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STOCHASTIC RESONANCE IN THE INFORMATION CAPACITY OF A NONLINEAR DYNAMIC SYSTEM

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We consider a nonlinear bistable dynamic system governed by the quartic potential with two-state quantization at the output — the earliest system to have revealed the phenomenon of periodic stochastic resonance. We devise a scheme in which this system is used to transmit a broadband aperiodic informative signal. With this scheme, we demonstrate that the system can be operated as a memoryless symmetric binary channel, and we develop the characterization of the transmission up to the evaluation of the input–output information capacity of this channel. We show that a regime exists where the information capacity can be increased by means of noise addition, a property we interpret as a form of aperiodic stochastic resonance. In addition, we demonstrate that a positive input–output gain in the efficacy of the signal recovery can be obtained with the stochastic resonator, compared to the recovery that would directly operate on the input signal-plus-noise mixture.

1. Introduction

Stochastic resonance (SR) is a phenomenon of noise-assisted signal transmission taking place in certain nonlinear systems [Moss *et al.*, 1994; Wiesenfeld & Moss, 1995; Anishchenko *et al.*, 1992, 1994; Pantazelou *et al.*, 1995; Chapeau-Blondeau & Godivier 1996]. Since its first observation some fifteen years ago [Benzi *et al.*, 1981; Nicolis, 1982], this phenomenon has mainly been studied and exploited to improve the transmission of a periodic signal, usually a sinusoid [Moss *et al.*, 1994; Wiesenfeld & Moss, 1995]. Such a situation, which has received considerable attention, essentially bears a conceptual significance. It shows how the transmission of a "coherent" signal of known form can be improved by noise addition, revealing a possibility of turning the noise from a nuisance into a benefit.

In order to improve the transmission of actual useful information via SR, one needs to substitute the periodic signal with a broadband aperiodic signal. The emphasis has only recently come to this question of aperiodic SR [Collins *et al.*, 1995a, 1995b; Kiss, 1996]. New measures were proposed to quantify the effect, based on a signal-to-noise ratio in the frequency domain [Kiss, 1996], or on cross-correlation measures [Collins *et al.*, 1995a, 1995b, 1996]. With broadband informative signals, particularly appropriate measures of SR are provided by information-theoretic quantities. An input–output transinformation has been considered in different reports. Such a measure is defined and studied in a realization of SR in a neuron that encodes an analog aperiodic input into an output spike train, with an experimental preparation in [Levin & Miller, 1996] and a theoretical model in [Heneghan *et al.*, 1996]. Bulsara and Zador [1996] also use an input–output transinformation, to quantify aperiodic SR in a simple threshold nonlinearity, and they establish a connection with the transcoding of an analog input into an output spike train by a neuron. For a comparable threshold nonlinearity, Chapeau-Blondeau [1997] develops the characterization up to the evaluation of the information capacity of the system, defined as the maximal achievable input–output transinformation occuring when the statistics of the aperiodic input signal is matched to the noise. Neiman *et al.* [1996] also employ information-theoretic measures but for SR with a periodic input signal.

In the present report we shall return to the earliest system to have revealed (periodic) SR, i.e. a nonlinear bistable dynamic system governed by the quartic potential, with two-state quantization at the output. A pioneering study on aperiodic SR [Hu *et al.*, 1992] has considered this system for the transmission of a binary input sequence of fixed length. Hu *et al.* [1992] use a method relying on repeated measurements of the output to average out the noise in order to correctly recover the input message, and they characterize SR through the portion of correctly received data. In principle, the method employed in [Hu *et al.*, 1992] requires a number of measurements tending to infinity, for an exact recovery guaranteed for every input binary datum. Here we devise a different scheme using this nonlinear bistable dynamic system for the transmission of a broadband aperiodic informative signal, and in which we demonstrate that the system can be operated as a memoryless symmetric binary channel. We then develop the characterization of the transmission up to the evaluation of the input–output information capacity of the channel. We show that this capacity can be increased by means of noise addition, a property we interpret as a form of SR. In this information-theoretic framework, with the knowledge of the capacity of the system and based on Shannon's second theorem [Cover & Thomas, 1991], we are sure of the possibility of a coding strategy that will have the minimal redundancy afforded by the optimal noise level at the maximum SR, and that will allow the recovery of the input message with an arbitrarily small probability of error, with only a single measurement per transmitted binary datum as used by our scheme.

2. The Nonlinear Information Channel

We consider $s(t)$ to be a coherent signal carrying useful information, and $\eta(t)$ is a stationary random noise. These two signals are applied to a nonlinear dynamic system whose internal state $x(t)$ evolves according to:

$$\tau_a \dot{x}(t) = x(t) - \frac{x^3(t)}{X_b^2} + s(t) + \eta(t)\,, \qquad (1)$$

with the parameters $\tau_a > 0$ and $X_b > 0$. We are in the presence of a forced (by $s(t) + \eta(t)$) bistable dynamic system, whose free relaxation $\tau_a \dot{x} = -dU/dx$ is governed by the so-called quartic potential $U(x) = -x^2/2 + x^4/(4X_b^2)$. This system has two stable equilibrium states $x = \pm X_b$ corresponding to the two minima of the potential $U(x = \pm X_b) = -X_b^2/4$.

The internal state $x(t)$ determines the output $y(t)$ of our system, through a "two-state quantization" [Moss *et al.*, 1994] expressed by

$$y(t) = \mathrm{sign}[x(t)]\,. \qquad (2)$$

The dynamics based on Eq. (1) has been the first type of system to reveal the phenomenon of SR. In this respect, this system has been the subject of numerous studies, in which SR is observed and analyzed mainly in the transmission of a periodic coherent signal $s(t)$, usually a sinusoid. The measure of the effect is usually a signal-to-noise ratio, obtained in the frequency domain, and which quantifies, at the frequency of the periodic input, the proportion of the output signal related to the periodic input among the part due to the noise [Moss *et al.*, 1994].

We shall show that another type of SR can be obtained with the system of Eqs. (1) and (2), in the transmission of a broadband aperiodic signal $s(t)$, and with the maximal achievable input–output transinformation as a measure of the benefit of adding noise.

The continuous-time coherent signal $s(t)$ will consist of an aperiodic "telegraph" signal of the form

$$s(t) = A \sum_{j=-\infty}^{+\infty} S_j \Gamma(t - jT)\,, \qquad (3)$$

where $A > 0$ is a constant amplitude, and $\Gamma(t)$ is a rectangular pulse of duration T and amplitude unity, i.e. $\Gamma(t) = 1$ for $t \in [0, T[$ and $\Gamma(t) = 0$ elsewhere. We have introduced a sequence of binary symbols $S_j = \pm 1$, j integer, where the S_j's are identically distributed and independent random variables.

In the absence of the noise $\eta(t)$, the minimal value of the coherent amplitude A that destroys bistability in Eq. (1) occurs when $x - x^3/X_b^2 + A = 0$

ceases to have three real roots, and it comes out as $A = 2X_b/\sqrt{27} \approx 0.38X_b$ [Moss *et al.*, 1994]. For $A < 0.38X_b$, the coherent input $s(t)$ alone is too small to induce transitions in the output $y(t)$. Addition of the noise $\eta(t)$ will then offer the possibility of inducing transitions in the output $y(t)$. We are interested in recovering the successive input symbols $S_j = \pm 1$, from the observation of the output signal $y(t)$. The input symbols S_j are emitted at a rate of one symbol every T, each new symbol starting at time $t_j = jT$ and lasting over a duration T. We introduce a scheme in which a symbol is decoded at the output, from a single observation of the signal $y(t)$, under the form $Y_j = y(t_j + T_{\text{trans}})$. The successive readings of the output signal $y(t)$ are done at the same rate of one reading every T, and they occur at times $t'_j = t_j + T_{\text{trans}}$. The phase T_{trans} is a fixed transmission time, appropriate to perform efficient readings of the output, and that we take to be $T_{\text{trans}} = T - \Delta t$, where $\Delta t > 0$ is the smallest possible time allowed by the resolution of the measurements. Such a choice of T_{trans} maximizes the time allowed for the state $x(t)$ to evolve to the vicinity of the stable state $x = \pm X_b$ corresponding to the current input symbol $S_j = \pm 1$ applied at t_j for a duration T, and $Y_j = y(t_j + T - \Delta t)$ will best reproduce S_j, just before a new input symbol S_{j+1} is emitted at time $t_{j+1} = t_j + T$.

We assume that, in the communication process, the interval T at which input symbols are emitted is known at the output where $y(t)$ is decoded. For the moment we shall further assume that the times $t'_j = t_j + T_{\text{trans}} = jT + T - \Delta t$ appropriate to perform efficient readings of the output $y(t)$, are also known at the output — a situation of external or remote synchronization for the output readings. Later, we shall consider that the times t'_j are not *a priori* known at the output, and we shall indicate a scheme whereby, with the sole observation of the output signal $y(t)$, an estimation can be done of the times $t'_j = t'_0 + jT$ efficient for the output readings — a situation of purely local synchronization at the output. It will amount to a method for estimating the unknown fixed phase t'_0 to place the successive readings of $y(t)$ separated by the known interval T.

Now, our system with input symbols $S_j = \pm 1$ and output readings $Y_j = \pm 1$, can be viewed as an information channel transmitting binary data. We shall show that this transmission of information can be assisted by noise addition, a property we interpret as a SR effect.

3. Noise-Enhanced Information Capacity

To illustrate this possibility of a noise-assisted transmission of information, we shall consider the case where the input noise $\eta(t)$ is a white noise with an even probability density function; this allows us to view our system as a *symmetric* binary channel. Further, we consider that the successive input symbols $S_j = \pm 1$ are applied for a time $T > \tau_a$ sufficiently larger than the interwell and the intrawell relaxation times of the dynamics of Eq. (1); and this allows us to view our system as a *memoryless* symmetric binary channel. We shall later produce a verification of the memoryless character of this symmetric binary channel.

For independent S_j's, with the input–output transmission probabilities

$$\Pr\{Y_j = 1 | S_j = 1\} = \Pr\{Y_j = -1 | S_j = -1\} = p\,, \tag{4}$$

it is possible to compute the input–output transinformation [Cover & Thomas, 1991]. For a memoryless symmetric binary channel, this transinformation is maximized with equiprobable values ± 1 for the input symbols, which yields the input–output information capacity C of the system under the form:

$$C = 1 + p\,\log_2(p) + (1-p)\,\log_2(1-p)\,. \tag{5}$$

We have realized a simulation of the present system, with a Euler discretization of Eq. (1) at a small time step $\Delta t = 10^{-2}$. The emission interval was kept fixed at $T = 500\Delta t$, and we took $X_b = 1$ as the unit of amplitude. Different values of the time constant τ_a with $\Delta t \ll \tau_a < T$, and of the signal amplitude A were tested. We chose $\eta(t)$ a zero-mean Gaussian noise. With our decoding scheme which samples the signal amplitude, the important characteristics of the noise is its rms amplitude σ_η, which remains finite for any realizable noise. If the power density D of the noise is introduced, then a physically realizable noise will have a short nonvanishing correlation time τ_c verifying $\sigma_\eta^2 \sim D/\tau_c$. When τ_c is much smaller than the time parameters τ_a and T, the white noise hypothesis can be expected to provide a correct description. In our discrete-time simulation, the successive values of $\eta(t)$ are generated as independent variables, making τ_c also no larger than Δt, and with the rms

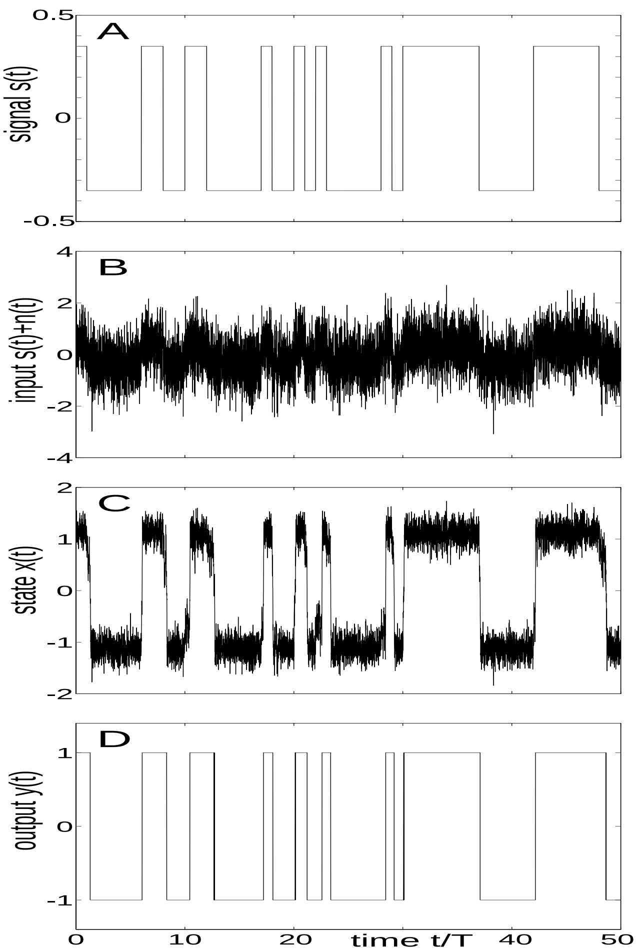

Fig. 1. Time evolution of the signals for the system of Eqs. (1) and (2) with $X_b = 1$ and $\tau_a = 10\Delta t$: Panel A shows the informative input $s(t)$ from Eq. (3) with $A = 0.35$ and the successive input symbols $S_j = \pm 1$ with equal probabilities occuring at times $t_j = jT$ with $T = 500\Delta t$. Panel B shows the input signal-plus-noise mixture $s(t) + \eta(t)$ with $\eta(t)$ a zero-mean Gaussian white noise of rms amplitude $\sigma_\eta = 0.65$. Panel C is the corresponding internal state $x(t)$ from Eq. (1). Panel D is the resulting output $y(t)$ from Eq. (2), which will then be read at times $t'_j = t_j + T_{\rm trans}$ to yield the output binary sequence Y_j.

amplitude σ_η as we want to impose it to the system. In such conditions, Fig. 1 shows a typical time evolution of the different signals.

We have performed an estimation of the probability p of Eq. (4) as the frequency of correct transmission over a large number of successive emissions of input symbols $S_j = \pm 1$ occuring as independent equiprobable random variables. The channel capacity C was then deduced according to Eq. (5). The variations of both p and C with the input noise level were examined.

Figures 2 and 3 represent the variations of the transmission probability p and of the information capacity C, as a function of the input noise rms amplitude σ_η, for different values of τ_a and A. We observe in Figs. 2 and 3 nonmonotonic evolutions of both p and C, with the noise level σ_η. When A is subliminal ($A < 0.38$ when $X_b = 1$), the signal $s(t)$ alone is unable to induce output transitions, and both p and C are strictly zero in the absence of the noise. Addition of the noise then allows an actual transmission of information through the system, with a maximum efficacy for a sufficient, optimal, noise level. We interpret this effect as a form of SR.

The transmission of the input sequence strongly relies upon the interwell dynamics of Eq. (1). The interwell transition times of Eq. (1) are not simply dependent upon τ_a, but also depend on the input amplitude. At a fixed τ_a, the interwell transition times increase as the input amplitude is reduced toward $0.38X_b$, and they can reach values well above τ_a. Accordingly in Fig. 2, at a fixed τ_a, the efficacy of the noise-assisted transmission increases as the signal amplitude A is increased (while remaining subliminal). At a fixed A subliminal, Fig. 3 shows that the efficacy of the noise-assisted transmission increases as the time constant τ_a is reduced relative to T, making the system more responsive. As visible in Figs. 2 and 3, parameter values are accessible that yield, at the resonance, an almost perfect transmission of information with both p and C very close to 1. This outcome is favored when τ_a is very small relative to T, but this is associated in Eq. (1) to values of $\dot{x}(t)$ that may become very large as τ_a goes to zero, making the system unstable. A finite nonzero value of τ_a has to be kept in order to keep $x(t)$ bounded. At any finite nonzero value of τ_a, the transmission probability p cannot be perfectly 1, in principle, because unexpectedly large deviations of the noise can always occur, with finite nonzero probabilities; yet these probabilities are in practice very, very small, leading, as shown in Figs. 2 and 3, to values of p and C coming very, very close to 1.

We can here verify the important assumption of a memoryless channel that allowed us to interpret C of Eq. (5) as the information capacity of the channel. We have computed the autocorrelation function $R_{YY}(k) = E(Y_j Y_{j+k})$ of the sequence of output symbols Y_j in response to an input sequence of independent symbols S_j. As visible in Fig. 4 representing $R_{YY}(k)$, there is practically no correlation between the successive output symbols

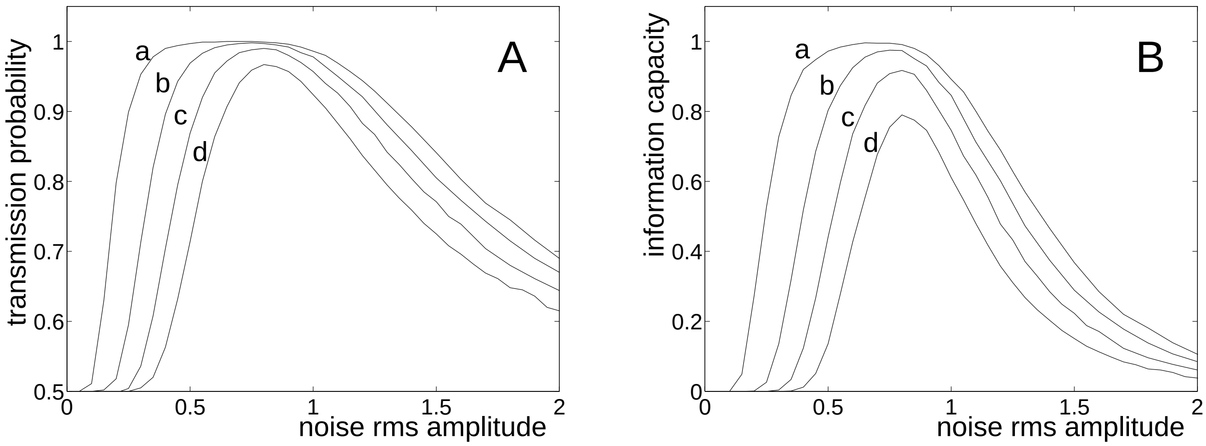

Fig. 2. Influence of the signal amplitude A: Probability of correct transmission p from Eq. (4) (panel A), and information capacity C from Eq. (5) (panel B), as a function of the rms amplitude σ_η of the input white noise $\eta(t)$ chosen to be zero-mean Gaussian. The time parameters are $\tau_a = 10\Delta t$ and $T = 500\Delta t$, and (a) $A = 0.35$, (b) $A = 0.3$, (c) $A = 0.25$, (d) $A = 0.2$.

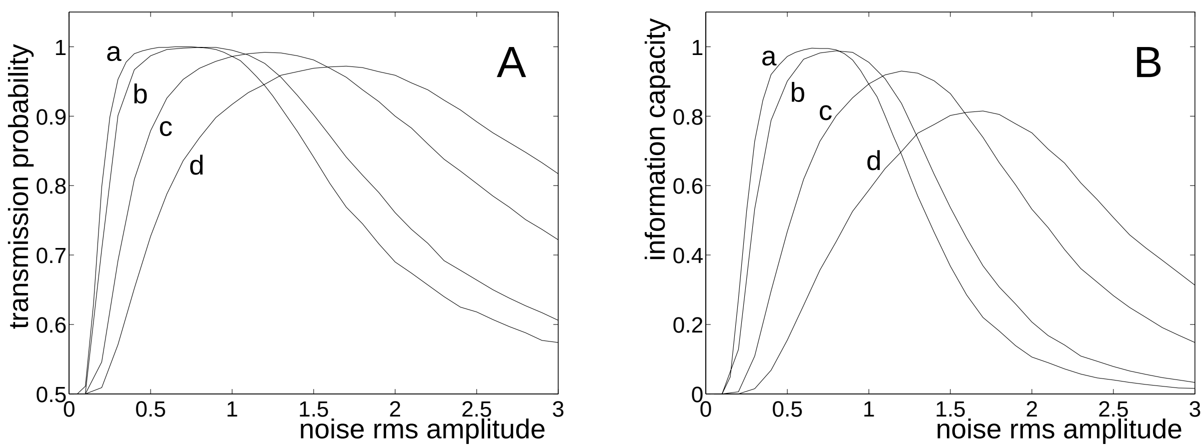

Fig. 3. Influence of the time constant τ_a: Probability of correct transmission p from Eq. (4) (panel A), and information capacity C from Eq. (5) (panel B), as a function of the rms amplitude σ_η of the input white noise $\eta(t)$ chosen to be zero-mean Gaussian. The signal amplitude is $A = 0.35$, and the time parameters are $T = 500\Delta t$ and (a) $\tau_a = 10\Delta t$, (b) $\tau_a = 12\Delta t$, (c) $\tau_a = 20\Delta t$, (d) $\tau_a = 30\Delta t$.

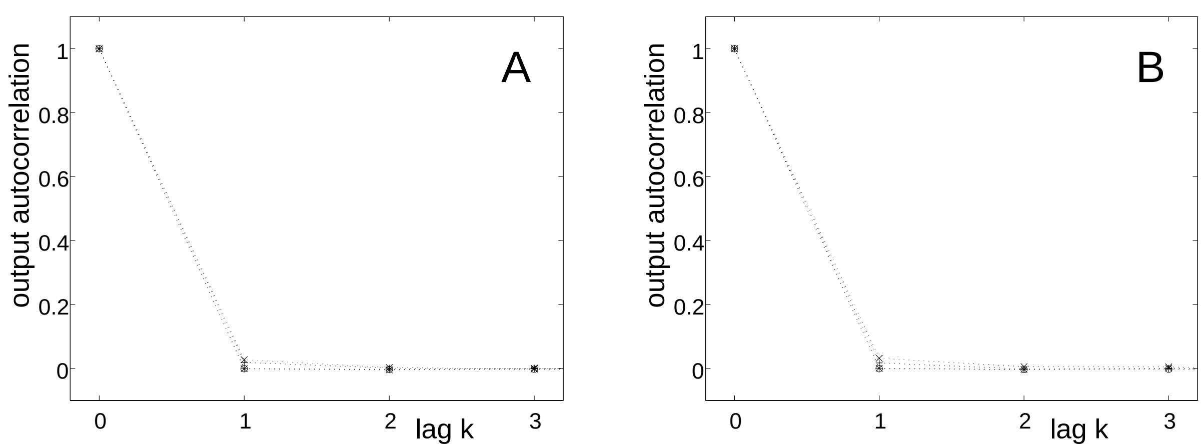

Fig. 4. Autocorrelation function $R_{YY}(k) = E(Y_j Y_{j+k})$ of the sequence of output symbols Y_j, as a function of the lag k, with $T = 500\Delta t$: Panel A is at fixed $\tau_a = 10\Delta t$, for ($\circ$) $A = 0.35$, ($*$) $A = 0.3$, ($+$) $A = 0.25$, ($\times$) $A = 0.2$, in each case at the location of the resonance of Fig. 2. Panel B is at fixed $A = 0.35$, for ($\circ$) $\tau_a = 10\Delta t$, ($*$) $\tau_a = 12\Delta t$, ($+$) $\tau_a = 20\Delta t$, ($\times$) $\tau_a = 30\Delta t$, in each case at the location of the resonance of Fig. 3.

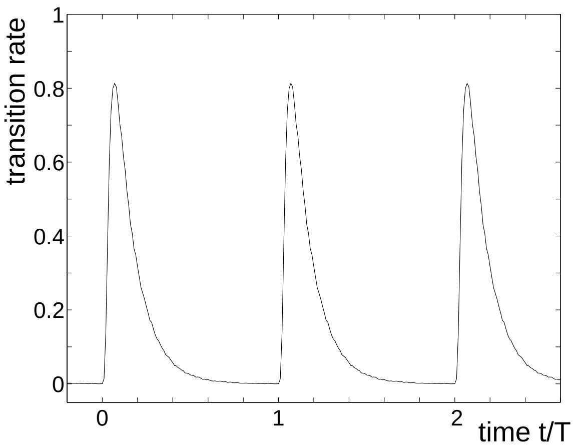

Fig. 5. Output transition rate $\lambda(t)$, estimated as the probability of a transition of $y(t)$ in the small interval $[t, t + dt[$, in the same conditions as in Fig. 1.

Y_j, which allows us to consider the channel as memoryless.

After having shown an effect of noise-enhanced capacity in the transmission, we return to the problem of estimating the constant phase T_{trans} with which to perform the output readings $y(jT + T_{\text{trans}})$ every T — the situation of local synchronization at the output. With the observation of the output signal $y(t)$ we have the possibility of estimating the transition rate $\lambda(t)$ at the output, where $\lambda(t)dt$ represents the probability of a transition of $y(t)$ in the small time interval $[t, t + dt[$. An estimation of $\lambda(t)$ is shown in Fig. 5 with the same conditions as in Fig. 1. The successive input symbols $S_j = \pm 1$ are emitted at times $t_j = jT$, but these times are ignored by the receiver. The output transition rate $\lambda(t)$ displays the period T, and $\lambda(t)$ is minimum when $y(t)$ is "maximally stabilized" in response to the current input symbol. This minimum identifies for the receiver at the output, the time $T_{\text{trans}} = T - \Delta t$ appropriate for the readings, as visible in Fig. 5. Thus, the local determination of the phase of the readings at the output, amounts to estimating the transition rate $\lambda(t)$ over one period T, say for t in $[0, T[$, identify in this period the time t_0 at which $\lambda(t)$ is minimum, and then perform the output readings at times $t_0 + j'T$. The estimation of $\lambda(t)$ for t in $[0, T[$, can be performed over a single realization of the output signal $y(t)$, via sample averages on N data points $y(t + nT)$ with the integer $n = 1$ to N.

4. Input–Output Gain in the Information Capacity

It is possible to bypass the stochastic resonator implemented by Eq. (1) and envisage the detection of the input symbols S_j directly from the input signal-plus-noise mixture $s(t)+\eta(t)$. In this condition, the output signal $y(t)$ to which the previous decoding procedure is applied is simply

$$y(t) = \text{sign}[s(t) + \eta(t)] \,, \tag{6}$$

in place of Eq. (2). We shall call this process linear decoding.

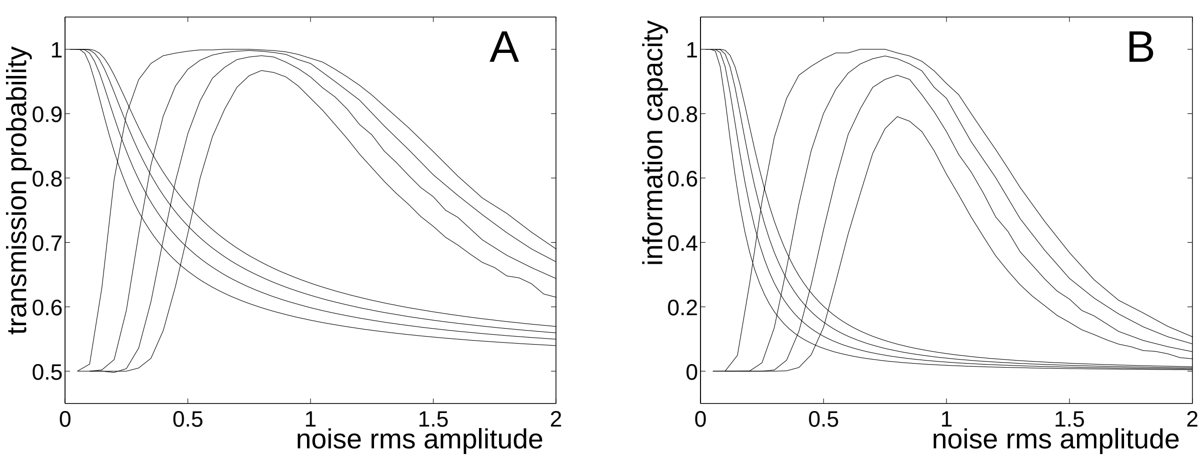

Fig. 6. Comparison of the linear and nonlinear decodings, with a zero-mean Gaussian input white noise $\eta(t)$ of rms amplitude σ_η in abscissa. Panel A shows the probability of correct transmission, and panel B the information capacity. The monotonically decaying curves are for the linear decoding, and the resonant curves for the nonlinear decoding with the stochastic resonator at $X_b = 1$, $T = 500\Delta t$ and $\tau_a = 10\Delta t$. The curves are parameterized, from the topmost to the lowest, by $A = 0.35$, 0.3, 0.25 and 0.2.

In such a case, the probability p of Eq. (4) reduces to

$$p_{\text{lin}} = \Pr\{A + \eta > 0\} = \Pr\{-A + \eta < 0\}, \quad (7)$$

which is analytically expressable under the form

$$p_{\text{lin}} = F_\eta(A), \quad (8)$$

with the statistical distribution function $F_\eta(u) = \Pr\{\eta(t) < u\}$ of the noise $\eta(t)$.

The probability p_{lin} of Eq. (8) used in Eq. (5) yields the information capacity C_{lin} in the linear decoding. This capacity C_{lin} can then be studied as a function of the input noise level, and also compared to the nonlinear capacity C in the presence of the stochastic resonator. For illustration, we chose again the case where the input white noise $\eta(t)$ is zero-mean Gaussian, with the rms amplitude σ_η, and

$$F_\eta(u) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{u}{\sqrt{2}\,\sigma_\eta}\right)\right], \quad (9)$$

with the error function $\operatorname{erf}(u) = 2\int_0^u \exp(-u'^2)du'/\sqrt{\pi}$. Figure 6 then compares the transmission probabilities and the information capacities for the linear and nonlinear decodings.

As visible in Fig. 6, when the noise is strictly zero, the linear decoding is perfect and completely dominates the nonlinear decoding which is inoperative without the noise. But as the noise rises above zero, the performance of the linear decoding monotonically decays, whereas that of the nonlinear decoding starts to rise. Rapidly, the nonlinear decoding outperforms the linear one and then permanently remains superior in efficacy. These evolutions of Fig. 6 demonstrate that, as soon as there are low levels of the noise, there is a positive benefit in passing the input signal-plus-noise mixture through the stochastic resonator of Eq. (1), rather than working directly on this input mixture. Moreover, for a given range of the noise level, with the stochastic resonator, there is a further additional benefit in increasing the noise level so as to maximize the performance.

5. Conclusion

We have shown that the nonlinear dynamic system of Eq. (1) can serve for the transmission of a broadband information-carrying signal, with assistance from the noise through SR. In practical use, the effect can be operated as follows. The system of Eq. (1) with fixed X_b and τ_a, receives a coherent signal $s(t)$ of subliminal amplitude $A < 0.38X_b$. In the absence of the noise, the system is completely unable to transmit any information. For a given $A < 0.38X_b$, an optimal nonzero noise rms amplitude exists that maximizes the information capacity of the system. At this optimal noise level, the synchronization scheme described in Sec. 3 allows a purely local decoding of the ouput, at the maximum efficacy of the transmission.

Now, with the strictly above-zero capacity afforded by the noise, and based on Shannon's second theorem [Cover & Thomas, 1991], we are sure of the possibility of a coding scheme that will allow the transmission of the input symbols S_j with an arbitrarily small probability of error. Moreover, this coding scheme will have the minimal redundancy afforded by the optimal noise level at the maximum of SR. For instance, in the conditions of Fig. 3(d), the maximum capacity at the resonance is around $0.8 = 4/5$ bit of information per transmitted binary symbol. Shannon's second theorem then guarantees the possibility of a coding scheme, whose effect will be to add an average redundancy of around 1 binary digit per 4 input symbols, in order to reduce the entropy of the input just below the channel capacity, which will be enough to overcome the noise and achieve a vanishing probability of error. Such conclusions are made possible here because we were able to define a communication channel for which we explicitly compute the information capacity. This contrasts the present results with the method in [Hu *et al.*, 1992], which uses multiple correlated output readings to reduce the noise through averaging, what requires an infinite number of readings to reach a vanishing probability of error in the recovery of each transmitted symbol. With a finite sampling rate, an infinite number of readings per transmitted symbol translates into a vanishing rate of information transfer in [Hu *et al.*, 1992], while with our transmission scheme the achievable rate is known to be the capacity of the channel. Also, in [Hu *et al.*, 1992], the important question of the synchronization of the ouput readings is not addressed, and the method ceases to be applicable if the times of emission of the input symbols are not known at the output.

In addition to showing this possibility of a noise-enhanced capacity, we have shown here the positive gain in the capacity that can be obtained in the presence of the stochastic resonator, compared to the situation where the resonator would be absent. This issue is similar to the problem of obtaining a signal-to-noise ratio larger at the output than at the input, in the context of periodic SR. It is known that severe constraints exist if one wants to obtain an input–output gain in the signal-to-noise ratio in periodic SR [Gong *et al.*, 1993; Dykman *et al.*, 1995; DeWeese & Bialek, 1995; Inchiosa & Bulsara, 1995]. We have shown here that an input–output gain in the measure of SR (the information capacity) can be obtained in aperiodic SR. This demonstrates a situation where there is an actual benefit in passing the input signal-plus-noise mixture through the stochastic resonator, rather than not passing it. We have to specify that we compare here the performance of the same type of signal recovery, with and without the stochastic resonator, as is also done in the studies on periodic SR interested in an input–output gain in the signal-to-noise ratio [Loerincz *et al.*, 1996; Chapeau-Blondeau & Godivier, 1997]. In each of these cases, a specific optimal linear filter can be conceived, which is not considered because usually it is assumed that the implementation of such a specific filter is not available. This is a reasonable assumption in systems having to operate with a pre-imposed "hardware", such as, for instance, neural systems, for which certain aspects were shown describable by bistable dynamic systems of the type of Eq. (1) [Bulsara *et al.*, 1991; Bulsara & Schieve, 1991; Longtin *et al.*, 1994], and for which the present extension of aperiodic SR may have relevance.

To summarize, the results of the present study (i) demonstrate a new form of aperiodic SR in a previously well-known periodic stochastic resonator, (ii) develop a characterization that goes up to the evaluation of the information capacity of the system, (iii) prove the possibility of an input–output gain in the efficacy of information transmission afforded by the stochastic resonator. These results contribute to the assessment of SR, and to the progression towards practical applications of SR for signal and information processing.

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