

# Extension of Chaos Anticontrol Applied to the Improvement of Switch-Mode Power Supply Electromagnetic Compatibility

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**Abstract**— We propose a feedback control method improving switch-mode power supplies electromagnetic compatibility (spectral peaks compliance). Indeed, the application of the classical method of chaos anticontrol to these systems leads the output voltage to have an exaggerated ripple or an undesirable spectrum, whereas these problems are solved with ours. To confirm the efficiency of this new and simple method, a comparison with the anticontrol method is included, together with a numerical example clearly showing the effect of this control.

**Index Terms**— anticontrol of chaos, Buck converter, power spectrum, output voltage ripple, A-switching diagram.

## I. INTRODUCTION

DC-DC converters are some of the most widely used circuits in power electronics. Switch-mode power supplies generate electromagnetic interference, generally consisting of the switching frequency and many harmonics. Electro-Magnetic Interference (EMI) emission is always a major concern for power electronic circuit designers. Such interference create significant electromagnetic compatibility (EMC) problems [4], particularly to comply with new regulations. Various methods of emission reduction, by modulating the PWM frequency of power supplies have been proposed, enabling to modify the noise emission spectrum [13]. The work of [4] presents the idea that chaos, a phenomenon which can occur naturally in switch-mode power supplies, might be used to improve their EMC by reducing spectral peaks: this leads to propose EMC compliance by chaos [5]. The process of chaos control is usually understood as a transition between chaos to order. After years of study of control (suppression) of chaos [17] [18] [19] [20], we are now interested in the study of transition between order to chaos. The task of purposely creating chaos, sometimes called chaotification or anticontrol of chaos, has attracted attention in recent years due to its great potential in nontraditional applications such as those found within the context of physical, chemical, mechanical, electrical, optical, biological, and medical systems. The application of the classical method [2] [3] of chaos anticontrol to switch-mode power supplies leads the output voltage to have an exaggerated output voltage ripple or an undesirable spectrum [10]. This is why we explored another direction, and we propose here a simple feedback control method which improves the EMC and eliminates these drawbacks. Also, simulation results of the output-feedback sinewave to distribute the power of the fundamental harmonics into frequency sidebands are presented.

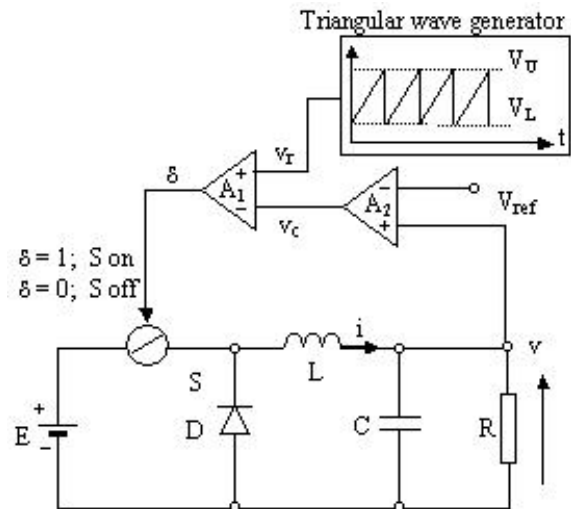


Fig. 1. Block diagram of the buck converter.

## II. STUDY OF THE BUCK CONVERTER

The elementary DC-DC converters such as Buck, Boost and Buck-Boost are a class of circuits which allows the conversion of energy from one DC level to another one with a minimum number of components. They are extensively used in power supplies for electronic circuits, in the control of the flow energy between DC-DC systems and in many industrial applications where there is a need of stabilizing a voltage to a desired value [16]. Figure 1 shows the block diagram of a Buck converter that uses a PWM voltage loop [6]. We assume throughout the paper that the components in the circuit are ideal. The circuit has two states: when the switch  $S$  is (either) closed or open. When  $S$  is closed, the input  $E$  provides energy to the load  $R$  as well as to the inductor  $L$ . During the interval when the switch  $S$  is open, the inductor current, which flows through the diode  $D$ , transfers some of its stored energy to the load  $R$ . Considering that the amplifier  $A_2$  has a gain  $a$ , we can write

$$v_c(t) = a(v(t) - V_{ref}). \quad (1)$$

$v_c(t)$  is applied to the inverting input of the comparator  $A_1$ ; the non-inverting input is fed by an independently generated ramp voltage  $v_r(t)$ , which periodically and linearly rises from a lower voltage  $V_L$  to an upper voltage  $V_U$  in a time  $T$ , and then instantaneously returns to  $V_L$ . The ramp voltage can be expressed as

$$v_r(t) = V_L + (V_U - V_L) \frac{t \bmod T}{T}. \quad (2)$$

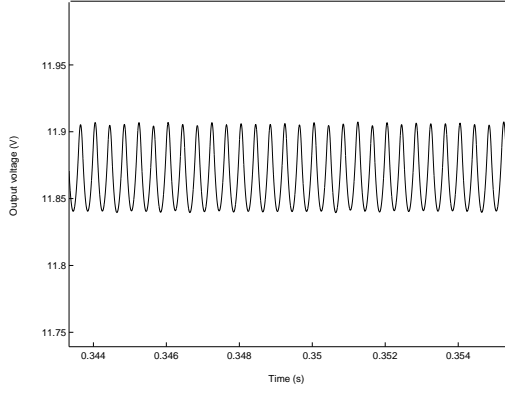


Fig. 2. Time-domain waveform of the output voltage  $v$  of the Buck converter (fundamental periodic operation).

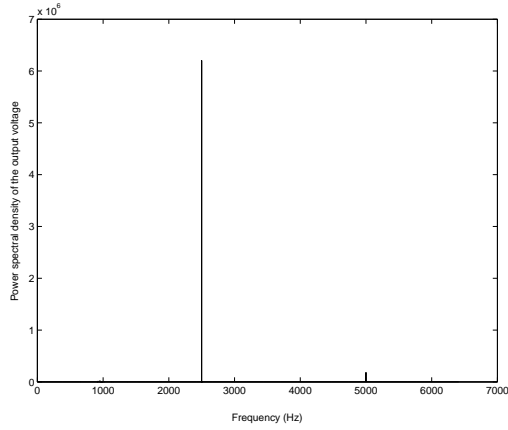


Fig. 3. Power spectrum of the output voltage  $v$  for the circuit (1).

Then, both  $v_c$  and  $v_r$  are applied to the comparator  $A_1$ , and every time the output difference changes its sign, the position of the switch  $S$  is commuted in such a way that  $S$  is open (and diode  $D$  conducts) when the control voltage exceeds the ramp voltage; otherwise  $S$  is closed (and  $D$  is blocked). The system is governed by two sets of linear differential equations pertaining to the on and off states of the controlled switch. The voltage  $v$  across the capacitor  $C$  and the inductance current  $i$  are taken as state variables [7]. The dynamic model can be written as

$$\frac{dv}{dt} = -\frac{1}{C}i(t) - \frac{1}{RC}v(t), \quad (3)$$

$$\frac{di}{dt} = -\frac{1}{L}v(t) + \frac{E}{L}\delta(t), \quad (4)$$

where  $E$  is a constant input voltage, and  $\delta$  the modulated signal which is zero when  $v_c(t) \geq v_r(t)$  and one when  $v_c(t) < v_r(t)$ . Elementary converters are second-order systems since they have two energy storage elements. Therefore, for any given switch condition, two first-order differential equations are required to describe the total behavior of the system. These circuits produce an average DC output voltage with periodical ripple (Fig. 2). Figure 3 presents the power spectrum of the output voltage, for the periodical wave-form, with a maximum of  $6.32 \times 10^6$ .

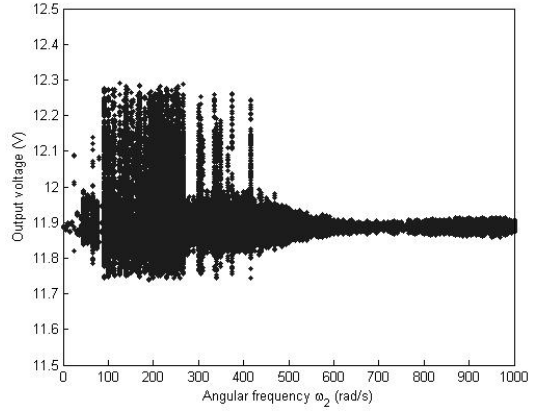


Fig. 4. Bifurcation diagram of the output voltage  $v$  obtained considering  $\omega_2$  as parameter.

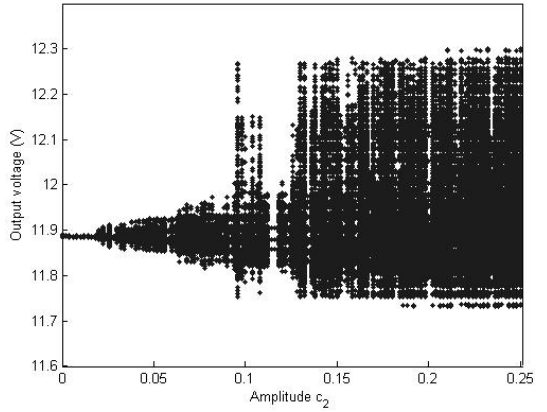


Fig. 5. Bifurcation diagram of the output voltage obtained considering  $c_2$  as parameter.

### III. GENERATING CHAOS

Chaotifying a system supposes to design a control input with a small amplitude such that the non-chaotic dynamical system becomes chaotic. The control law

$$v_c(t) = v_{c1}(t) + v_{c2}(t) \quad (5)$$

is a sum of the control law (1) and the new control law  $v_{c2}$ , which has the expression:

$$v_{c2} = c_2 \sin[\omega_2(v(t) - V_{ref})] \quad (6)$$

We choose as bifurcation parameter the amplitude of sinus  $c_2$  and the angular frequency  $\omega_2$ . Qualitatively, by variation of  $c_2$  and  $\omega_2$  we can observe the way the circuit changes its behavior from a fundamental steady state to a chaotic state. The A-switching bifurcation diagrams [14] for the Buck converter are computed when the amplitude  $c_2$  and the angular frequency  $\omega_2$  are varied in the range  $(0, 0.25)$ , respectively  $(0, 1000)$  with the following values of the remaining parameters:  $L = 20$  mH,  $C = 47$   $\mu$ F,  $R = 22$   $\Omega$ ,  $a = 8.4$ ,  $V_{ref} = 11.3$  V,  $V_L = 3.8$  V,  $V_U = 8.2$  V,  $T = 400$   $\mu$ s,  $E = 16$  V, and  $\omega_0 = 911$  rad/s. These diagrams can be obtained by looking at the dynamical system every moment when there is an equality between the generator ramp voltage  $v_r(t)$  and the control signal  $v_c(t)$ .

$$\begin{aligned} a(v(t) - V_{ref}) + c_2 \sin[\omega_2(v(t) - V_{ref})] = \\ = V_L + \frac{V_U - V_L}{T}(t \bmod T). \end{aligned} \quad (7)$$

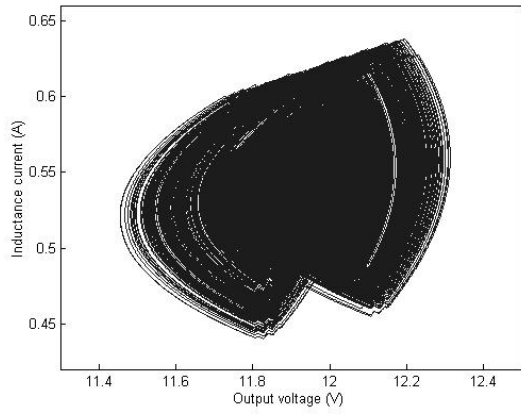


Fig. 6. Phase portrait with  $\omega_2=250$  rad/s and  $c_2=0.5$  V.

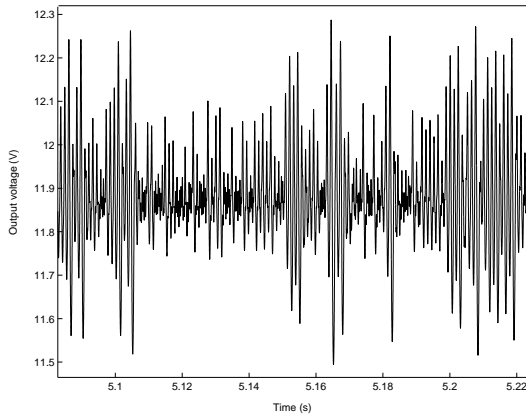


Fig. 7. Time-domain waveform with  $\omega_2=250$  rad/s and  $c_2=0.5$  V.

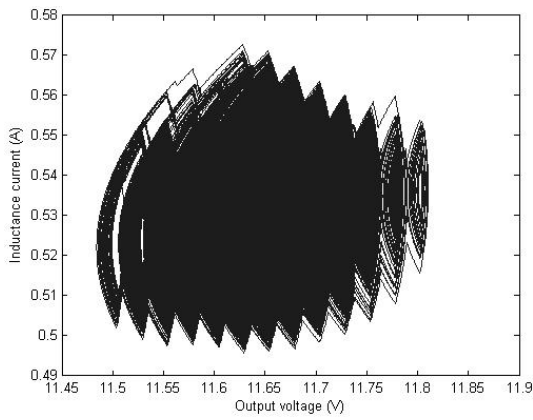


Fig. 8. Phase portrait with  $\omega_2=250$  rad/s and  $c_2=5$  V.

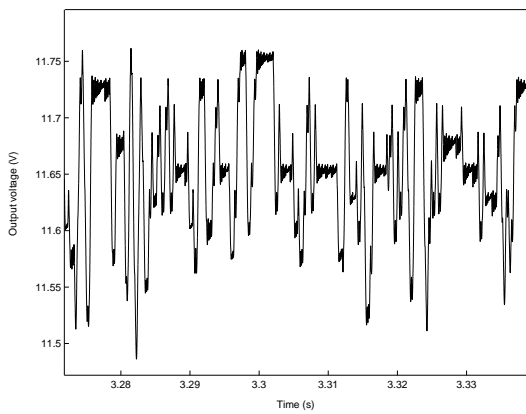


Fig. 9. Time-domain waveform with  $\omega_2=250$  rad/s and  $c_2=5$  V.

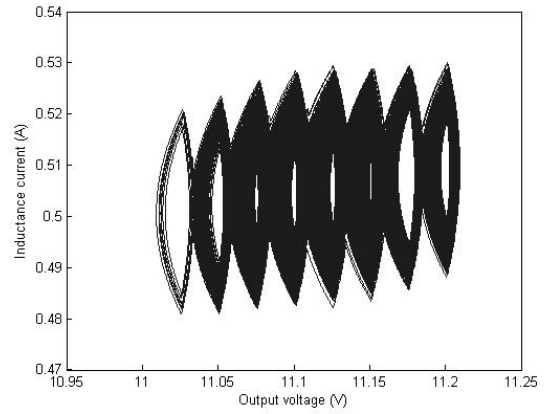


Fig. 10. Phase portrait with  $\omega_2=250$  rad/s and  $c_2=13$  V.

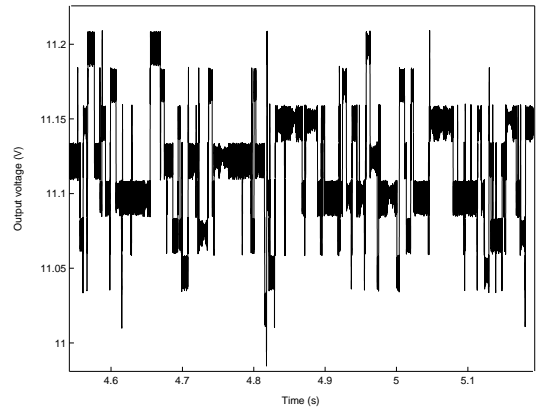


Fig. 11. Time-domain waveform with  $\omega_2=250$  rad/s and  $c_2=13$  V.

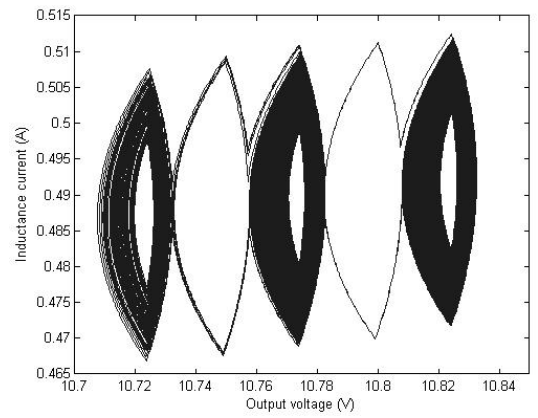


Fig. 12. Phase portrait with  $\omega_2=250$  rad/s and  $c_2=20$  V.

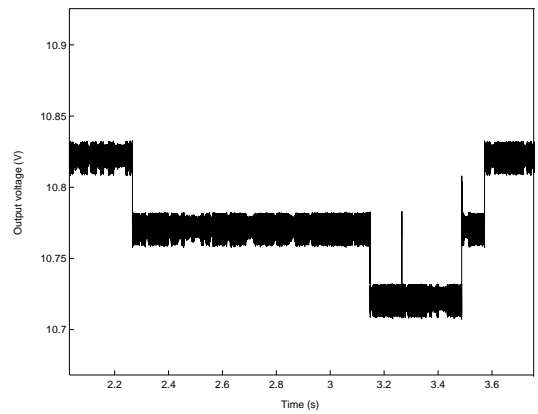


Fig. 13. Time-domain waveform  $\omega_2=250$  rad/s and  $c_2=20$  V.

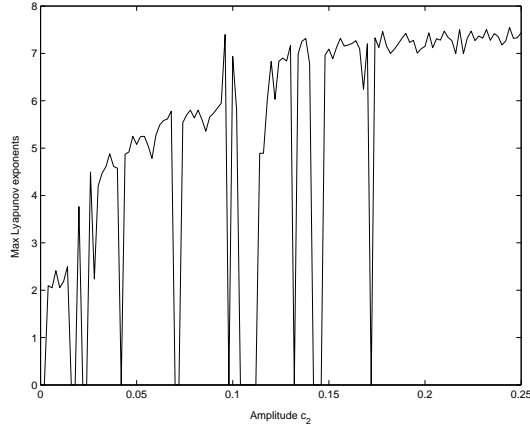


Fig. 14. The output voltage Lyapunov exponents and ripple for  $c_2$  variation ( $\omega_2=250$  rad/s).

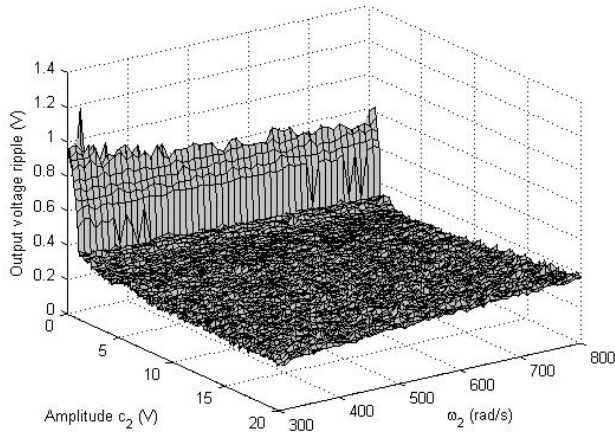


Fig. 15. The output voltage ripple.

The results of the computer simulations of the bifurcation diagrams of the output voltage obtained by considering A-switching moments are shown in Figures 4 and 5. The Buck converter with the nonlinear feedback controller presents a chaotic behavior for different angular frequencies  $\omega_2$  and amplitudes  $c_2$ , as reported in [3].

Figures 6, 8, 10 and 12 show different attractors of the controlled system, with  $c_2 = 0.5$  V, 5 V, 13 V and 20 V. Corresponding time-domain wave forms are presented in the figures 7, 9, 11 and 13.  $c_2$  clearly has an influence on the shape of the phase portrait attractor ( $\omega_2 = 250$  rad/s).

The anticontrol method designs a simple nonlinear feedback controller  $\sin[\omega_2(v(t) - V_{ref})]$  with an arbitrarily small amplitude ( $c_2$ ) to make the Lyapunov exponents of the controlled system strictly positive. The presence of chaos is proved by the positive Lyapunov exponents (Fig. 14), having  $c_2$  as parameter. The application of the classical method of anticontrol of chaos to switch-mode power supplies leads the output voltage to have a exaggerated output voltage ripple (Fig. 15) and in consequently an uninteresting spectrum [10] [15].

Unfortunately, this study with the classical method disables to choose the coefficient  $c_2$  and the angular pulsation

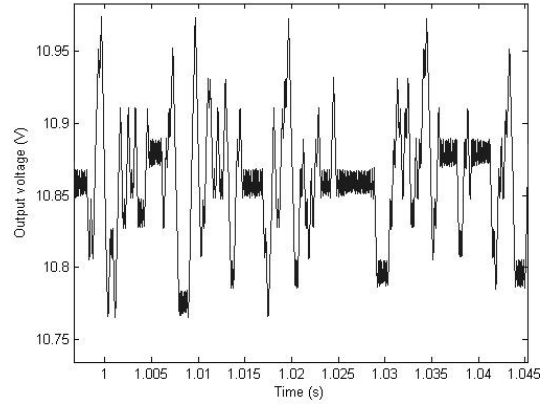


Fig. 16. The output voltage using the control law 5.

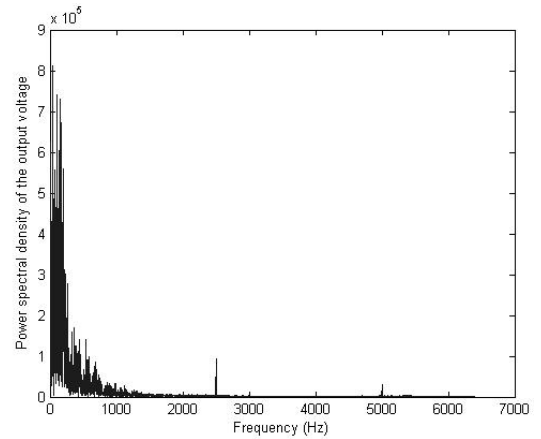


Fig. 17. Power spectrum of the output voltage.

$\omega_2$  because of the large ripple, which reduce the efficiency of the DC-DC converter. Even if many other frequencies appear on the frequency-domain, on the process of chaotifying the system, the voltage ripple influences the amplitude of the power spectral density. The best solution (200 mV) from the ripple point of view, is obtained for  $c_2=18$  V and  $\omega_2=300$  rad/s. Figure 16 and 17 present the time-domain wave-form and power spectral density of the output voltage, using these particular values for the parameters  $c_2$  and  $\omega_2$ . The improvement of power spectrum (Fig. 17) compared with figure 3 did not justify the application of this method (ripple performance). This is why we need to find another solution which substitutes the control law  $v_{c2}$ .

#### IV. BOUNDING THE OUTPUT VOLTAGE RIPPLE

Let us study another nonlinear controller, which has the form

$$v_{c3} = c_3 v(t) \sin(\omega_3 t). \quad (8)$$

So the new control law is:

$$v_c(t) = v_{c1}(t) + v_{c3}(t) \quad (9)$$

For this nonlinear controller, the higher switching frequency, much greater than the frequency of the triangular wave generator ( $1/T$ ) is the solution of reducing the ripple. Therefore, the amplitude and angular frequency of  $v_{c3}$  influence the number of switchings,  $S$  switches many times during  $T$  of the ramp (Fig. 18 and 19).

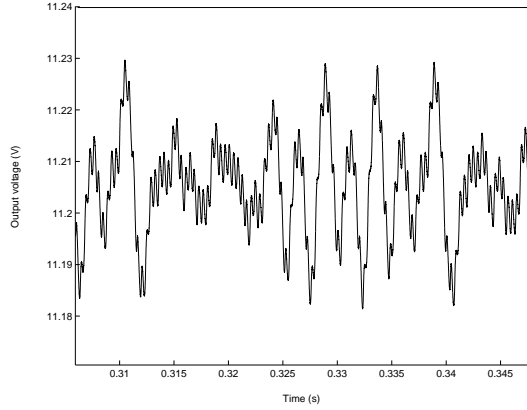


Fig. 18. Output voltage for the circuit with the control law (9) and (1).

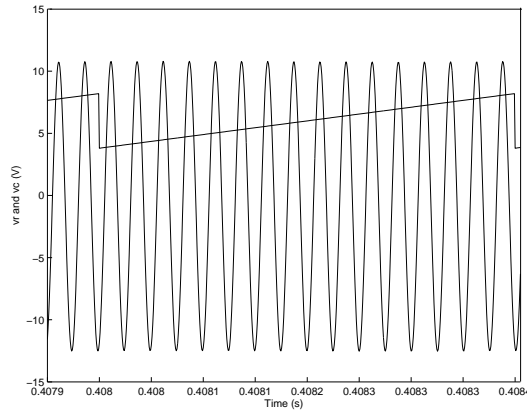


Fig. 19. Triangular wave and control signal for the circuit with the control law (9) and (1).

Because of these multiple switching, the output voltage does not have enough time to rise or to decrease too much. Figure 18 shows the variation of the output voltage: it is 50 mV, for the particular case  $c_3 = 1.04$  and  $\omega_3 = 250000$  rad/s. Let us vary the angular frequency  $\omega_3$  and the amplitude  $c_3$ , in order to find the best possible power spectrum and ripple. Figure 20 shows very clearly that high frequencies have no more influence on the ripple. Even if the frequency rises, the output voltage ripple does not decrease and, on the other way, the switching frequency of the switch  $S$  is limited to 80 kHz - 100 kHz, in practice.

Therefore, choosing  $\omega_3$  is very simple. Any value on the domain  $(2 \times 10^5, 5 \times 10^5)$  is appropriate for a small ripple. This corresponds to the interval (32 kHz, 80 kHz) for the frequency of  $v_{c3}$ . Our choice is  $\omega_3 = 250000$  rad/s ( $f_3 \approx 40$  kHz).

On the contrary, the choice of  $c_3$  could only be done to minimize the power spectrum. Figure 23 shows the power spectrum amplitude of the output voltage. The minimum value is obtained for  $c_3 = 1.04$ . The importance of the switch frequency begins to decrease when the amplitude of sinus  $c_3$  increases. For high values of  $c_3$  ( $c_3 > 1.2$ ) low frequencies are favored. The power spectral density of the output voltage (Fig. 22) for the circuit with the control law (9), obtained with  $c_3 = 1.04$ , is reduced to a maximum of  $9.43 \times 10^4$ . Adding the simple control law (8) as feedback of the sys-

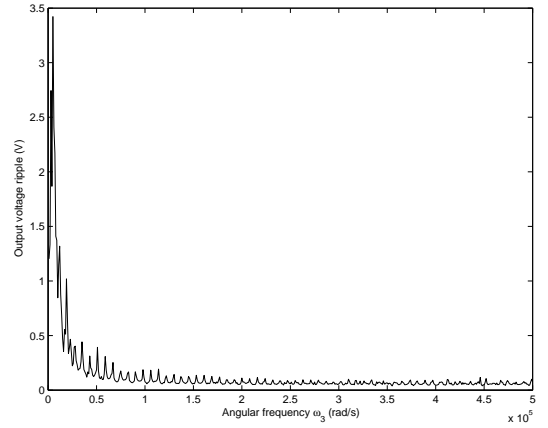


Fig. 20. The output voltage ripple in function of  $\omega_3$ .

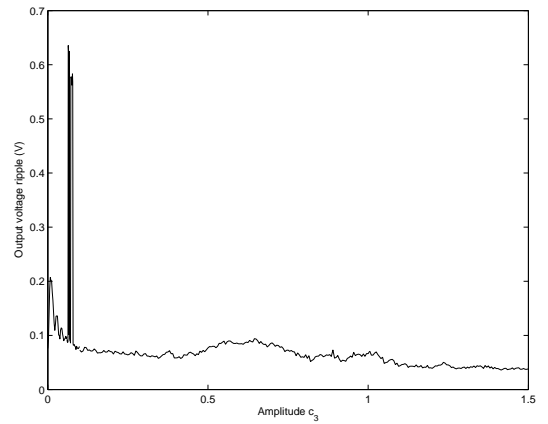


Fig. 21. The output voltage ripple in function of  $c_3$ .

tem decreases the amplitude of the spectrum and widens the spectrum for low frequencies. The output voltage wave-form of the Buck converter with the control law (1) has the ripple of the same order as the system with the control law (1) and (9) for a large variation domain of  $c_3$  (0.1 - 1.5). As we mentioned in abstract and introduction, the control law (9) is introduced to keep the ripple bounded. The spectacular decrease of the power spectrum density is a consequence of the increase of the switching frequency from 2.5kHz to 40kHz.

## V. CONCLUSION

We proposed a feedback control method improving switch-mode power supplies EMC. To confirm the efficiency of this new method, a comparison with the anticontrol method is included. Indeed, the application of the classical method of chaos anticontrol to these systems leads the output voltage to have an exaggerated output voltage ripple and an undesirable spectrum, whereas these problems are solved with ours.

The new controller proposed in this paper improves the time-domain (ripple) performance and the frequency-domain (spectral) performance. We are now working on a combination of our method with the anticontrol method: the partial results are promising.

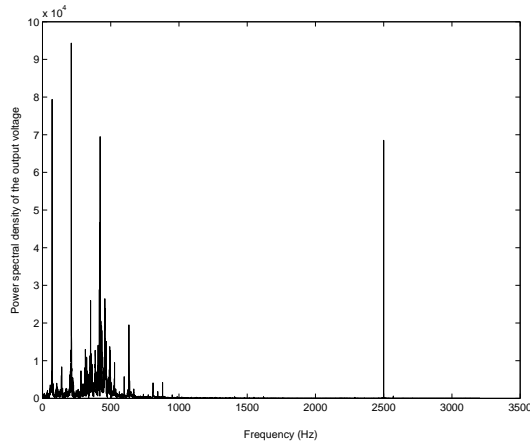


Fig. 22. Power spectrum of the output voltage for the circuit obtained with  $c_3=1.04$

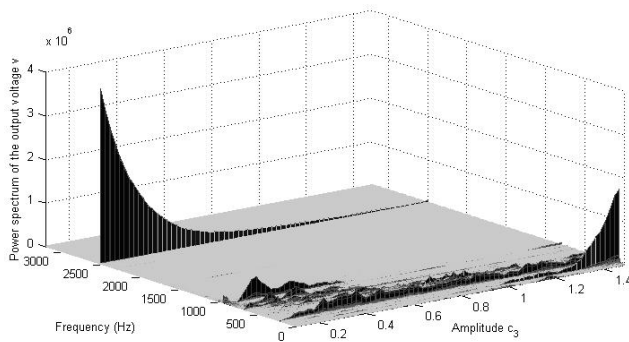


Fig. 23. Power spectrum with the control law (9) and (1).

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