatrix} 9 & 16 \\ 8 & 14 \end{pmatrix}$

$\begin{pmatrix} x_1 \succeq 20 - 9 & or & x_2 \succeq 20 - 16 \\ x_1 \succeq 12 - 8 & or & x_2 \succeq 12 - 4 \end{pmatrix}$

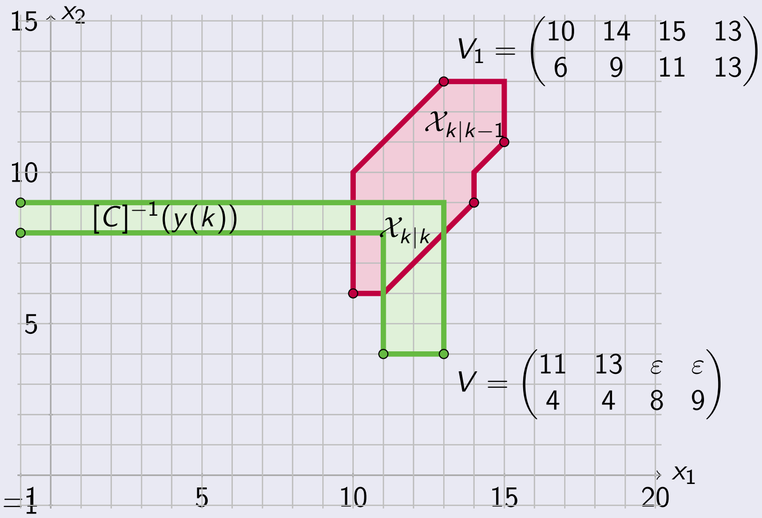


Q2 : Computation $[C]^{-1}(y(k)) = \overline{\mathcal{X}} \cap \underline{\mathcal{X}}$

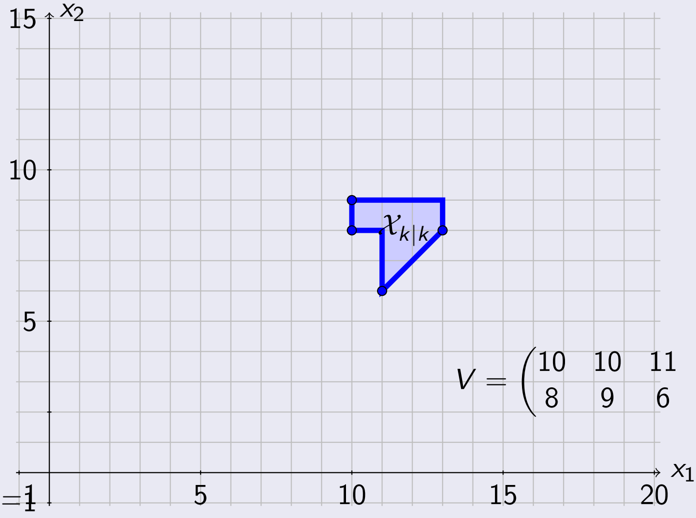


State Estimation : Set-membership approach

Q3 : Is it possible to obtain the intersection
$\mathcal{X}_{k|k} = [C]^{-1}(y(k)) \cap \mathcal{X}_{k|k-1}$



Q3 : Is it possible to obtain the intersection
$\mathcal{X}_{k|k} = [C]^{-1}(y(k)) \cap \mathcal{X}_{k|k-1}$



$$V = \begin{pmatrix} 10 & 10 & 11 & 13 \\ 8 & 9 & 6 & 8 \end{pmatrix}$$

Filtering algorithm :

Require : $\mathcal{X}_{k-1|k-1}, y(k)$

Ensure : $\mathcal{X}_{k|k}$

$$\mathcal{X}_{k|k-1} = [\underline{A}, \overline{A}]\mathcal{X}_{k-1|k-1} \qquad (\textit{prediction})$$

$$\underline{\mathcal{X}} = \{x | x \preceq \underline{C} \diamond\!\!\!\setminus y(k)\}$$

$$\overline{\mathcal{X}} = \{x | y(k) \preceq \overline{C}x\}$$

$$[C]^{-1}(y(k)) = \underline{\mathcal{X}} \cap \overline{\mathcal{X}} \qquad (\textit{likelihood})$$

$$\mathcal{X}_{k|k} = \mathcal{X}_{k|k-1} \cap [C]^{-1}(y(k)) \qquad (\textit{estimation})$$

Filtering algorithm :

Require : $\mathcal{X}_{k-1|k-1}, y(k)$ — n, N, q

Ensure : $\mathcal{X}_{k|k}$

$$\mathcal{X}_{k|k-1} = [\underline{A}, \overline{A}]\mathcal{X}_{k-1|k-1} \qquad \mathcal{O}(2Nn^2)$$

$$\underline{\mathcal{X}} = \{x | x \preceq \underline{C} \circ\!\!\!\!\setminus y(k)\} \qquad \mathcal{O}(nq)$$

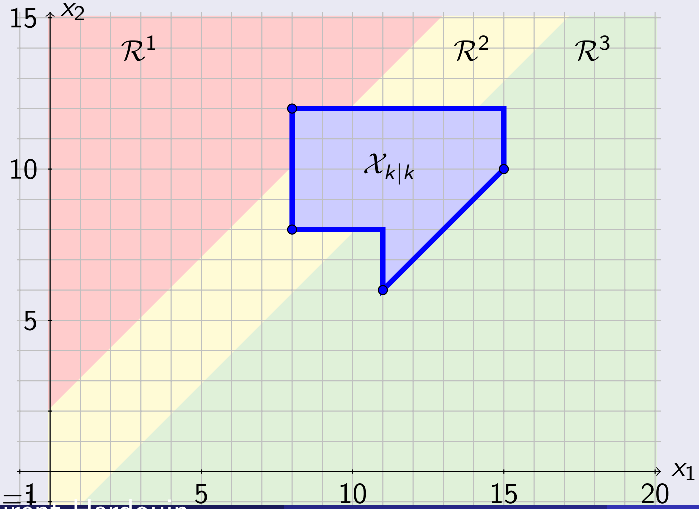
$$\overline{\mathcal{X}} = \{x | y(k) \preceq \overline{C} x\} \qquad \mathcal{O}(nq)$$

$$[C]^{-1}(y(k)) = \underline{\mathcal{X}} \cap \overline{\mathcal{X}}$$

$$\mathcal{X}_{k|k} = \mathcal{X}_{k|k-1} \cap [C]^{-1}(y(k)) \qquad \mathcal{O}(n^n)$$

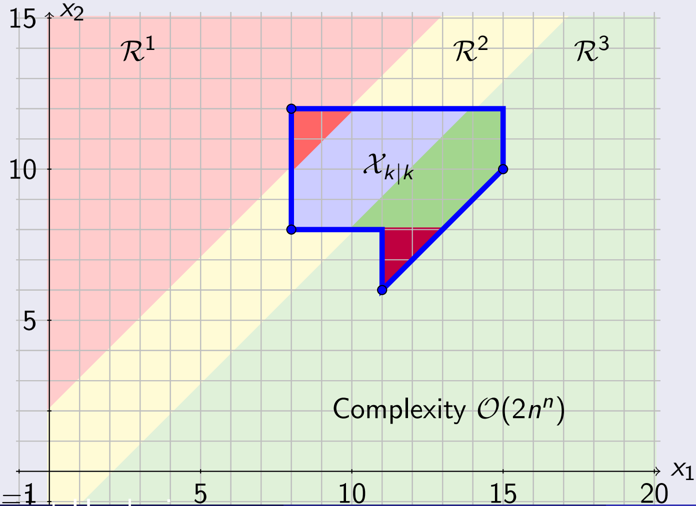
State Estimation : Set-membership approach

Alternative approaches : Decomposition in PWA (Adzkiya et al. Automatica 2015)

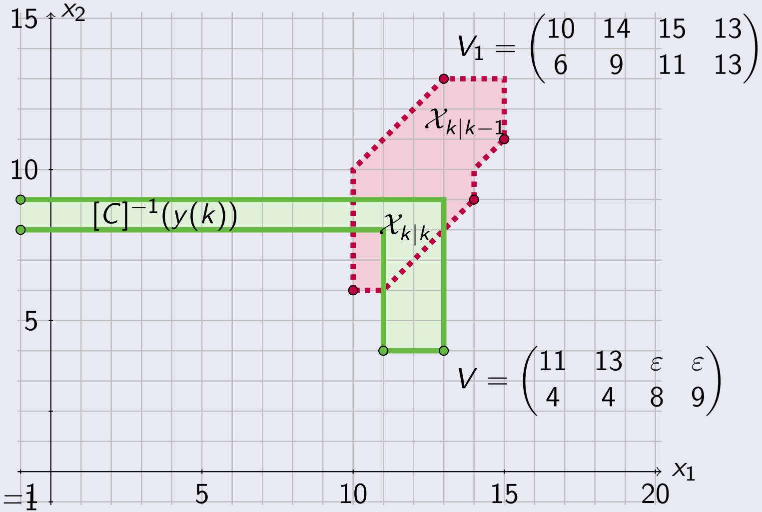


State Estimation : Set-membership approach

Alternative approaches : Decomposition in PWA (Adzkiya et al. Automatica 2015)

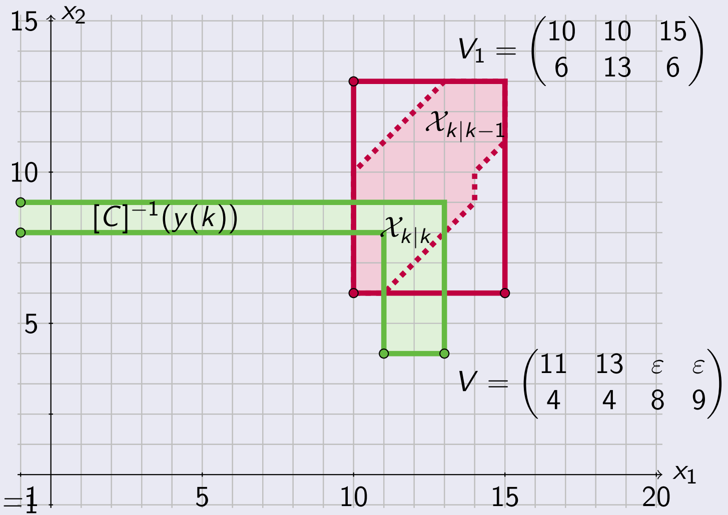


Alternative approaches : Interval analysis (Winck PhD 2022)



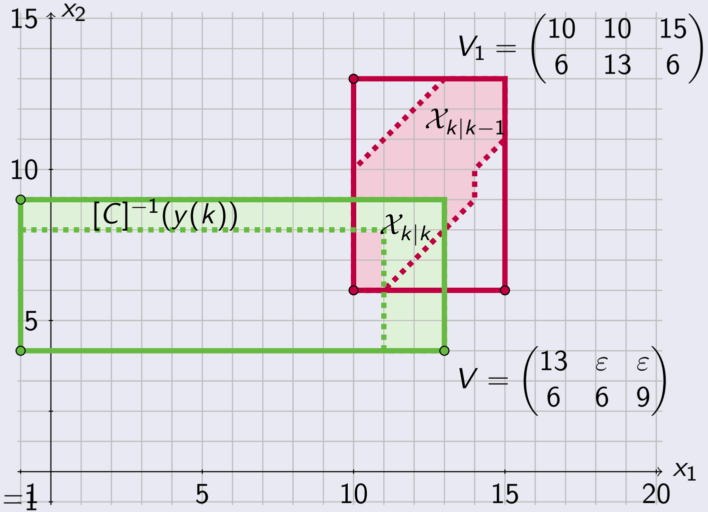
State Estimation : Set-membership approach

Alternative approaches : Interval analysis (Winck PhD 2022)



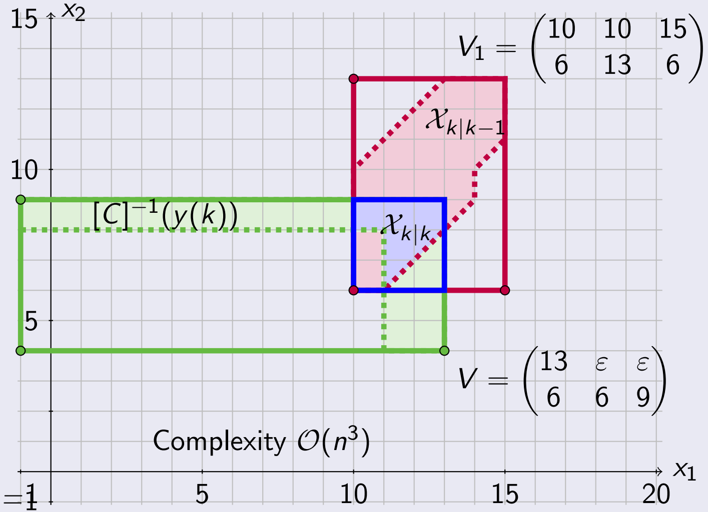
State Estimation : Set-membership approach

Alternative approaches : Interval analysis (Winck PhD 2022)



State Estimation : Set-membership approach

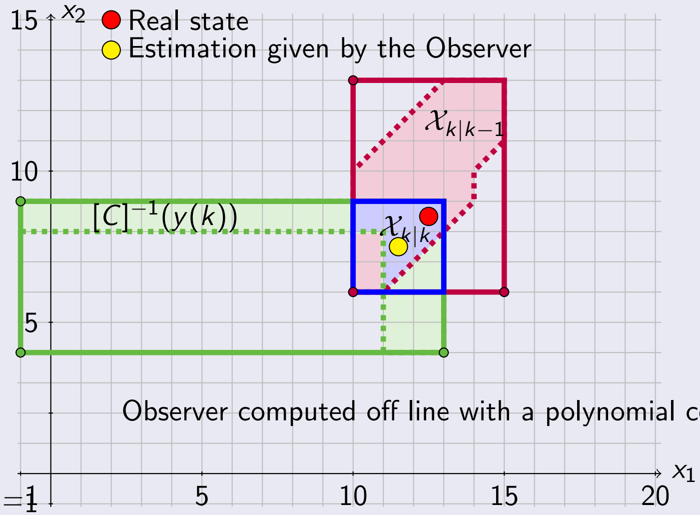
Alternative approaches : Interval analysis (Winck PhD 2022)



Performances comparison

- Using tropical polytope approach, the set of all possible solution is obtained, the complexity is $\mathcal{O}(n^n)$.
- Using DBM the same set is obtained (Adzkiya et al. Automatica 2015),with an exponential complexity also, but practically worst.
- Using Box an overapproximation is obtained with a polynomial complexity (Winck, PhD 2022).
- Using SMT (Satisfability Modulo theory) solver (e.g., z3 solver) (Mufid et al. IEEE TAC, 2022) is equivalent to keep the $\mathcal{H}$-form of the tropical polytope. This is suitable when a point included in the estimation set is desired (check a solution). But needs to keep all the constraints on the horizon of estimation, which growth at each step.

Where is the estimation given by the observer ?



Observer computed off line with a polynomial complexity

Conclusion

State Estimation

- An efficient observer exists, the greatest possible solution is obtained
- A set-membership approach based on max-plus polytope is the most efficient to obtain the set of all possible solutions, even if the complexity is still exponential.
- Interval analysis yields an over estimation of the solution set with a polynomial complexity

Conclusion

Open problems to address

- Developing an interval observer to compute on-line an upper bound
- Developing more efficient algorithms to compute intersection of max-plus polytope
- Developing method to obtain underestimation set (set included in the solution set), (Barnhill et al., arxiv.org, 2023).
- Selecting a point in the solution set (support) by considering stochastic approach (Santos-Mendes et al. IEEE TAC, 2019, Winck et al. IEEE TAC 2022).
- Considering only $\mathcal{H}$-form to avoid the costly transposition to $\mathcal{V}$-form.
- Developing state estimation method for systems involving resource sharing (Schafaschek et al. 2020).

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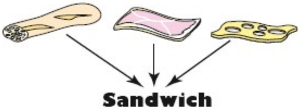

Sandwiches Algebra [Cohen et al.]

1 piece of Bread + 1 slice of ham + 1 slice of cheese is equal to 1 sandwich. Another way of counting !