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An anti-pulsatory device used as an active noise control system in a duct

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Abstract. — This paper presents an original device for reducing the noise produced by the periodic internal flow fluctuations. It is a movable valve which regulates the flow and then attenuates the noise radiated at the mouth. Assuming plane waves and making use of the transfer matrices and electroacoustic analogies, a standing wave analysis of this system is first presented. This analysis reveals the influence of this original source as a serial generator which can be used in an active noise control system, in particularly at the engine exhaust system. Considering the periodicity of the flow, a frequencial control strategy is developed. It is an extremal control which adjusts the complex command parameter in order to minimise a quadratic criterion. On-line identification procedure, according to a black box model, makes it possible to track the unsteady characteristics of the primary source. Finally some experimental results show the efficiency of this device and the validity of our model.

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1. Introduction

The noise reduction of an engine exhaust system has been a topic of great interest for many years (Alfredson and Davies, 1971, Prasad and Croker, 1983). As a result great improvements have been achieved in the design of mufflers. Nevertheless, reflective as well as dissipative mufflers generally have poor performance at low frequencies and for a good wide band response, they would be very large. These limitations in passive mufflers have given rise to the concept of active attenuation which consists in superposing an anti-phased noise over the unwanted one in order to cancel the sound field at the downstream of an auxiliary source (Roure 1985). Classically, the auxiliary sources are loudspeakers, but the strength of loudspeakers decreases with decreasing frequency. This gives a particularly poor response at low frequencies and they are also susceptible to damage from dirty and hot environments. These drawbacks make it difficult to use a loudspeaker in thermal engine exhausts which are characterised by low frequency noise radiation and the high temperature of the medium.

The L.E.A. (Laboratoire d'Etudes Aérodynamiques) has developed an original auxiliary source, which is robust and particularly effective at low frequencies (Tartarin et al., 1991; Laumonier, 1990; Peube et al., 1987). It consists in a fluctuating head-loss which regulates the pulsed flow to attenuate its low frequency radiation.

The first part of this paper presents the basic concept and technology of this device. Afterwards, assuming plane waves and making use of electroacoustic analogies, an analytical model of the device is developed. This work makes it possible to describe the device as a particular acoustic generator which can be used as auxiliary source in an active control system applied to the pulsed flow. The specific control strategy developed to reach the optimal command law is then presented. It is an extremum control which consists of a quadratic criterion minimisation obtained with an iterative adjustment of the command parameters.

In the last part, some experimental results obtained in the laboratory are presented. They make possible to validate the model and show the efficiency of the chosen adaptive control. For this, we successively present the influence of the source measured in a duct with mean flow and a number of attenuation results. In particular the insertion losses of the active device and of a classical passive muffler are compared. The adaptive behaviour of the command is finally shown to prove the industrial viability of the system.

2. System presentation

Aeroacoustic theory (Lighthill, 1952; Curle, 1955) shows that the flow periodic fluctuations, which characterise engine exhaust systems, generate unipolar radiation at the mouth. Therefore the attenuation of these flow fluctuations reduces, by causality, the noise radiated at the mouth.

Usually the regulation of a flow is provided with a device which modifies the head-loss (valve, diaphragm...). The synchronous variation of the head loss with the flow pulsation cancels the perturbation at the downstream.

To verify this principle, the L.E.A. has developed a mechanical system. The simplest device for instanta-

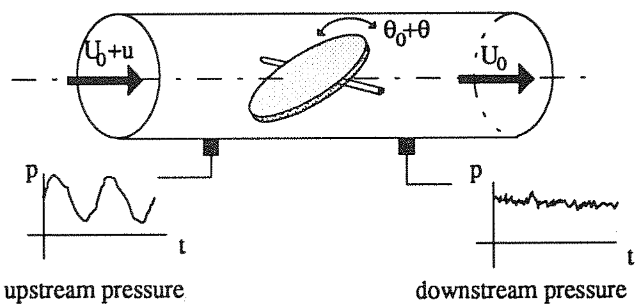

Figure 1. The principle of the anti-pulsatory device: An oscillating valve around a mean incidence.

neously modifying the head-loss is a motor driven valve (see Fig. 1).

The movement of this oscillating valve synchronised with the local flow fluctuation cancels them out. Downstream of the valve the flow becomes steady and sound radiated is reduced. The valve characteristics which can be adjusted to obtain the flow regulation are the mean incidence θ_0 and the oscillating movement θ.

The periodicity of the exhaust flow has suggested a frequencial command law for the electrical actuator, consisting in a calculation of the amplitude movement and phase for each harmonic, that we chose to reduce. This choice makes it possible to define a quadratic criterion according to the control strategy. To obtain this criterion, the pressure is picked up by a sensor downstream of the valve. This error signal is synchronously sampled by means of an optical encoder fixed on the pulsed flow generator. Furthermore this optical encoder gives a phase reference which makes it possible to adjust the command parameter.

The self acting system can be described in three steps:

- Synchronous sampling and frequency analysis of the error signal.
- Harmonic synthesis of the command.
- Running of the new command law.

Nevertheless, the search for the optimal command, which minimises the fluctuating pressure, needs identification of the transfer function between the command parameter and the action obtained at the error sensor location, the so-called secondary path. This is constituted by the frequency response of the electrical driver, the law between the valve movement and the pressure generated, the transfer function between this pressure and the pressure picked up at the error sensor location and the frequency response of this sensor. The complex parameters of this transfer function are obtained by means of a specific on-line identification procedure using the command and error signal increments.

3. Acoustic analysis of the device

The analysis of an engine exhaust system can be approached in two ways, by finite pulse analysis or standing wave analysis. The first is carried out by the method of characteristics which is a time domain approach, the second is developed in the frequency domain and is faster and less tedious than the method of characteristics.

The engine exhaust flow being periodic, the active control system with the oscillating valve is described by means of the standing wave analysis.

In this approach, a source of acoustic energy, such as an engine, a compressor, or a turbine is characterised by means of the electroacoustic analogies. This approach makes it possible to represent the acoustic source as a Thévenin generator applied to electrical systems (Doige and Alves, 1989).

Figure 2 describes two basic forms where voltage and current are replaced by pressure and mass fluctuation respectively. The acoustic source can effectively be replaced either by a pressure source driving a load impedance in series with an internal source impedance (Fig. 2b) or by a fluctuating mass source driving parallel- connected source and load impedance (Fig. 2c).

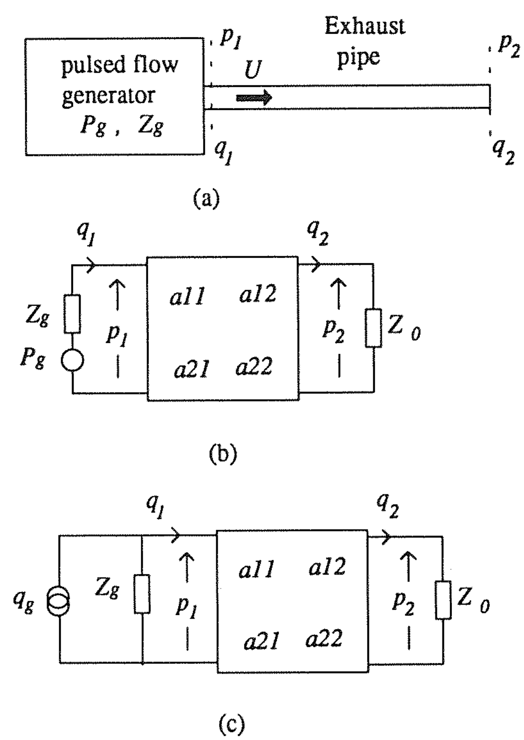

Figure 2. Electroacoustic analogy of an exhaust system. b) pressure source, c) mass source.

In fact these two models are equivalent. The passive and active constituents downstream of the source represent the load impedance of this generator. In our case, it concerns the duct, the valve and the radiation impedance of an open ended duct with mean flow.

Considering the thermodynamic process to be almost isentropic in a uniform duct and adopting fluctuating pressure p and mass velocity q as the two state variables, the harmonic solutions for the one dimensional wave equation with mean flow give the transfer matrix relation between the state variables at each point in the duct. Munjal (1987) gives a particular transfer matrix which incorporates the convective and dissipative effect of the mean flow (see Appendix).

This four pole matrix relates the state variables at locations 1 and 2 in a duct separated by a distance L.

The electroacoustic analogy and this transfer matrix approach make it possible to calculate the state variables at each location in the duct. In order to evaluate the valve characteristics and to introduce them into the foregoing model, the two state variables p and q are used.

The passive and active influence of the valve can be successively characterised using the linear acoustics principle of superposition. Considering the one-dimensional relations of energy, mass and momentum, the four pole matrix of the fixed valve in an unsteady flow is obtained first, then the electroacoustic analogy of the oscillating valve in a steady flow is described. Finally the superposition of the two solutions gives the behaviour of the active control system.

3.1. *Fixed valve representation*

Considering the valve as an obstacle within a material domain D we sought to determine, the transfer matrix relation between the state variable at sections S_1 and S_2, respectively upstream and downstream boundaries of the domain D (Fig. 3).

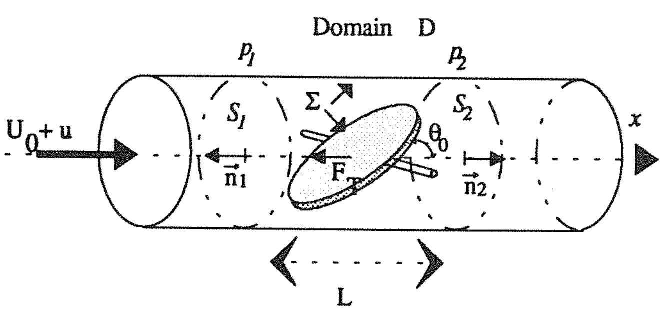

Figure 3. Fixed valve in a pulsed flow.

The basic equations of continuity, momentum and energy are used in their integral form in the domain D.

If the tube diameter is short enough and the length of the domain L is considered equal to the diameter of the duct, we can suppose that the wavelength is quite significant compared with the domain length. This assumption is valid in the majority of industrial exhaust systems which generate low frequency pulsations. It makes it possible to take account for the basic equations under quasi-steady conditions, therefore the one-dimensional equation of continuity in its integral form is given by:

$$\int_{\partial \mathrm{D}} \rho_{\mathrm{T}}\; u_{\mathrm{T}}\; \mathbf{n}\; \mathrm{d}\sigma = 0 \qquad (1)$$

Splitting the axial velocity and the density into mean and fluctuation components, we respectively obtain:

$$u_{\mathrm{T}} = U_0 + u \qquad (2)$$

$$\rho_{\mathrm{T}} = \rho_0 + \rho \qquad (3)$$

Integrating the relation (1) on the domain D, and subtracting from it the corresponding unperturbed steady flow equation, yields the first order linearized equation of continuity:

$$\rho_0(u_1 - u_2)S + U_0(\rho_1 - \rho_2)S = 0 \qquad (4)$$

where the subscripts 1 and 2 refer respectively to upstream and downstream conditions. S is the section of the duct which is taken as constant ($S_1 = S_2 = S$). On these boundary sections flow is considered uniform and homogeneous.

Under the same assumption (the domain length is small in comparison with the wave length), the one dimensional momentum equation in its integral form is given by:

$$\int_{\partial D} \rho_{\mathrm{T}}(u_{\mathrm{T}}\; \mathbf{n})u_{\mathrm{T}}\; \mathrm{d}\sigma = \int_{\partial D} \mathbf{T}\; \mathrm{d}\sigma \qquad (5)$$

Where $\int_{\partial D} \mathbf{T}\, \mathrm{d}\sigma$ is the contact resultant force, ∂D is the domain area and $\mathrm{d}\sigma$ an element of it. Considering the classical decomposition for the internal movement of this surface

$$\partial D = S_1 \cup S_2 \cup \Sigma \qquad (6)$$

and neglecting viscous stress, we obtain the contact resultant force:

$$\int_{\partial D} \mathbf{T}\; \mathrm{d}\sigma = -\int_{S_1} p_{\mathrm{T}}\; \mathbf{n}_1\; \mathrm{d}\sigma - \int_{S_2} p_{\mathrm{T}}\; \mathbf{n}_2\; \mathrm{d}\sigma - \mathbf{F}_{\mathrm{T}} \qquad (7)$$

$\mathbf{F}_{\mathrm{T}}$ is the contact resultant force on the area Σ (valve and duct surface). Introducing the equation (7) in (5), integrating the result and splitting density, pressure ($p_{\mathrm{T}} = P + p$), and axial velocity, we obtain the force one-dimensionnal expression of F_{T} linearized to the first order. Introducing (4), F_{T} is simplified as:

$$F_{\mathrm{T}} = F_0 + f = \rho_0 S U_0(u_1 - u_2) + (P_1 + p_1 - P_2 - p_2)S \qquad (8)$$

The resultant F_{T} is considered as the drag on the valve in the Douerin (1992) and Gervais et al. (1993) approach. Separation of the stationary and fluctuating parts gives:

$$F_0 = S(P_1 - P_2) \qquad (9)$$

$$f = \rho_0 S U_0(u_1 - u_2) + (p_1 - p_2)S \qquad (10)$$

The mean force F_0 appears as the product of the duct section by the head-loss in a steady flow due to the valve. No theoretical formulation can describe this head-loss, nevertheless we classically define a non-dimensional head-loss coefficient Λ which relates the difference between the static pressure on both sides of the valve and the dynamic pressure. This coefficient is obtained experimentally and is defined as follows:

$$P_1 - P_2 = \Lambda \rho_0 U_0^2/2 \qquad (11)$$

Perturbing this equation and splitting mean and fluctuating expressions, we obtain the linearized relation between equation (8) and head-loss in a pulsed flow:

$$\frac{F_0}{S} = \Lambda \frac{1}{2} \rho_0 U_0^2 = P_1 - P_2 \quad (12)$$

$$\frac{f}{S} = \frac{1}{2} \Lambda \rho_2 U_0^2 + \Lambda \rho_0 U_0 u_2 = \rho_0 U_0 (u_1 - u_2) + (p_1 - p_2) \quad (13)$$

If the simple isentropic relation can be adopted on the upstream side of the valve ($p_1 = \rho_1 c^2$), such is not the case at section S_2 because insertion of the valve produces turbulent separation and consequently dissipation. The entropy is fluctuating downstream of the valve. From the first law of thermodynamics expressed through the differential Gibbs relation, and after manipulation, the governing relation between fluctuating pressure, density and entropy is:

$$\rho_2 = \frac{p_2}{c^2} - \frac{\rho_0}{C_p} s_2 \quad (14)$$

where c is the local sound speed, C_p the specific heat at constant pressure and s_2 the fluctuating entropy downstream of the valve.

Considering an adiabatic evolution and then no decrease in stagnation enthalpy, we have:

$$\mathrm{d}h_\mathrm{T} + u_\mathrm{T}\, \mathrm{d}u_\mathrm{T} = T_\mathrm{T}\, \mathrm{d}S_\mathrm{T} + \frac{\mathrm{d}p_\mathrm{T}}{\rho_\mathrm{T}} + u_\mathrm{T}\, \mathrm{d}u_\mathrm{T} = 0 \quad (15)$$

where $h_\mathrm{T}, T_\mathrm{T}$ and S_T are the enthalpy, temperature and entropy of the medium.

Integrating this relation on the domain D, using the state relation for the perfect gas ($p_\mathrm{T} = r \rho_\mathrm{T}\, T_\mathrm{T}$) and splitting entropy, pressure, density and axial velocity in mean and perturbed contributions, we obtain:

$$P_1 - P_2 = \frac{P_2 S_2}{r} = \Lambda \frac{1}{2} \rho_0 U_0^2 \quad (16)$$

$$p_1 - p_2 - \frac{U_0^2}{2} (\rho_1 - \rho_2) = \frac{P_2}{r}\, s_2 + \frac{S_2}{r}\, p_2 \quad (17)$$

Using the relations (4,13,14,16,17), we obtain the four pole matrix which relates the state variables p and q on the both sides of the valve. For a Mach number such as $M^2 \ll 1$, it can be shown that this transfer matrix is advantageously approximated as following:

$$\begin{bmatrix} p \\ q \end{bmatrix}_1 = \begin{bmatrix} 1 & \Lambda M Z_{c_0} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}_2 \quad (18)$$

This representation is valid under the assumption that the head-loss coefficient is not too large, in other words that the valve is not completely closed. The electroacoustic analogy of this quadripole is classically a serial impedance. Therefore the fixed valve in a pulsed flow can be represented as an impedance in series with the duct, which is expressed:

$$Z_\mathrm{f} = \Lambda M Z_{\mathrm{c}0} \quad (19)$$

where $Z_{\mathrm{c}0} = c_0/S$ is the characteristic impedance of the uniform duct without flow.

It may be noted that Z_f is a linear function of the head-loss coefficient and of the Mach number; therefore, without flow the impedance is zero and, causally, the valve insertion does not perturb acoustic propagation.

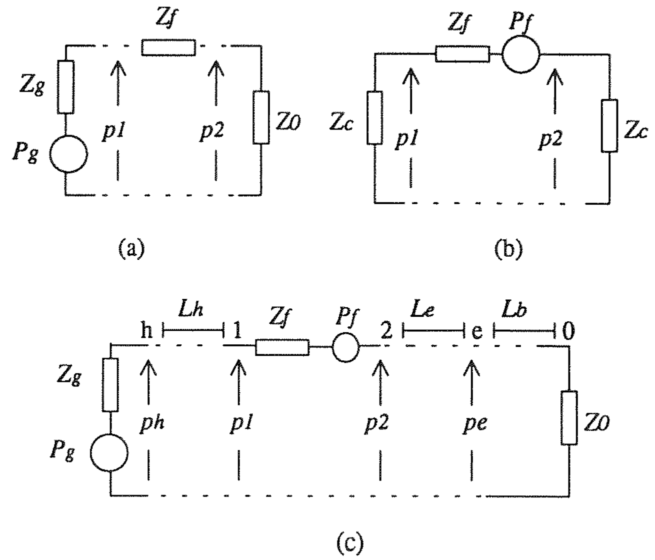

Figure 4. Electroacoustic representation of the device. a) Fixed valve in a pulsed flow, b) oscillating valve in a steady flow, c) active device with the two sources.

The electroacoustic representation of an exhaust system with a fixed valve is presented in Figure 4a.

3.2. *Oscillating valve in a steady flow*

In this part we consider the oscillating valve in an infinite length duct with mean flow. The head-loss produced by the valve is modified when it oscillates around its mean incidence. Therefore the flow is perturbed and the head-loss coefficient can be split into mean and fluctuating parts:

$$\Lambda = \Lambda_0 + \lambda \quad (20)$$

For this perturbed flow we can consider the previous quasi-steady approximation. The equations (4,14,16,17) are then unchanged and the fluctuating part of the resultant force, linearized to the first order, becomes:

$$\begin{aligned} \frac{f}{S} &= \rho_0 U_0 (u_1 - u_2) + (p_1 - p_2) \\ &= \frac{1}{2} \Lambda_0 \rho_2 U_0^2 + \Lambda_0 \rho_0 U_0 u_2 + \lambda\, \frac{1}{2} \rho_0 U_0^2 \end{aligned} \quad (21)$$

This relation is identical to equation (13), with an additional linear term which takes account of the head-loss coefficient fluctuation ($\lambda \rho_0 U_0^2/2$). Under the same assumption of a small Mach number ($M^2 \ll 1$), we obtain the characteristic relation of the oscillating valve:

$$p_1 - p_2 = \lambda \rho_0\, \frac{U_0^2}{2} + M Z_{\mathrm{c}0} \Lambda_0 q_2 \quad (22)$$

where $Z_{\mathrm{c}0} = c_0/S$ and $q_2 = \rho_0 S u_2$.

The electroacoustic analogy of this relation is a serial Thévenin generator with an infinite length duct. The figure (4.b) presents the electrical analogies of this generator with the characteristic impedance Z_c of the duct as upstream and downstream boundary conditions. The characteristics of this generator are:

$$\text{source strength :} \qquad P_{\mathrm{f}} = \lambda \rho_0 \, \frac{U_0^2}{2} \tag{23}$$

$$\text{internal impedance :} \qquad Z_{\mathrm{f}} = M Z_{\mathrm{c}0} \Lambda_0 \tag{24}$$

The source strength (23) is the product of the fluctuating head-loss coefficient with dynamic pressure, and it rises with the square of the flow. It is a particularly interesting characteristic of the oscillating valve, because its efficiency is not dependent on frequency. Consequently this auxiliary generator can attenuate very low frequencies in a mean flow. But, in the absence of mean flow, the generator is ineffective.

The internal impedance of this generator (24) expresses the mean head-loss produced by the mean incidence of the valve. It must be as small as possible in the experimental prototype to limit the back pressure created by the device. This is the main drawback of this secondary source in an active control system. The mean incidence of the valve is in fact a function of the electromechanical and energetic characteristics of the electrical motor which drives the valve.

An interesting perspective is the fact that mean flow increases with the frequency of the engine exhaust for the majority of pulsed flow generators (thermal engines, compressors,...). The generator strength of our oscillating valve rises with the square of the frequency. Moreover the power required by the electrical actuator increases also with the square of the frequency due to the valve's inertia. These two characteristics balance each over, therefore the electrical power consumed will be quasi-constant over the whole frequency range.

3.3. *Acoustic behaviour of the anti-pulsatory device*

The linear superposition of the two previous solutions makes it possible to describe the behaviour of the active device. Therefore the upstream boundary conditions are the primary generator and its internal impedance. Downstream of the valve, the boundary condition is the open ended duct impedance (Fig. 4c).

This superposition makes it possible to also describe the active device by means of a block approach. Under this formalism, the pressure picked up at the error sensor location can be expressed as:

$$p_{\mathrm{e}} = P_{\mathrm{f}} \, Ge + P_{\mathrm{g}} \, Fe \tag{25}$$

where Fe is the transfer function between the pressure at the error sensor location generated by the auxiliary source and Ge is the transfer function between the pressure picked up at the same location and the primary source strength (primary path).

If we consider that the duct is characterised by the four pole matrix given in the appendix, we obtain the analytical transfer functions Fe and Ge which are expressed as follows:

$$Fe = \frac{-Z_{\mathrm{e}} \mathrm{e}^{-jKML_{\mathrm{e}}}}{\left[\left(Z_f + Z_{\mathrm{e}} + Z_{\mathrm{g}_1}\right)\cos\, KL_{\mathrm{e}} + j\left(Z_{\mathrm{c}} + \frac{Z_{\mathrm{e}}}{Z_{\mathrm{c}}}\left(Z_{\mathrm{f}} + Z_{\mathrm{g}_1}\right)\right)\sin\, KL_{\mathrm{e}}\right]} \tag{26}$$

$$Ge = \frac{Z_{\mathrm{e}} \mathrm{e}^{jKM(\mathrm{L_e}+\mathrm{L_h})} / \left(\cos\, KL_{\mathrm{h}} + j\frac{Z_{\mathrm{g}}}{Z_{\mathrm{c}}}\sin\, KL_{\mathrm{h}}\right)}{\left[\left(Z_{\mathrm{f}} + Z_{\mathrm{e}} + Z_{\mathrm{g}_1}\right)\cos\, KL_{\mathrm{e}} + j\left(Z_{\mathrm{c}} + \frac{Z_{\mathrm{e}}}{Z_{\mathrm{c}}}\left(Z_{\mathrm{f}} + Z_{\mathrm{g}_1}\right)\right)\sin\, KL_{\mathrm{e}}\right]} \tag{27}$$

where

$$Z_{\mathrm{e}} = \frac{Z_0 + jZ_{\mathrm{c}} \tan KL_{\mathrm{b}}}{1 + j\frac{Z_0}{Z_{\mathrm{c}}} \tan KL_{\mathrm{b}}} \tag{28}$$

and

$$Z_{\mathrm{g}_1} = \frac{Z_{\mathrm{g}} + jZ_{\mathrm{c}} \tan KL_{\mathrm{h}}}{1 + j\frac{Z_{\mathrm{g}}}{Z_{\mathrm{c}}} \tan KL_{\mathrm{h}}} \tag{29}$$

Z_{g_1} being the upstream impedance at station 1 (Fig. 4c), is a function of the primary source impedance, and Z_{e} is the downstream impedance at location "e" . Thus the two transfer functions Fe and Ge depend on the boundary conditions and on the characteristic duct lengths which are:

L_{h} between the primary source and the valve,
L_{e} between the valve and the error sensor,
L_{b} between the error sensor and the termination.

Considering a feedback controller which is expressed as $P_{\mathrm{f}} = Hc \; p_{\mathrm{e}}$, one obtains the classical error signal expression in terms of the primary source strength:

$$\frac{p_{\mathrm{e}}}{P_{\mathrm{g}}} = \frac{Ge}{1 - Hc\, Fe} \tag{30}$$

The transfer function of this filter (Hc) would lead to a decrease in error signal. It can be obtained by means of a pole-zero placement control strategy for example. Theoretically the relations (26) and (27) can be used to calculate this transfer function. But the transfer functions Ge and Fe depend, as has already been shown, on the boundary conditions which have no analytical representation. Particularly, the internal impedance of the primary source (Z_{g}) needs an experimental investigation to obtain its complex representation. Furthermore the thermal condition of the medium is rather unsteady in the pulsed flow generator, this makes it impossible to use these transfer functions to synthesise the command law of the actuator. It is then necessary to use an adaptive command with an on-line identification to realise this synthesis.

4. Adaptive extremal control

Usually, control strategies are based on a self-tuning feedforward control approach using a L.M.S. algorithm to update the controller coefficients (Nelson and Elliott, 1992; Guicking et al., 1991). But, as has already mentioned, the noise signal in our study is periodic, this making it easy to obtain frequency information from the mechanical system. Therefore, to evaluate the physical principle, a frequencial approach was used to drive the auxiliary source. The interesting results obtained by Laumonier (1990) led us to develop an industrial prototype.

This prototype has to be robust and self-governing, in spite of the unsteady characteristics of the medium. An adaptive control technique with on-line estimation of the system transfer function guarantees these properties.

An extremum control algorithm was then developed to find the optimal amplitude and phase command which minimises each harmonic level of the pulsed flow. The on-line identification of the secondary path is obtained with a recursive least squares algorithm with a forgetting factor μ (RLS-μ).

The linear acoustics principle of superposition used in the foregoing model, present the error signal as the sum of the unwanted pressure fluctuations coming from the primary source and the opposite ones generated by the auxiliary source (the oscillating valve). The block diagram of this system is shown in Figure 5: $H(f)$ is the secondary path transfer function defined in the preliminary part, $c(t)$ is the periodic command of the actuator, $p(t)$ is the unwanted noise and $n(t)$ the turbulence which is considered as a white noise.

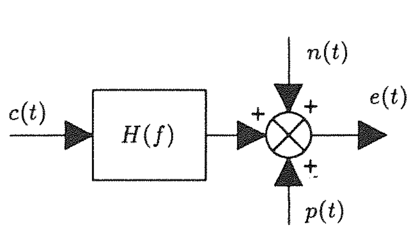

Figure 5. Error signal model.

A periodic control $c(t)$ is completely described by its amplitude A_n and phase φ_n at each harmonic nF_0, which are memorised in a parameter vector $\mathbf{C}$:

$$\mathbf{C} = [A_1\ e^{j\varphi_1},\ A_2\ e^{j\varphi_2}, \ldots,\ A_n\ e^{j\varphi_n}]^{\mathrm{T}} \tag{31}$$

The error signal $e(t)$ is also periodic and can be described, like $c(t)$, by a vector $\mathbf{E}$ composed of the amplitude B_n and phase φ_n at each harmonic:

$$\mathbf{E} = [B_1\ e^{j\phi_1},\ B_2\ e^{j\phi_2}, \ldots,\ B_n\ e^{j\phi_n}]^{\mathrm{T}} \tag{31}$$

According to these notations, the error parameters are functions of the command parameters as follows:

$$\mathbf{E} = \mathbf{P} + [\mathbf{H}]\mathbf{C} \tag{33}$$

The relation (33) describes the error parameters $\mathbf{E}$ resulting from the interference between the noise parameters $\mathbf{P}$ and the anti-noise parameters $[\mathbf{H}]\mathbf{C}$. The matrix $[\mathbf{H}]$ represents the action of each harmonic of the command signal on the corresponding error signal harmonic. For a linearized model, the matrix $[\mathbf{H}]$ is diagonal, there is no coupling between the harmonics:

$$\mathbf{H} = \mathrm{diag}\ (H_1,\ H_2,\ \ldots,\ H_N), \tag{34}$$

The elements H_n of $[\mathbf{H}]$ are equal to the complex gain of the transfer function $H(f)$ at the n^{th} harmonic. Under this formalism, each complex element of the error vector $\mathbf{E}$ characterises a harmonic of the flow which will be attenuated. This frequencial selective approach is particularly interesting because it makes it possible to modify the tone of an engine exhaust system, to comply with psychoacoustic requirements, for example.

4.1. *Criterion expression*

The measured signal at the downstream pressure sensor $e(t)$ is the error signal for the incidence control $c(t)$. It must be as small as possible. But turbulence excludes choosing $e(t) = 0$ as an objective of optimal control, we therefore propose to minimise the following quadratic criterion:

$$J = \int_{T_0} \mathrm{e}^2(t)\ \mathrm{d}t \tag{35}$$

T_0 being the period of the pulsed flow. An iterative technique, also known as extremal control, is used to reach the control law $\mathbf{C}_{\mathrm{opt}}$ which minimises J (Wellstead and Scotson, 1990; Micheau et al., 1992).

4.2. *Newton Raphson algorithm*

The criterion is totally defined within the set of parameters associated with the real signals. With criterion expression (35), the Newton-Raphson algorithm is used to minimise the criterion:

$$\mathbf{C}_{m+1} = \mathbf{C}_m - \eta\Gamma_m^{-1}\mathbf{G}_m \tag{36}$$

where $\mathbf{G}_m$ is the Hessien matrix at the point $\mathbf{C}_m$, $\mathbf{G}_m$ the gradient at the point $\mathbf{C}_m$, and η the convergence coefficient. Following the relation (33), $\Gamma_m^{-1}\mathbf{G}_m$ can be approximated by $[\hat{\mathbf{H}}]^{-1}\mathbf{E}_m$ [14], with $[\hat{\mathbf{H}}]$ an estimation of the matrix $[\mathbf{H}]$ then:

$$\mathbf{C}_{m+1} = \mathbf{C}_m - \eta[\hat{\mathbf{H}}]^{-1}\mathbf{E}_m \tag{37}$$

The extremum control is equivalent to an integrator controller in the set of parameters. An estimation of the matrix $[\mathbf{H}]$ is necessary to uncouple the real and imaginary parts of the error parameters, which is equivalent to compensating the phase-shift of the transfer function. However, the estimation of $[\mathbf{H}]$ is biased because the transfer function is very sensitive to physical conditions. Thus, the choice of the convergence coefficient η influences algorithm behaviour according to the bias on $[\mathbf{H}]$. The theoretical path of the control $\mathbf{C}_m$ for the linear model (31) is:

$$\mathbf{C}_m = -[\mathbf{H}]^{-1}\mathbf{P} + (\mathbf{C}_0 + [\mathbf{H}]^{-1}\mathbf{P})(\mathbf{I} - \eta[\mathbf{H}][\hat{\mathbf{H}}]^{-1})^m \tag{38}$$

where $m \in [0; +\infty[$ and $\mathbf{C}_0$ is the control vector at the time origin. The algorithm converges if the coefficient η respects the following condition:

$$0 < \eta < 2\eta_c$$

$$\eta_c = \mathrm{Max}\ (\mathrm{Re}(\frac{\hat{H}_n}{H_n}))\ \text{ with }\ n \in [1, N]$$

Re(.) being the real part.

In spite of a rough estimation of the transfer function, this algorithm converges. Nevertheless a good estimation of H_n makes it possible to increase the convergence speed of the algorithm.

4.3. *On-line identification*

Fundamentally, the command law is elaborated using a black box concept. The transfer function represented by the matrix [**H**], depends on the operating point and is very sensitive to physical conditions. As we have shown, the convergence is guaranteed if there is an identification of this matrix. Therefore an on-line identification process was developed, for which we consider that the variation of the gain matrix [**H**] and of the perturbation **P** are slow. Under this assumption, the representation can be written as a function of the control and error increments:

$$\mathbf{\Delta E}_m = [\mathbf{H}]\mathbf{\Delta C}_m$$

with

$$\begin{aligned}\mathbf{\Delta C}_m &= \mathbf{C}_m - \mathbf{C}_{m-1} \\ \mathbf{\Delta E}_m &= \mathbf{E}_m - \mathbf{E}_{m-1}\end{aligned} \tag{39}$$

A recursive least squares algorithm with forgetting factor μ (RLS μ) is used to estimate each component $\hat{H}_n$ of the diagonal matrix $[\hat{\mathbf{H}}]$ on-line:

$$\begin{aligned}&\hat{H}_{n,m} = \hat{H}_{n,m-1} + \frac{\Delta C^*_{n,m}}{R_{n,m}}(\Delta E_{n,m} - \hat{H}_{n,m-1}\Delta C_{n,m}) \\ &\text{and} \quad R_{n,m} = \mu R_{n,m-1} + ||\Delta C_{n,m}||^2\end{aligned} \tag{40}$$

with * the conjugate of the complex term.

Due to the independence of the parameters, this algorithm is scalar. During the searching phase of extremal control, increments $\mathbf{\Delta C}_m$ provide persistent excitation for on-line transfer function identification. But at the optimum, control increments disappear, and it is then necessary to freeze the identification algorithm. This on-line identification procedure is particularly interesting because it is not necessary to add a white noise signal to the command as is the case for on-line identification with a temporal algorithm (Eriksson and Allie, 1989). This is one of the main advantages of this frequencial approach, furthermore the turbulence in the duct having a wide band spectrum, this selective approach gives good noise immunity.

5. Experimental procedures and results

In the two previous parts of this paper, we have presented a theoretical model of the oscillating valve, and the control strategy adopted to command such an auxiliary source in order to attenuate flow fluctuation in a duct. The aim of the work described in the present part, is firstly to experimentally confirm the analytical model developed and secondly to prove the ability of the active device to attenuate unwanted noise.

We successively present the transfer function Fe measured and calculated with our model (26), attenuation results obtained with the active device compared to the one obtained with a passive muffler, and finally the adaptive behaviour of the control algorithm. Before presenting results, a brief description of the experimental system is given.

5.1. *Instrumentation and equipment*

The arrangement of equipment used for the measurement is shown in Figure 6. A steady flow is generated in the duct by a fan. This flow can be adjusted to fix the mean velocity wanted and becomes unsteady by means of a specific system which makes it possible to fix independently the fluctuating pressure level generated and its frequency. Stopping this system makes it possible to conserve a steady flow. This experimental device is particularly adapted to reproduce almost of the configurations which can be found in an industrial system.

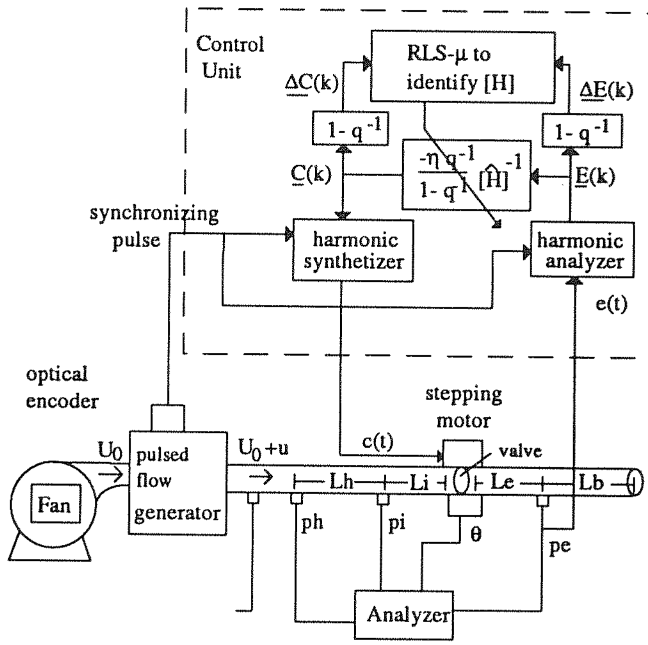

Figure 6. Scheme of the experimental system.

The error signal is obtained from the synchronising pulse with the aid of an optical encoder which also delivers the necessary frequency information.

Three piezoresistive pressure sensors are used: p_h and p_i to obtain the upstream impedance at the location h, and p_e as error sensor for the controller.

For all measurements, the pressure levels are obtained with a spectral analyzer. The valve is driven by a stepping motor, the command signal of which is generated by a control unit which can simultaneously realise the synchronous acquisitions of the error sensor.

L_e, L_i, L_h and L_b are the characteristic lengths used to obtain the theoretical expression of the transfer function Fe. A position sensor gives the instantaneous position of the valve.

A hot-wire anemometer gives the information on the mean flow and then on the dynamic pressure.

5.2. *Transfer function Fe*

In this experimental procedure, the valve is sinusoidally driven by the stepping motor in an initially steady flow. The experimental transfer function measurement requires knowledge of the pressure level p_e, which is nondimensionalized with the steady flow dynamic pressure $(\rho_0 U_0^2/2)$ and multiplied by the fluctuating head loss coefficient λ obtained by mean of the fluctuating and mean position of the valve (θ_0, θ).

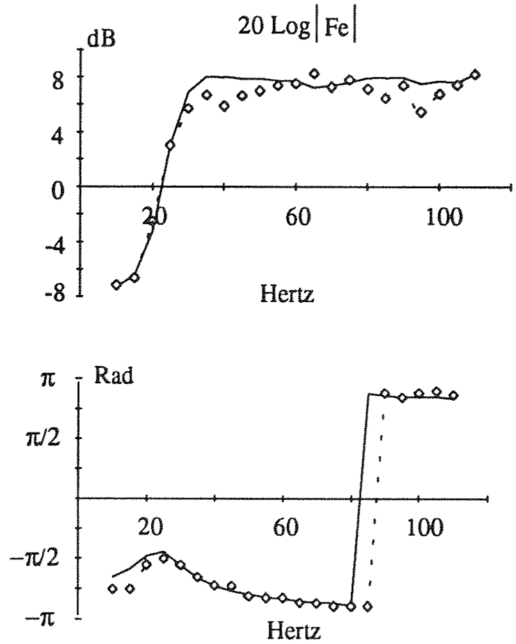

Figure 7. Experimental (◇ − ◇ − ◇) and theoretical (—) evolution of the transfer function Fe. L_h = 0.49 m, L_i = 0.3 m, L_e = 0.35 m, L_b = 0.6 m, M = 0.09, θ_0 = 28.8° (Λ_0 = 3.11) and θ = 5°.

The matrix (A.2), presented in the Appendix, and the boundary conditions are necessary to calculate the theoretical function Fe. The boundary condition used downstream is the radiation impedance of an open ended duct with mean flow proposed by Ingard (1974). The upstream boundary condition is experimentally evaluated by means of the pressure p_h and p_i, which give the complex reflection coefficient $R_h(\omega)$ and from it the impedance Z_h is evaluated,

$$R_h(\omega) = \frac{H_{hi}\, e^{jKML_h} - e^{-jKL_h}}{e^{jKL_h} - H_{hi}\, e^{jMKL_h}}$$

and $$Z_h = Z_c\,\frac{1 + R_h(\omega)}{1 - R_h(\omega)} \qquad (41, 42)$$

Here $H_{hi} = p_h(\omega)/p_i(\omega)$ is the measured acoustic transfer function, L_h is the spacing between the two sensors, Z_c and K (defined in the Appendix) are respectively the characteristic impedance of the duct and the wave number for a viscous moving medium.

The results obtained are presented in Figure 7. The good agreement between theory and experiment confirms the validity of the model proposed in section 3.

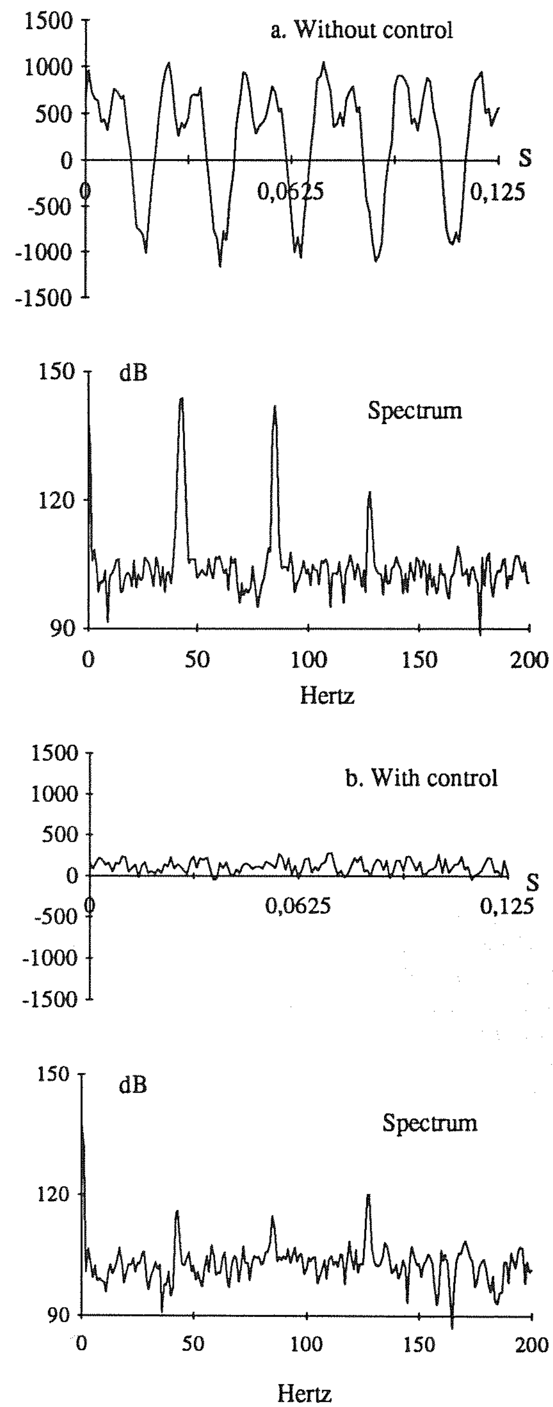

Figure 8. a) Without control: 42 Hz (148 dB) and 84 Hz (145 dB). b) With control: 42 Hz (120 dB) and 84 Hz (119 dB).

5.3. *Attenuation result*

Some typical results obtained on the previous system are presented in this section. Figure 8a shows the fluctuating pressure measured with the error sensor without control, the periodic character of the flow appears clearly on the spectrum of this signal.

Figure 8b exhibits the reduction obtained with the active device. In this example the error vector **E** is composed of the first two harmonics, the reduction obtained is about 28 dB on the fundamental and 26 dB on the harmonic. In fact these two frequencies drop down to almost background noise level. In this case the mean incidence of the valve is about $\theta_0 = 32.4°$ which produces a mean head loss coefficient $\Lambda_0 = 4.4$. This result clearly shows the selectivity of the device developed, and its capacity to attenuate high pressure levels.

Figure 9 compares the insertion loss obtained with the active device and with a classical passive muffler for different frequencies of the primary source. Insertion loss (Prasad and Croker, 1983) is classically defined as the difference between pressure levels measured at a station point, with and without the device tested in place. The pressure is measured at the error sensor location. This result clearly shows the capacity of our device to cancel low frequency noise, and the relative inefficiency of the muffler in this frequency range. We observe that below 20 Hz the muffler does not attenuate but in fact amplifies the noise. Our active device is thus a vast improvement over the passive muffler in this frequency range.

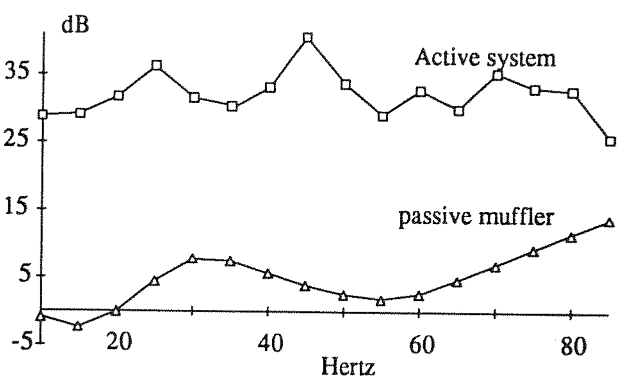

Figure 9. Insertion loss comparison between our device and a passive muffler.

5.4. *Adaptive behaviour of the controller*

The adaptive character of the active device is shown through a particular experimental procedure. It consists in suddenly increasing (or decreasing) the mean flow to change the operating point, and in looking at the evolution of the error signal and the transfer function $[\hat{\mathbf{H}}]$ identified for each iteration. A typical result is shown in Figure 10, where we choose to cancel a pure tone at 42 Hz. The device is started at the first iteration, $[\hat{\mathbf{H}}]$ is initialised to $(-1, 0)$ and the command is naught. The error signal then rapidly decreases and the transfer function estimation is stabilised after 3 iterations, optimal command and error minimisation is obtained after 6 iterations. At iteration 8, we increase the mean flow of the primary source. Dynamic pressure is then increased by a factor of about ten. As we have seen, the auxiliary source strength equation (23) is equally increased by the same factor. The curves presented show the ability of the algorithm to track the new operating point. After two iterations the new value of $[\hat{\mathbf{H}}]$ is estimated and then the

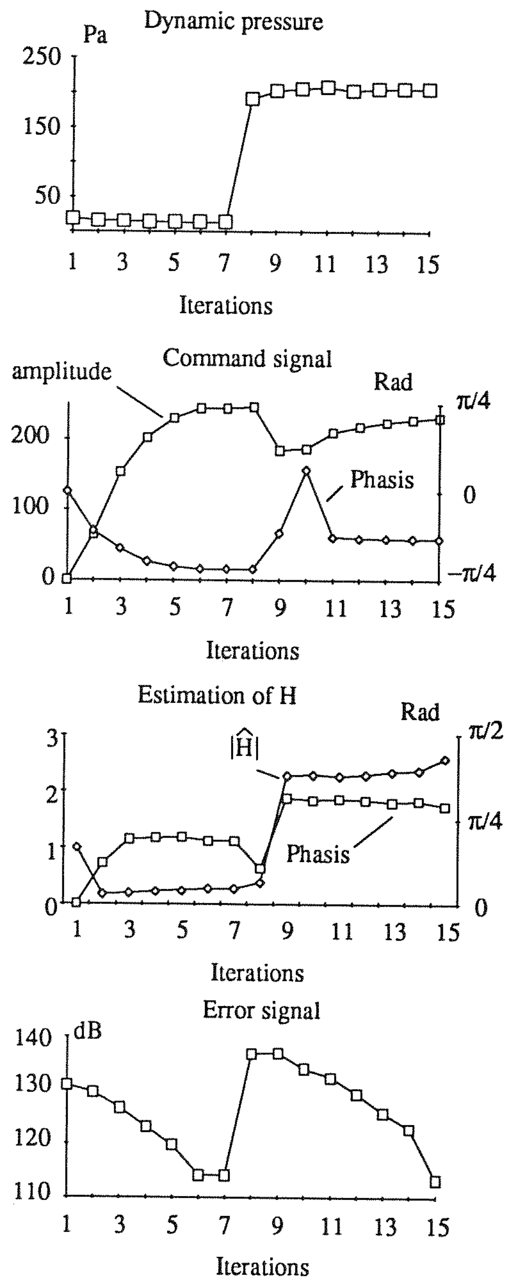

Figure 10. Algorithm behaviour.

algorithm converges to minimise error signal. We see that the transfer estimation obtained is in accordance with the physical model of the device, since $[\hat{\mathbf{H}}]$ increases by the same factor as dynamic pressure and source strength.

6. Conclusion

The work presented in this paper concerns recent developments in the use of anti-pulsatory devices as engine exhaust mufflers. The model developed to characterise this device shows that it can be compared to a serial generator. Its internal impedance (Z_f) is a mean head-loss function and its source strength (P_f) is a dynamic pressure ($\rho_0 U_0^2/2$) function. It is interesting to note that source strength does not depend on frequency but on the square of the mean flow. Assuming sufficiently high dynamic pressure, the source can produce significant fluctuating pressure at low frequencies.

This device has been used as an auxiliary source in the active control of noise both on laboratory flow benches and industrial exhaust systems. An original adaptive control has been developed to drive this secondary source. It is an extremal control which requires a frequency analysis of the error signal obtained by means of an optical encoder fixed on the primary generator. On-line identification allows the controller to track the unsteadiness of the system.

Experiments have validated the analytical model and shown the efficiency of the control algorithm, in particular its ability to adjust the command parameter in the presence of instability.

This theoretical and experimental work has proved the efficiency of this device in attenuating the noise produced by an engine exhaust system.

The mechanical robustness of the valve (a metallic disk), which makes it possible to use this device in a dirty and hot environment, is the main advantage of the device. Such an auxiliary source could prove to be an interesting alternative to loudspeakers in active noise control systems in engine exhaust ducts.

Appendix. Transfer matrix and load impedance

The state variables acoustic pressure p and mass velocity q at the two stations 1 and 2 in a duct can be related in terms of the four pole matrix as:

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ q_2 \end{bmatrix} \tag{A.1}$$

then $Z_1 = \frac{p_1}{q_1} = \frac{a_{11}Z_2+a_{12}}{a_{21}Z_2+a_{22}}$, and $Z_2 = \frac{p_2}{q_2}$. For a straight duct element (area S, length L) with a mean flow Mach number M and with damping effects, Munjal (1987) gives a particular relation:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \mathrm{e}^{-jKML} \begin{bmatrix} \cos KL & jZ_c \sin KL \\ j/Z_c \sin KL & \cos KL \end{bmatrix} \tag{A.2}$$

where $K = (k - j\alpha)/(1 - M^2)$
k being the wave number of the medium and an attenuation constant tacking account the viscothermal effect and the friction factor,

and $$Z_c = \frac{c_0}{S}\left(1 - \frac{\alpha c_0}{\omega} + j\,\frac{\alpha c_0}{\omega}\right) \tag{A.3}$$

Z_c being the characteristic impedance of the duct in the moving viscous flow, c_0 the sound speed, and ω the flow pulsation.

For a finite length duct, Z_1 will be the load impedance and the impedance Z_2 will be the radiation impedance. Thus using equations (A.2), one has:

$$Z_1 = \frac{(Z_2 + jZ_c \tan KL)}{(1 + j\frac{Z_2}{Z_c} \tan KL)} \tag{A.4}$$

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