INFORMATION PROCESSING IN NEURAL NETWORKS BY MEANS OF CONTROLLED DYNAMIC REGIMES

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ABSTRACT

This paper is concerned with the modeling of neural systems regarded as information processing entities. I investigate the various dynamic regimes that are accessible in neural networks considered as nonlinear adaptive dynamic systems. The possibilities of obtaining steady, oscillatory or chaotic regimes are illustrated with different neural network models. Some aspects of the dependence of the dynamic regimes upon the synaptic couplings are examined. I emphasize the role that the various regimes may play to support information processing abilities. I present an example where controlled transient evolutions in a neural network, are used to model the regulation of motor activities by the cerebellar cortex.

1. INTRODUCTION

At a certain level, biological neural systems can be considered as very complex physico-chemical plants, and accordingly be described with appropriate physical and chemical concepts. However, at a higher level, these systems have to be considered as information processing entities, and here one must come to a description involving information processing concepts (such as representations, codes, computations, algorithms, signal-noise dilemma, reliability, *etc*). This constitutes the perspective of the discipline known as computational neuroscience (Sejnowski, 1988). There already exist important theoretical models that can be viewed as paradigms for the information processing performed by biological neural systems. This is the case for the Hopfield neural network (Hopfield, 1982, 1984), that proposes a framework where attractor dynamics toward controlled stable steady states are used to represent memory processes in the nervous system. After a brief review of the Hopfield model, emphasizing its neurobiological information-processing orientation, I shall further develop in this paper the possibility to use various dynamic regimes in neural networks to support information processing abilities. This development will be organized in two steps. In the first step, I shall show that, beyond stable steady regimes, other types of dynamic regimes (oscillatory, chaotic, transient) are accessible and controllable in neural systems. In the second step, I shall demonstrate, in the

continuation of the Hopfield scheme, the possibility to exploit these regimes for information processing. For this purpose, I shall rely on an example, that illustrates how it is possible to model the ability of the cerebellar cortex to control motor activities, through the use of transient evolutions in a neural network.

2. A CLASSICAL PARADIGM FOR INFORMATION PROCESSING IN NEURAL NETWORKS

The Hopfield neural network (Hopfield, 1982) is a model that assigns a specific role to certain dynamic evolutions of neuron activities, in order to store and retrieve information.

The model is composed of N neurons, with binary output activities $S_i(t) = 0$ or 1, indicating whether neuron i is silent or active at time t. These N neurons are fully connected through N^2 synapses modeled by multiplicative coefficients w_{ij} for the link from neuron j to neuron i. At time t, the membrane potential of neuron i is defined as:

$$V_i(t) = \sum_{j=1}^{N} w_{ij} S_j(t).$$ (1)

To model the nonlinear input-output relationship of the neuron, $V_i(t)$ is compared to a threshold potential V_{th}. Within a discrete-time dynamics, it is prescribed that if $V_i(t) > V_{th}$ then $S_i(t+\Delta t) = 1$, otherwise $S_i(t+\Delta t) = 0$.

A state $S(t)$ for the neural network at time t is then introduced as:

$$S(t) = [S_1(t), S_2(t), ..., S_N(t)].$$ (2)

This state of equation (2) is the entity that supports the information that is processed by the network. It can be compared to a binary word in an electronic computer. The state $S(t)$ can experience very different time evolutions, that are controlled by the synaptic couplings w_{ij}. Depending on the w_{ij}'s, one can observe with this model, either steady evolutions, or periodic evolutions with possibly very long recurrence times in large networks.

Hopfield gives conditions on the w_{ij}'s to obtain the two following important properties:

i) the evolutions of $S(t)$ will always be convergent, toward stable steady states;

ii) these stable steady states of the evolutions can be imposed (as fixed points or point attractors of the dynamics).

The prescription stipulates that the imposition of M stable steady states S^p, $p = 1$ to M, is obtained by setting the synaptic couplings w_{ij} according to:

$$w_{ij} = \sum_{p=1}^{M} (2S_i^p - 1)(2S_j^p - 1), \quad \text{and} \quad w_{ii} = 0.$$ (3)

There are of course limitations relating the number M of imposed steady states, to the number N of neurons in the network. But I will not go, here, into the discussion of such issues. Rather, I want to emphasize the general significance of the Hopfield neural network for neurobiological information processing.

The Hopfield model provides a framework to represent and interpret memory processes in the nervous system. Its content can be abstracted as follows. Neural networks are nonlinear dynamic systems, whose states of activity are capable of complex time evolutions. These evolutions critically depend upon the strengths of the synaptic couplings, that are plastic or adaptive elements. Learning consists in adjusting the synaptic couplings, in order

to implement attractor dynamics, toward imposed stable steady states of activity of the network. These stable steady states represent the patterns of information that the network has memorized, and they are endowed with controlled basins of attraction that determine the conditions of stability and retrieval of the learned patterns.

I wish to point out that the emergent properties for information processing of the Hopfield network, come as a consequence of a few basic elements, that can be considered as some of the most essential characteristics of biological neurons. Namely, the model incorporates neurons that have a threshold nonlinearity, these neurons are massively connected into a network, through the medium of plastic synapses. This basis confers to the model a real and profound biological significance (in addition to the technological applications that it entails). These fundamental characteristics of biological neurons are included in a very schematic way in the original Hopfield model, where the focus is placed on the collective properties at the network level, with an interpretation in information processing terms. But an interesting fact, that strengthens the status of the Hopfield model as an important paradigm, is that the general scheme that it establishes appears to be robust, and is preserved when more and more biological details are added to describe individual neurons (see for instance Amit, 1990, 1991).

3. DYNAMIC VARIABILITY IN NEURAL NETWORKS

I examine in this section a neural network model, slightly different from Hopfield's, although of the same inspiration. This model stands for a minimal framework that allows the observation of the complete range of dynamic regimes that are accessible for neural networks. Namely, these observed regimes can be steady, oscillatory (periodic or quasiperiodic), chaotic (from low-dimensional to high-dimensional chaos).

The neural network model incorporates N neurons, with output activities $S_i(t)$ which are allowed to continuously vary in the finite interval [0, 1]. The variable S_i is interpretable as the mean firing rate of neuron i.

These N neurons are fully connected through N^2 synaptic coefficients w_{ij}. At time t, the membrane potential of neuron i is again defined by equation (1). In the context of a discrete-time dynamics, the neuron activities evolve according to:

$$S_i(t+\Delta t) = f[V_i(t)]. \tag{4}$$

The function $f(.)$ describes the nonlinear input-output transfer of the neurons. As often done in neural modeling, it is given here a sigmoidal shape, with threshold θ and slope β, of the form:

$$f(V_i) = \frac{1}{1+\exp[-\beta(V_i-\theta)]}. \tag{5}$$

In the resulting neural network, the activities $S_i(t)$ can experience very different time evolutions or regimes, that depend upon the adjustable parameters of the model. For the networks studied here I took $\theta = 0$ and $\beta = 5$, these values being by no means critical to obtain the reported properties. The remaining parameters are the synaptic efficacies w_{ij}, that were used as control parameters to change the dynamic regime of the network. The unit of time is such that $\Delta t = 1$.

As an illustration, figure 1 represents three typical time evolutions of the activities in
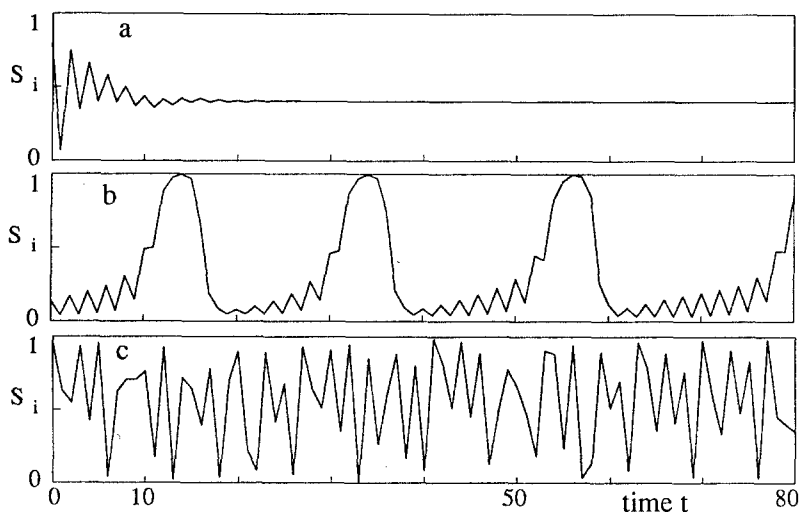
Fig. 1. Three time evolutions of a neuron activity $S_i(t)$ in a neural network with three different sets of synaptic couplings: a) convergence to a steady state, b) oscillatory pattern, c) chaotic evolution.

small networks incorporating no more than 15 neurons. With three different sets of w_{ij}, we observe in figure 1a an evolution that converges to a steady state, in figure 1b an evolution with an oscillatory pattern, and in figure 1c a chaotic evolution.

The unsteady evolutions, as in figure 1c, can be shown to exhibit classical characteristics that are attached to deterministic chaos (Bergé, 1986). For instance, it is possible to represent the attractor reached by a neuron activity, in the plane spanned by the quantities $\{S_i(t), S_i(t+\Delta t)\}$. Figure 2 depicts two such phase-plane attractors. The attractor of figure 2a, which is a one-dimensional close curve, is associated to a quasiperiodic regime. In contrast, the attractor of figure 2b, which presents a thinly folded structure, is an example of a fractal attractor, with dimension between 1 and 2. It characterizes a chaotic regime.

The presence of chaos can be observed in small neural networks. Figure 3 depicts two phase-plane attractors described by two different neuron outputs $S_i(t)$, in a five-neuron network. These two attractors, although different, bear some similarity. In fact, they represent two projections, in two different planes, of the complete attractor involving all the variables $S_i(t)$, with $i = 1$ to 5. The existence of such complex attractors reveals subtle correlations that can occur between two neuron activities in a network in the presence of specific sets of synaptic couplings.

For these neural networks, it is also possible to characterize the dynamics of the neuron activities in a phase plane spanned by two phase coordinates $S_i(t)$, $S_j(t)$, provided by two arbitrary outputs evaluated at the same time t. Figure 4 gives two examples of such attractors. The qualitative structure of attractors in $\{S_i(t), S_i(t+\Delta t)\}$ or in $\{S_i(t), S_j(t)\}$ are quite similar. Each of them can reveal chaotic dynamics, and can serve to extract fractal dimensions for the attractor of the dynamics. The attractor of figure 4a with its fine structure is characteristic of low-dimensional chaos. The attractor of figure 4b, that densely fills finite regions of the phase plane, is characteristic of high-dimensional chaos.
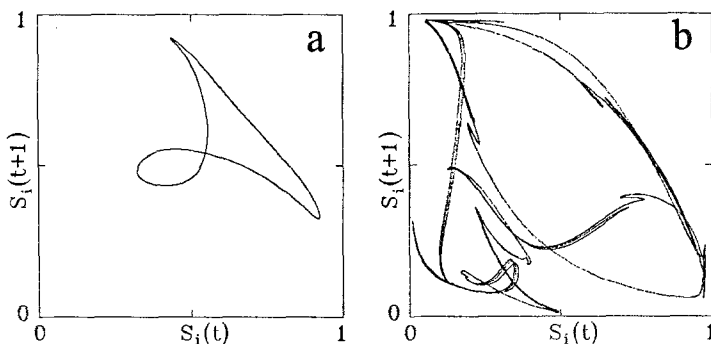
Fig. 2. Attractors in phase plane $\{S_i(t), S_i(t+1)\}$ for a neuron activity in two neural networks: a) quasiperiodic regime, b) low-dimensional chaos.
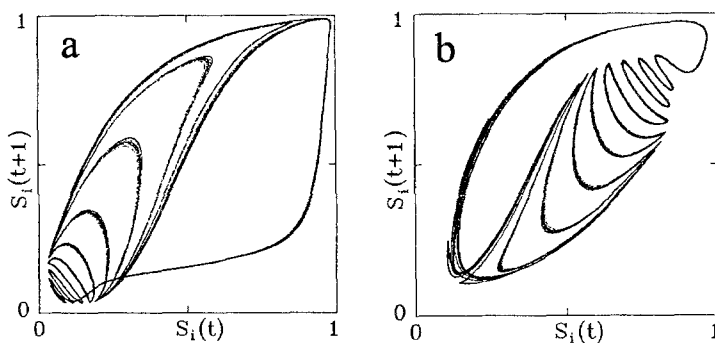
Fig. 3. Attractors in phase plane $\{S_i(t), S_i(t+1)\}$ for two neuron activities in a 5-neuron network in a chaotic regime: a) for the neuron i=1, b) for the neighbouring neuron i=2.
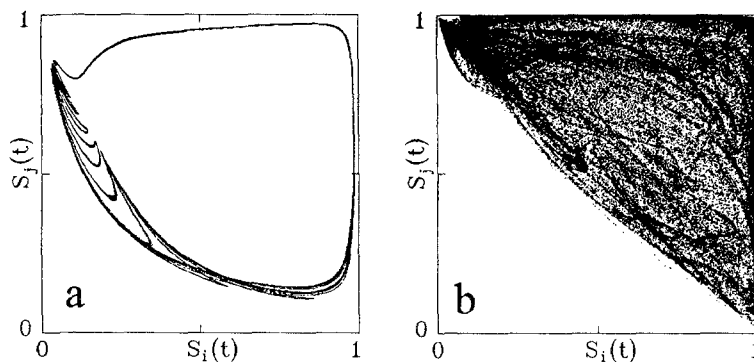
Fig. 4. Attractors in phase plane $\{S_i(t), S_j(t)\}$ for neuron activities in two neural networks: a) in the 5-neuron network of figure 3 with here $i=1$ and $j=2$ (the same two neurons are observed) and characterizing a low-dimensional chaotic regime, b) in a 15-neuron network and characterizing a high-dimensional chaotic regime.

4. CONTROL OF THE DYNAMIC REGIME THROUGH LEARNING

4.1. Steady versus unsteady evolutions

When no specific rules are implemented to assign values to the w_{ij}, or in other words, when the w_{ij}'s are random, I observed that evolutions converging to a steady state (as in figure 1a) become less probable as the network size N increases. At the same time, unsteady evolutions (as in figure 1b-c) become more probable. This point is illustrated in figure 5. First, we consider networks whose synaptic efficacies w_{ij} are randomly drawn with uniform probability out of the interval $[-1, 1]$. Figure 5a shows, for these unconstrained networks, the probability that all the neurons of a network reach a steady state, as a function of the size N of the network. The values of the probability were evaluated as frequencies of occurrence over many networks of size N. We observe in figure 5a that, with no synaptic constraints, this probability decreases when the networks become more complex, that is when their number of neurons N increases. Figure 5b represents this same probability, but for networks whose synaptic efficacies are constrained by the Dale principle. This principle (Eccles, 1977), originating in biological observations, stipulates that, in general, the synapses emitted by a given neuron are all of the same type (either excitatory, or inhibitory). The Dale principle is not of thorough validity in biological neural networks. Nevertheless, it can be envisaged as a possible constraint whose impact is worth examining, when testing schemes of organization for the synapses in a neural network model. So, for the networks of figure 5b, the synapses w_{ij} were randomly drawn with uniform probability either in $[0, 1]$ or in $[-1, 0]$, according to a toss-up choice fixed for each emitting neuron j. The curve of figure 5b shows that the probability of reaching a steady state can be significantly increased when an appropriate structure is conferred to the network through synaptic modification. This possibility forms the basis of the information storing ability of the Hopfield neural network, where conditions are given on the synapses in order to ensure convergence to steady states representing memories. However, the probability curve of figure 5a shows that, in general, convergence to steady states is far from being a guaranteed property in neural networks, especially when their size become large. Explicit mechanisms have to apply for organizing the synaptic connections, if steady states are required, to implement memory processes for instance.

4.2. Destabilizing steady evolutions

In the opposite direction, some authors have proposed that unsteady or chaotic dynamics as in figure 1c, can be a desirable feature to support certain cognitive processes (Skarda, 1987). In a similar way as before, it is possible to devise synaptic plasticity mechanisms that will destabilize a network away of any steady state of activity, or even provoke the onset of a chaotic regime with controlled dimensionality (McGuire, 1991).

For instance, to destabilize a given neuron k it is possible to introduce a cost function $E_k(t)$ that will tend to assume high values whenever activity $S_k(t)$ sits in a steady state. At time t, the synaptic efficacies w_{ij} are then adjusted in order to minimize $E_k(t)$. The adjustment of the w_{ij}'s can be performed through a gradient descent of the form:

$$w_{ij}(t+\Delta t) = w_{ij}(t) - \alpha \frac{\partial E_k(t)}{\partial w_{ij}} . \qquad (6)$$

In general, $E_k(t)$ will be a state function of the neural network, whose dependence upon the
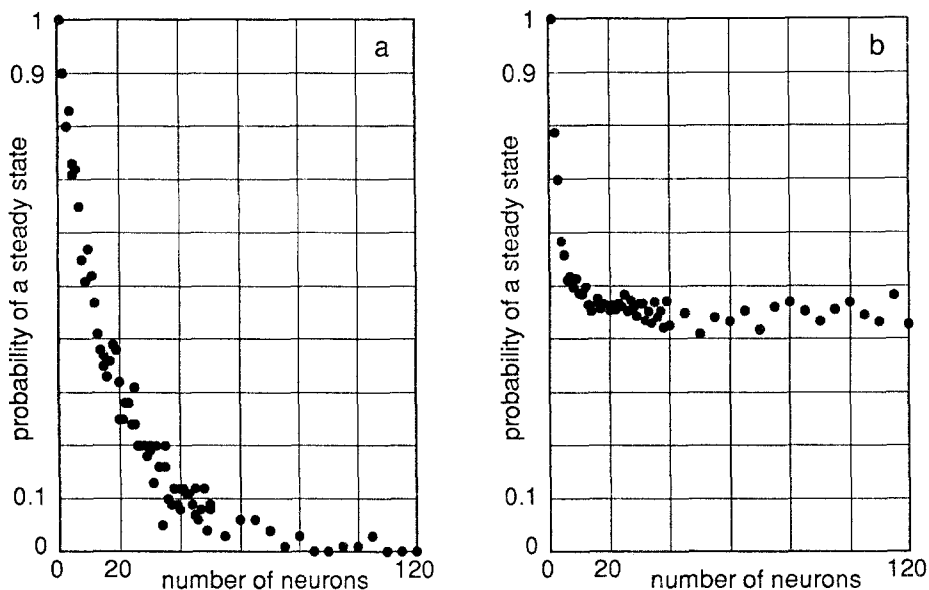
Fig. 5. Probability of convergence to a steady state in neural networks with a) unconstrained synapses, b) synapses constrained by the Dale principle.

w_{ij}'s and t will be brought in as a consequence of the unique dependence upon the state $S_k(t)$. The performance of the gradient descent of equation (6) thus requires, in parallel to the dynamics of the neural network, to maintain an additional set of dynamic variables $p_{ij}^m(t)$ defined as (Williams, 1989):

$$p_{ij}^m(t) = \frac{\partial S_m(t)}{\partial w_{ij}} . \tag{7}$$

The introduction of the p_{ij}^m's comes about as a consequence of the chain rule while differentiating to compute the gradient. In the context of this model, one finds that their time evolution is governed by:

$$p_{ij}^m(t) = f'[V_m(t-\Delta t)] \times [\delta_{mi} S_j(t-\Delta t) + \sum_{n=1}^{N} w_{mn} p_{ij}^n(t-\Delta t)], \tag{8}$$

where δ_{mi} is the Kronecker delta, and $p_{ij}^m(0) = 0$, $\forall i,j,m$.

The derivative $f'[V_i(t-\Delta t)]$ for the sigmoid of equation (5) can be written as:

$$f'[V_i(t-\Delta t)] = \beta f(V_i(t-\Delta t))[1-f(V_i(t-\Delta t))] = \beta S_i(t)[1-S_i(t)]. \tag{9}$$

A more general cost function can be made up by linear superposition of terms similar to $E_k(t)$ and defined for several neurons k, and then summed up over a given time window $]t-T, t]$. In such conditions, the resulting adjustments of the w_{ij}'s as given by equation (6) just add up linearly.

A possible expression for the cost function $E_k(t)$ is:

$$E_k(t) = [S_k(t)-0.5][S_k(t-\Delta t)-0.5]. \tag{10}$$

Minimizing the cost $E_k(t)$ of equation (10) tends to destroy short-term correlations in $S_k(t)$, preventing this signal to settle in a constant value. The corresponding synaptic adjustment law, as expressed by equation (6), follows here with:

$$\frac{\partial E_k(t)}{\partial w_{ij}} = [S_k(t)-0.5]p_{ij}^k(t-\Delta t) + p_{ij}^k(t)[S_k(t-\Delta t)-0.5].$$ (11)

Application of the synaptic plasticity law (11) for a few time steps, was very often found sufficient to drive the neural network away of a steady state forth into an unsteady regime persisting permanently after the synaptic plasticity was stopped. Other simple cost functions were found to be efficient in destabilizing steady activities, for instance:

$$E_k(t) = \tfrac{1}{2}[(S_k(t)S_k(t-\Delta t)]^2 + \tfrac{1}{2}[S_k(t)-1]^2.$$ (12)

The construction of sets of synaptic couplings that implement specific dynamic behaviours of the neuron activities in a network, stands as a fundamental operation in neural modeling, and bears biological significance. The minimization of a cost function in order to realize that construction, although practically efficient, may appear more difficult to justify on biological grounds. However, at the least, this methodology can be interpreted as a constructive proof that demonstrates the existence of a set of synaptic couplings that do realize a specific dynamic evolution of the activities. The possibility of the existence of such sets of synapses is in itself an important finding. The means of elaboration of these sets in actual biological neural networks can be addressed as a separate issue. It may happen that synaptic values that are now theoretically found through a minimization process (that appears global and finalist), can identically be reached through another, strictly local, process. Classical mechanics provides an example of such a situation, where the motion of a mobile can either be deduced from the least-action principle (finalist and global minimization), or from Newton's laws (strictly local).

4.3. Structure in chaos

In a chaotic regime, due to the presence of sensitive dependence on initial conditions, the neuron activities $S_i(t)$ as such, do not appear as good candidates to support pieces of information. Indeed, the precise macroscopic values of these activities, critically depend upon uncontrollable microscopic fluctuations. In contrast, the chaotic attractors of figures 2 to 4 exhibit remarkable structural stability. The question arises whether they could constitute a substrate for mental images. Chaotic time evolutions may embed some internal coherence. For instance, let us consider the phase plane attractor of figure 3a. This attractor is chaotic, and it is easy to check that two trajectories, initiated very close to one another on the attractor, exponentially diverge to become completely different after a short time. However, for an arbitrary trajectory of $S_i(t)$ on the attractor, one can compute the time autocorrelation function $C_{ii}(\tau)$ (normalized and centered), or correlation coefficient defined as the normalized covariance:

$$C_{ii}(\tau) = \frac{\langle [S_i(t)-\langle S_i \rangle][S_i(t-\tau)-\langle S_i \rangle] \rangle}{\langle [S_i(t)-\langle S_i \rangle]^2 \rangle}$$ (13)

The notation $\langle x(t) \rangle$ denotes the time average of signal $x(t)$. This autocorrelation function $C_{ii}(\tau)$ is plotted in figure 6. The first maximum peak after $\tau = 0$ occurs at $\tau_M = 23$, where the normalized autocorrelation is higher than 0.5. Moreover, the time decay of
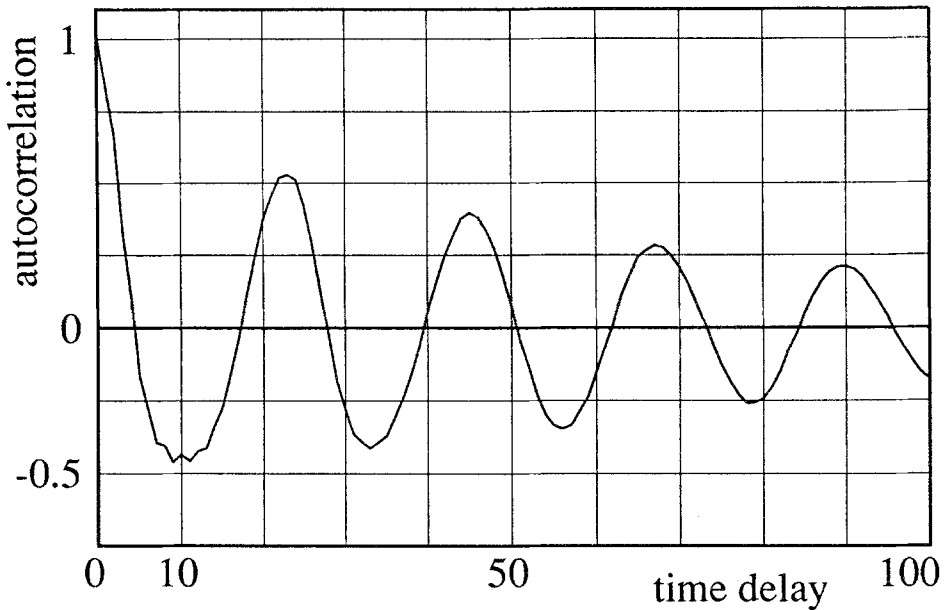
**Fig. 6.** Time autocorrelation function of an evolution $S_i(t)$ on the attractor of figure 3a.

$C_{ii}(\tau)$ is slow. I have verified that the long-time tail of the autocorrelation $C_{ii}(\tau)$, when delays above $\tau = 100$ are also considered, exhibits exponential decay with a time constant τ_{decay} around 85. These elements point to strong correlations that are preserved in signal $S_i(t)$ over long time scales, and this property holds for any trajectory of $S_i(t)$ on the attractor of figure 3a. A time delay of $\tau_M = 23$, or a correlation time of $\tau_{decay} = 85$, represent long intervals compared to the typical evolution time $\Delta t = 1$ of single neurons. In this small network with $N = 5$ neurons, these two intervals are even large times compared to the travelling time across the whole network, which is of the order of 5. In general, coherent oscillations will display periods of the order of a few Δt in unstructured networks, or of the order the travelling time across the network when a loop structure prevails among all the neurons. Beyond these limits, in some cases, specific sets of synaptic couplings exist, that allow to sustain coherent reverberation of activity over longer times, as for the case of figure 6.

5. MOTOR CONTROL BY THE CEREBELLAR CORTEX

I shall present in this section a neural network model of the cerebellar cortex that is able to learn and retrieve trajectories, or time sequences of activity patterns in a neural network.

The cerebellar cortex plays a role in the control of motor activity and coordination of movements. Its anatomy reveals the presence of five types of neurons, organized in a very regular layered structure, depicted in figure 7.

The cerebellar cortex receives two external input pathways, formed by the mossy fibres, and by the climbing fibres. In broad terms, the mossy fibres are known to carry information coming from the motor cortex and concerning the intention of the movement, as well as sensory information related to the physical environment of the movement. The climbing fibres carry an error signal controlling learning and adjustment of performance. In contrast, there is a single output pathway, formed by the axons of the Purkinje cells, that emit motor
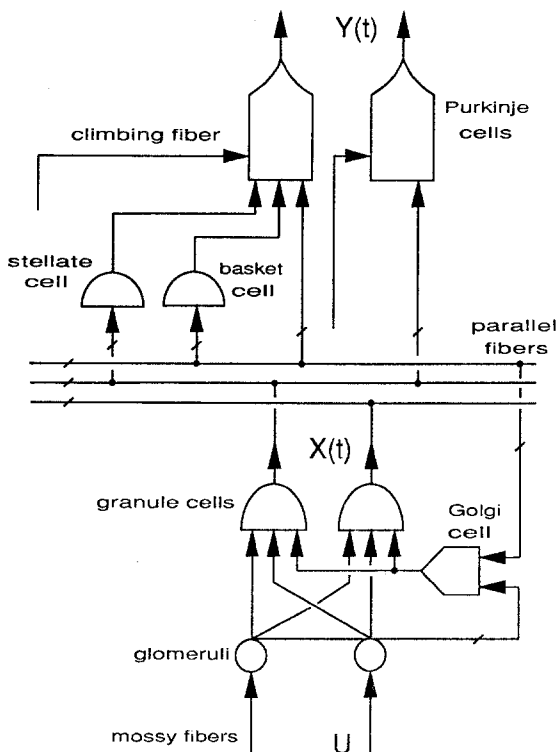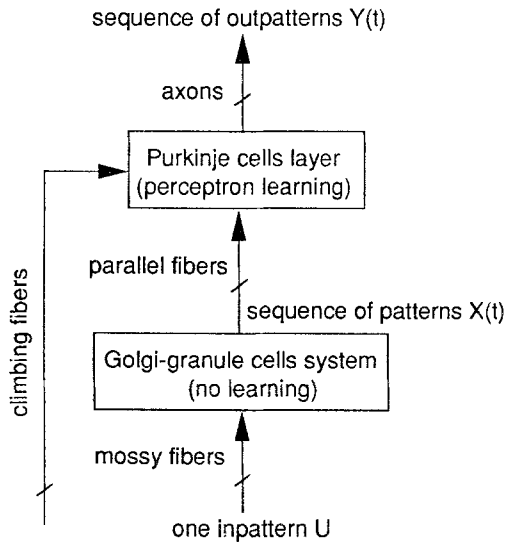
Fig. 7. Diagrammatic representation of the anatomy of the cerebellar cortex.

commands for motoneurons via cerebellar nuclei.

A classical theory (Marr, 1969; Albus, 1971) proposes a perception model of the cerebellar cortex that performs a one-to-one association of static patterns (one input pattern on the mossy fibres is associated to one output pattern on the Purkinje cells axons). There is a possibility to expand this scheme, in order to endow the model with dynamic association performances. This can be achieved by taking advantage of the feedback loop formed in the layered architecture by the Golgi cells (Chapeau-Blondeau, 1991). This feedback allows an autonomous dynamics of the system, which becomes able to generate, in response to a static input pattern, not just a static output pattern (as common layered neural networks), but a time sequence of different output patterns. This output sequence is in fact labelled by the current static input, and can be retrieved upon its presentation.

More specifically, a static input pattern **U** is applied on the mossy fibres. As a response, owing to the feedback loop via the Golgi cells, a sequence of patterns $X(t)$ evolving at each time step, is produced on the granule cells outputs. If it is admitted (based on experimental evidence) that there is no significant plasticity for the synapses of the Golgi-granule cells circuit, the sequence of $X(t)$ cannot be externally specified. It depends upon synaptic efficacies that cannot be controlled and that are assumed here to be fixed. The role of assigning specific values to the patterns of the sequence is devoted to the Purkinje cells layer. This layer acts as a perception, which associates to any fixed unspecified pattern **X**,

sequence of outpatterns Y(t)

axons

Purkinje cells layer
(perceptron learning)

parallel fibers

sequence of patterns X(t)

Golgi-granule cells system
(no learning)

climbing fibers

mossy fibers

one inpattern U

Fig. 8. Functional diagram of the model for the cerebellar cortex.

an output pattern **Y** that can be exactly specified through a classical perception learning supervised by the error signals carried by the climbing fibres. This specification aims at generating the motor commands that are appropriate to implement a given movement in a given environment (as coded by input **U**).

It is known that a one-layer perception is not able to realize any arbitrary set of pair associations. A limitation arises from the necessity of linear separability of the set. However, if a given set is not linearly separable, it is always possible, by increasing the dimensionality of the representation, to make it linearly separable. This possibility provides an interpretation to a unique feature of the cerebellar cortex, mainly the presence of the very prominent number of granule cells, involving a divergence of the order of 1 to 10,000 of the fibres at this level. In the framework of the model, the role of the huge number of granule cells is to ensure correct association in the perception learning, by increasing the dimensionality of the representation of the intermediate patterns **X**. Furthermore, I have shown that the additional degrees of freedom provided by the numerous granule cells, also allow to incorporate biological constraints on the signs of the synapses, without losing the possibility of perception associations (Chapeau-Blondeau, 1991).

The global structure of the neural network model that results for the cerebellar cortex is depicted in figure 8. Upon presentation of a static input pattern **U** that codes both the intention and the context of the movement, the neural network generates the sequence of output patterns **Y**(t) that has been learned as appropriate for the current motor control task.

6. CONCLUSION

In this paper, I have demonstrated the dynamic variability that can be achieved with neural networks, considered as nonlinear adaptive dynamic systems. Steady, oscillatory and chaotic regimes can be easily observed, and critically depend upon synaptic couplings.

Under certain conditions, when no specific synaptic constraints operate, unsteady evolutions tend to prevail. The different accessible dynamic regimes can form the substrate of various cognitive functions. Stable steady states are frequently relied upon to implement memory processes. Oscillatory evolutions, as in figure 1b, can play a role in the control of rhythmic activities. I have shown (Chapeau-Blondeau, 1992a) that the period of oscillation in an oscillatory regime can be continuously varied through the alteration of a synaptic efficacy or of an external neuron input, thus offering internally or externally regulated neural pacemakers. The control of transient trajectories from specified starting points in a neural network can also contribute to information processing for motor tasks. This ability has been illustrated here in a neural network model of the cerebellar cortex.

Concerning chaotic regimes in neural systems, it seems that their existence is now established, both from experimental evidence (Babloyantz, 1985, 1986; Mpitsos, 1988; Cerf, 1990; Doyon, 1992) and theoretical arguments (Harth, 1983; Guevara, 1983; Chapeau-Blondeau, 1992b, 1993). However, they role for information processing in the nervous system is far from being fully understood, although some propositions are already debated (Skarda, 1987; Yao, 1990; Hansel, 1992). A challenge ahead is, beyond the analysis of chaotic dynamics, to uncover how to control and exploit them (Hayes, 1993; Shinbrot, 1993).

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