International Conference on Complex Systems, Intelligence and Modern Technology Applications,
Cherboug, France, 19–22 Sept. 2004; pp. 192–197.

VARIOUS FORMS OF IMPROVEMENT BY NOISE IN NONLINEAR SYSTEMS

David ROUSSEAU, François CHAPEAU-BLONDEAU
Laboratoire d'Ingénierie des Systèmes Automatisés (LISA), Université d'Angers
62 avenue Notre Dame du Lac, 49000 Angers, France.
E-mail : david.rousseau@univ-angers.fr, chapeau@univ-angers.fr

ABSTRACT
In nonlinear systems, signal and noise can cooperate constructively. To illustrate this possibility, this paper presents, in a coherent perspective, various forms of stochastic resonance or improvement by noise in nonlinear systems. Especially, it is shown that the constructive role played by the noise can take place through several distinct mechanisms, either in single nonlinearities or in nonlinear arrays.

KEY WORDS
Nonlinear systems, signal, noise, stochastic resonance, array.

1 Introduction

In complex systems with nonlinear interactions, noise is not necessarily a nuisance but can sometimes play a beneficial role. Stochastic resonance is a nonlinear phenomenon which expresses the possibility of exploiting the noise in order to improve the transmission or the processing of a signal [1, 2, 3]. This paradoxical phenomenon was introduced some twenty years ago in the context of geophysical dynamics [4]. Stochastic resonance has since gradually been observed in a growing variety of processes, including electronic circuits [5, 6], optical devices [7, 8], chemical reactions [9, 10], neurons [11, 12]. A stochastic resonance phenomenon usually occurs in the presence of an information-carrying signal, associated to a noise, which both interact with a nonlinear transmission or processing system. An appropriate measure of performance is introduced, and stochastic resonance takes place when the performance measure can be improved by means of an increase in the amount of noise.

Stochastic resonance has now been shown feasible with various types of information-carrying signals, including deterministic periodic or nonperiodic signals, or random signals. Also, various types of nonlinear systems, interacting with the information-carrying signal in the presence of noise, have been shown to give way to stochastic resonance. Essentially, stochastic resonance has been reported in nonlinear systems incorporating thresholds or potential barriers [1, 2]; here the mechanism of improvement, qualitatively, is that the noise will assist small signals in overcoming the thresholds or barriers. More recently, another form of stochastic resonance was reported for saturating nonlinearities, where the noise is able to reduce the distortion experienced by large signals [13]. Also recently, another new form of stochastic resonance was reported with arrays of nonlinear sensors [14, 15]; the mechanism of improvement here is that independent noises added on the sensors induce more variability and a richer representation capability in the individual responses collected over the array.

In the present paper, we will consider the transmission of a periodic information-carrying signal, in order to illustrate, in a coherent perspective, diverse forms of stochastic resonance demonstrating various possible mechanisms of improvement by noise. We will also focus on the more recent forms of stochastic resonance, with saturating nonlinearities and with nonlinear arrays, and will provide new examples in these conditions.

2 Nonlinear transmission of a periodic signal

An information-carrying signal $s(t)$, and a noise $\eta(t)$, form the inputs to a nonlinear system which responds with the output signal $y(t)$, according to the setting of Fig. 1.

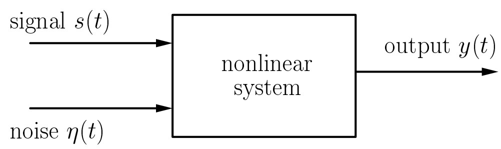

Figure 1: Nonlinear transmission process.

We will address here the situation where the information-carrying input $s(t)$ is a periodic signal, transmitted by a nonlinear system under the form of a static or memoryless nonlinearity $g(.)$ realizing

$$y(t) = g[s(t) + \eta(t)] \ . \tag{1}$$

Our aim will be to quantitatively analyze the efficacy of transmission of the periodic input $s(t)$ onto the output $y(t)$ in the presence of the noise $\eta(t)$. We will investigate the possibility of increasing the transmission efficacy through an increase of the level of the noise $\eta(t)$, for various types of nonlinearities $g(.)$, i.e. stochastic resonance.

In the case of a periodic input $s(t)$, the classical measure [1] which is appropriate to quantify a stochastic resonance effect is a signal-to-noise ratio, defined in the frequency domain, and which measures, in the output signal

$y(t)$, the part contributed by the periodic input and the part contributed by the noise [1, 16]. When $s(t)$ in Eq. (1) is deterministic periodic with period T_s, the output signal $y(t)$ of Eq. (1) generally is a cyclostationary random signal, with a power spectrum containing spectral lines at integer multiples of $1/T_s$ emerging out of a continuous noise background [16]. A standard measure of similarity of $y(t)$ with the T_s-periodic input $s(t)$, is a signal-to-noise ratio (SNR) defined as the power contained in the output spectral line at the fundamental $1/T_s$ divided by the power contained in the noise background in a small frequency band ΔB around $1/T_s$.

For the input–output relationship of Eq. (1), the power contained in the output spectral line at the frequency n/T_s is given [16] by $|\overline{Y}_n|^2$, where $\overline{Y}_n$ is the order n Fourier coefficient of the T_s-periodic nonstationary output expectation $\mathrm{E}[y(t)]$, i.e.

$$\overline{Y}_n = \left\langle \mathrm{E}[y(t)] \exp\Bigl(-\imath n \frac{2\pi}{T_s} t\Bigr) \right\rangle \;, \tag{2}$$

with the time average defined as

$$\langle ... \rangle = \frac{1}{T_s} \int_0^{T_s} ... \, dt \;. \tag{3}$$

From Eq. (1), it follows that the output expectation $\mathrm{E}[y(t)]$ at a fixed time t is computable as

$$\mathrm{E}[y(t)] = \int_{-\infty}^{+\infty} g(u) f_\eta[u - s(t)] du \;, \tag{4}$$

with $f_\eta(u)$ the probability density function of $\eta(t)$.

The magnitude of the continuous noise background in the output spectrum is measured [16] by the stationarized output variance $\langle \mathrm{var}[y(t)] \rangle$, with the nonstationary variance $\mathrm{var}[y(t)] = \mathrm{E}[y^2(t)] - \mathrm{E}[y(t)]^2$ at a fixed time t, and

$$\mathrm{E}[y^2(t)] = \int_{-\infty}^{+\infty} g^2(u) f_\eta[u - s(t)] du \;. \tag{5}$$

An SNR $\mathcal{R}_n$, for the harmonic n/T_s in the ouput $y(t)$, follows as

$$\mathcal{R}_n = \frac{|\overline{Y}_n|^2}{\langle \mathrm{var}[y(t)] \rangle \, \Delta t \, \Delta B} \;, \tag{6}$$

where Δt is the time resolution of the measurement (i.e. the signal sampling period in a discrete-time implementation); we take here throughout $\Delta t \, \Delta B = 10^{-3}$.

3 Nonlinear sensors

A first example of stochastic resonance, or noise-improved signal transmission, can be obtained with the simple case of a two-state quantizer with threshold unity realizing the nonlinearity

$$g(u) = \mathrm{sign}(u-1) = \begin{cases} -1 & \text{for} \quad u \le 1 \;, \\ \;\;1 & \text{for} \quad u > 1 \;, \end{cases} \tag{7}$$

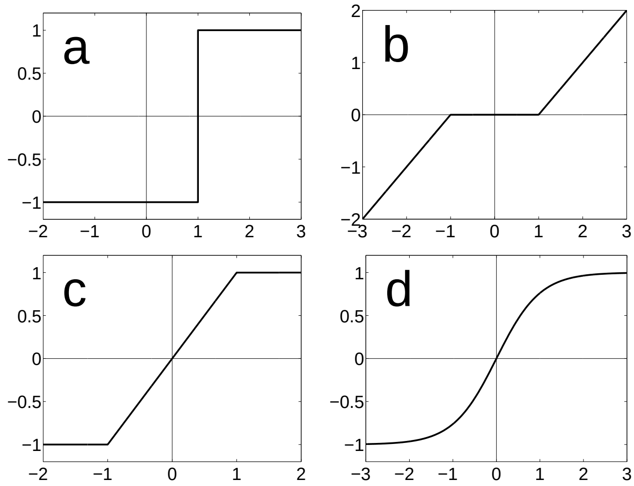

Figure 2: Various forms for the nonlinearity $g(u)$ in Eq. (1): (a) two-state quantizer of Eq. (7), (b) threshold-linear nonlinearity of Eq. (10), (c) linear-saturating nonlinearity of Eq. (11), (d) smooth saturating nonlinearity of Eq. (12).

as depicted in Fig. 2(a).

For the nonlinearity of Eq. (2(a)), equations (4) and (5) lead to

$$\mathrm{E}[y(t)] = 1 - 2F_\eta[1 - s(t)] \;, \tag{8}$$

and

$$\mathrm{var}[y(t)] = 4F_\eta[1 - s(t)] \Bigl\{ 1 - F_\eta[1 - s(t)] \Bigr\} \;, \tag{9}$$

with $F_\eta(u)$ the cumulative distribution function of $\eta(t)$.

The signal-to-noise ratio $\mathcal{R}_1$ that follows from Eq. (6) is represented in Fig. 3 in the case of the transmission of the periodic input $s(t) = A\cos(2\pi t/T_s)$. In the regime of a small amplitude $0 < A < 1$, the input signal $s(t)$ alone is unable to trigger transitions of the ouput $y(t)$ of the quantizer of Fig. 2(a). The presence of $s(t)$ at the input is therefore completely invisible at the output. This translates into a zero output SNR $\mathcal{R}_1$ at zero noise in Fig. 3. When some input noise $\eta(t)$ is added, a cooperative effect can take place where the noise $\eta(t)$ is able to assist the small signal $s(t)$ so as to overcome the quantization threshold and elicit transitions in the output $y(t)$. This gives way to an output SNR $\mathcal{R}_1$ which increases as the level σ_η of the input noise $\eta(t)$ is raised. The output SNR $\mathcal{R}_1$ culminates at a maximum for a nonzero optimal level of the input noise $\eta(t)$, where the cooperative behavior is at its best efficacy. When more noise is further added, the detrimental influence of the noise resumes to degrade the output SNR $\mathcal{R}_1$ down to zero for very high noise levels. This is an instance of the stochastic resonance effect, with no transmission at zero noise and a nonzero optimal amount noise that maximizes the transmission efficacy.

Another example of stochastic resonance can be obtained with the nonlinearity

$$g(u) = \begin{cases} u+1 & \text{for} \quad u \le -1 \;, \\ \;\;0 & \text{for} \quad -1 < u < 1 \;, \\ u-1 & \text{for} \quad u \ge 1 \;, \end{cases} \tag{10}$$

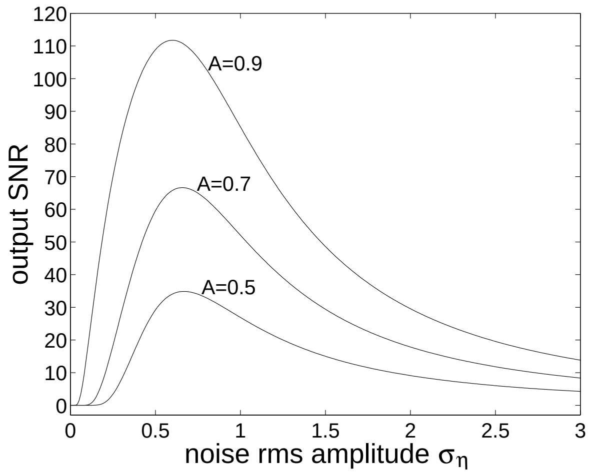

Figure 3: Output signal-to-noise ratio $\mathcal{R}_1$ from Eq. (6) as a function of the rms amplitude σ_η of the input noise $\eta(t)$ chosen zero-mean Gaussian, for the transmission of the periodic input $s(t) = A\cos(2\pi t/T_s)$ by the two-state quantizer of Fig. 2(a).

depicted in Fig. 2(b).

The corresponding SNR $\mathcal{R}_1$ from Eq. (6) is represented in Fig. 4 in the case of the transmission of the periodic input $s(t) = A\cos(2\pi t/T_s)$. Again, in the regime of a small amplitude $0 < A < 1$, the input signal $s(t)$ alone is unable to induce variations of the output $y(t)$. It is only when the noise $\eta(t)$ is applied at the input that a transmission of $s(t)$ can take place, with a maximum efficacy for a nonzero amount of noise, as expressed by the resonant evolution of the output SNR $\mathcal{R}_1$ in Fig. 4.

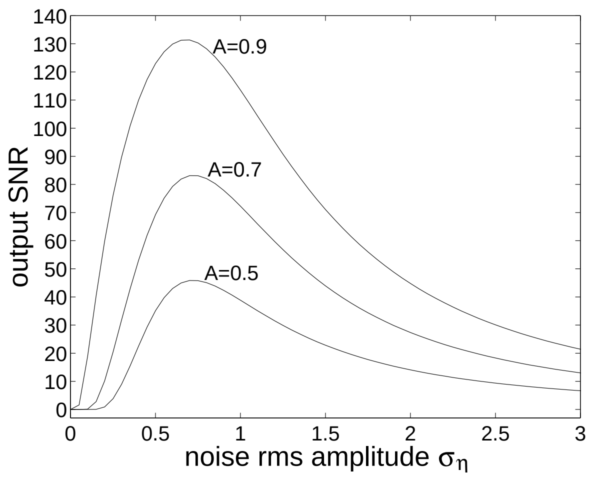

Figure 4: Output signal-to-noise ratio $\mathcal{R}_1$ from Eq. (6) as a function of the rms amplitude σ_η of the input noise $\eta(t)$ chosen zero-mean Gaussian, for the transmission of the periodic input $s(t) = A\cos(2\pi t/T_s)$ by the nonlinearity of Fig. 2(b).

It can be noted in Figs. 3 and 4 that the optimal noise level maximizing the SNR $\mathcal{R}_1$ is usually different in every condition of signal amplitude A and nonlinearity $g(.)$. Yet, in each condition, the present theory allows an explicit computation of the optimal noise level.

In addition to threshold nonlinearities in the style of Figs. 2(a) and 2(b), stochastic resonance can also take place with saturating nonlinearities. We consider for illustration the nonlinearity, very common for sensors with saturation,

$$g(u) = \begin{cases} -1 & \text{for} \quad u \leq -1 \\ u & \text{for} \quad -1 < u < 1 \\ 1 & \text{for} \quad u \geq 1 \,, \end{cases} \tag{11}$$

as depicted in Fig. 2(c).

The resulting SNR $\mathcal{R}_1$ from Eq. (6) is represented in Fig. 5 for the transmission of the periodic input $s(t) = A + \cos(2\pi t/T_s)$ with an offset A. When this offset A is large relative to 1 (here when $A \geq 2$), the input signal $s(t)$ alone solicits the nonlinearity of Fig. 2(c) always in its positive saturation region. In this case, the output $y(t)$ remains stuck at $+1$; no variation of $y(t)$ is ever induced by $s(t)$, and therefore the variations of $s(t)$ at the input are completely invisible at the output. When the noise $\eta(t)$ is added at the input, again a cooperative effect can take place between $s(t)$ and $\eta(t)$. Here the mechanism is that the fluctuations of the noise are able, on occasions, to bring the nonlinearity of Fig. 2(c) to operate in its linear part. This enables some transmission of information between the input $s(t)$ and the output $y(t)$ with assistance from the noise $\eta(t)$. This cooperative mechanism translates in Fig. 5, into a resonant evolution of the output SNR $\mathcal{R}_1$ as the level σ_η of the noise $\eta(t)$ is raised, with a maximum of $\mathcal{R}_1$ at a nonzero optimal noise level. This is a form of stochastic resonance, or noise-aided signal transmission through saturating nonlinearity, with a novel example in Fig. 5 not treated in [13].

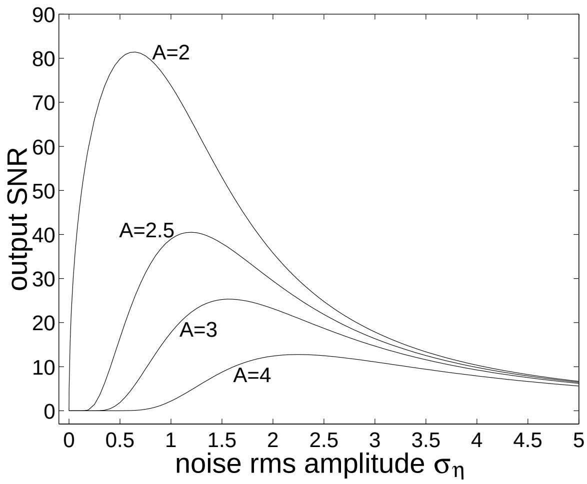

Figure 5: Output signal-to-noise ratio $\mathcal{R}_1$ from Eq. (6) as a function of the rms amplitude σ_η of the input noise $\eta(t)$ chosen zero-mean Gaussian, for the transmission of the periodic input $s(t) = A + \cos(2\pi t/T_s)$ by the saturating nonlinearity of Fig. 2(c).

Another example of stochastic resonance with saturating nonlinearities can be obtained with the smooth nonlinearity

$$g(u) = \tanh(u) \tag{12}$$

depicted in Fig. 2(d).

The resulting output SNR $\mathcal{R}_1$ from Eq. (6) is represented in Fig. 6 for the transmission of the periodic input $s(t) = A + \cos(2\pi t/T_s)$. At zero noise in Fig. 6, the output SNR $\mathcal{R}_1$ is infinite. This is due to the smooth character of the saturating nonlinearity of Fig. 2(d) as opposed to that of Fig. 2(c). For large offset A, although the periodic component is very small in the output $y(t)$, the noise component is absent, whence the infinite SNR $\mathcal{R}_1$. When some noise $\eta(t)$ is added at the input, Fig. 6 shows that the output SNR $\mathcal{R}_1$ starts to degrade rapidly. Yet, this degradation does not develop monotonically. When more noise is added, a constructive action of the noise is recovered. This is conveyed in Fig. 6 by a range of the noise level σ_η where the output SNR $\mathcal{R}_1$ improves as σ_η grows. This nonmonotonic evolution of $\mathcal{R}_1$, as the noise grows, instead of a monotonic degradation, is another form of stochastic resonance, or improvement by noise. When a small amount of noise pre-exists, further addition of noise may bring improvement to the transmission near saturation. A qualitative explanation is again that the added noise, on average, has the ability to shift the operating zone of the nonlinearity of Fig. 2(d) away from the saturating part and towards the linear region more favorable to an efficient transmission of the signal $s(t)$.

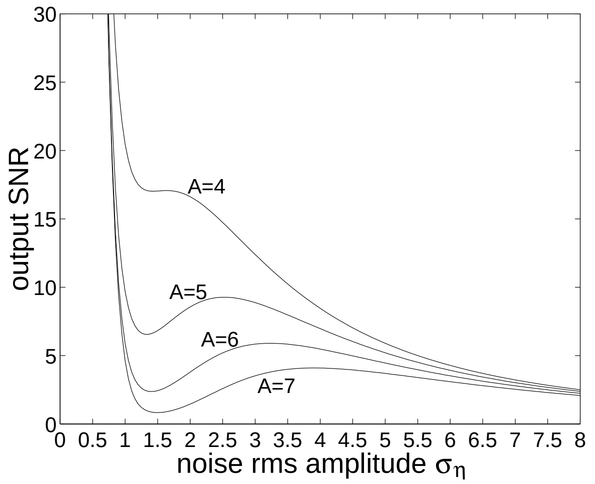

Figure 6: Output signal-to-noise ratio $\mathcal{R}_1$ from Eq. (6) as a function of the rms amplitude σ_η of the input noise $\eta(t)$ chosen zero-mean Gaussian, for the transmission of the periodic input $s(t) = A + \cos(2\pi t/T_s)$ by the smooth saturating nonlinearity of Fig. 2(d).

4 Nonlinear arrays

The forms of improvement by noise we have shown in Section 3 essentially involve an input signal $s(t)$ which is by itself not optimally positioned in relation to a transmitting nonlinearity. In such condition, the beneficial role of noise can be described as a means of displacing the operating zone of the nonlinearity into a region more favorable to the signal transmission. We shall now illustrate another mechanism of improvement by noise, which applies also when the input signal $s(t)$ is optimally positioned in relation to the nonlinearity. This new form of stochastic resonance takes place when the nonlinearities are replicated and associated in a parallel array [14, 17] as in Fig. 7.

An input signal $x(t)$ is applied onto a parallel array of N identical nonlinear sensors $g(.)$ as in Fig. 7. Arrays as in Fig. 7 can serve as models for various existing systems such as flash analog-to-digital converters, sonar arrays, networks of sensory neurons, and also for future intelligent sensing networks, possibly with neuronal inspiration [14, 15, 17]. A noise $\eta_i(t)$, independent of $x(t)$, can be added to $x(t)$ at each sensor i so as to produce the output

$$y_i(t) = g[x(t) + \eta_i(t)], \qquad i = 1, 2, \dots N \ . \tag{13}$$

The N noises $\eta_i(t)$ are white, mutually independent and identically distributed (i.i.d.) with probability density function $f_\eta(u)$. The response $y(t)$ of the array is obtained by averaging the outputs of all the sensors, as

$$y(t) = \frac{1}{N}\sum_{i=1}^{N} y_i(t) \ . \tag{14}$$

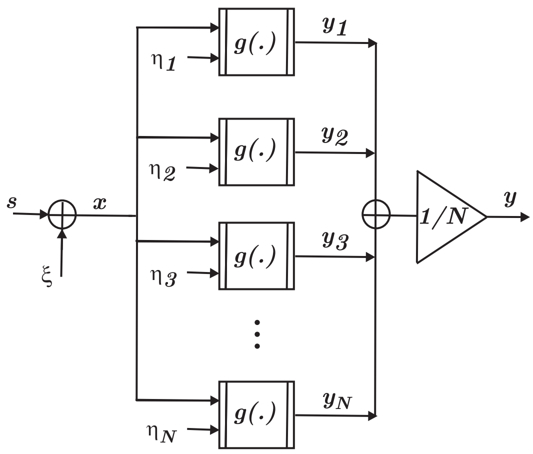

Figure 7: Parallel array of N identical nonlinearities $g(.)$.

The input $x(t)$ is formed by the signal-plus-noise mixture $x(t) = s(t) + \xi(t)$, where $s(t)$ is our deterministic periodic component as in Section 3, and $\xi(t)$ is a stationary white noise, independent of both $s(t)$ and the $\eta_i(t)$, and with probability density function $f_\xi(u)$. At time t, for a fixed given value x of the input $x(t)$, one has, according to Eq. (14), the conditional expectations $\mathrm{E}[y(t)|x] = \mathrm{E}[y_i(t)|x]$ and $\mathrm{E}[y^2(t)|x] = \mathrm{E}[y_i^2(t)|x]/N + \mathrm{E}^2[y_i(t)|x](N-1)/N$ which are independent of i since the $\eta_i(t)$ are i.i.d. Since $x(t) = s(t) + \xi(t)$, the probability density for the value x is $f_\xi\big(x-s(t)\big)$, and Eqs. (4) and (5) become

$$\mathrm{E}[y(t)] = \int_{-\infty}^{+\infty} \mathrm{E}[y(t)|x] f_\xi\big(x-s(t)\big) dx \ , \qquad (15)$$

and

$$\mathrm{E}[y^2(t)] = \int_{-\infty}^{+\infty} \mathrm{E}[y^2(t)|x] f_\xi\big(x-s(t)\big) dx \ . \qquad (16)$$

Because of Eq. (13), one has for any i,

$$\mathrm{E}[y_i(t)|x] = \int_{-\infty}^{+\infty} g(x+u) f_\eta(u) du \qquad (17)$$

and

$$\mathrm{E}[y_i^2(t)|x] = \int_{-\infty}^{+\infty} g^2(x+u) f_\eta(u) du \ . \qquad (18)$$

From the above equations, the output signal-to-noise ratio $\mathcal{R}_n$ of Eq. (6) can be computed for transmission of $s(t)$ by the array.

For an array of N identical two-state quantizers as in Fig. 2(a), transmitting the periodic input $s(t) = 1 + \cos(2\pi t/T_s)$, Fig. 8 shows the output SNR $\mathcal{R}_1$ from Eq. (6). In these conditions, the input $s(t) = 1 + \cos(2\pi t/T_s)$ is optimally positioned in relation to the threshold 1 of the quantizers; $s(t)$ by itself is exactly centered at the threshold and is no longer permanently subthreshold as it was in Fig. 3. As a consequence, with a single quantizer $N = 1$ in Fig. 8, the ouput SNR $\mathcal{R}_1$ degrades monotonically as the level σ_η of the array noise $\eta_1(t)$ is increased. Noise addition here brings no improvement to the transmission of the optimally positioned input $s(t)$. Yet, as soon as $N \geq 2$ in Fig. 8, it is observed that the output SNR $\mathcal{R}_1$ can be improved by increasing the level σ_η of the array noises $\eta_i(t)$, with a nonzero optimal value of σ_η that maximzes $\mathcal{R}_1$. An array of $N \geq 2$ nonlinearities in parallel with added noises $\eta_i(t)$, is more efficient than a single nonlinearity with no added noise, for the transmission of the noisy input $s(t)$. A qualitative explanation is that the independent noises $\eta_i(t)$ injected in the array, bring diversity to the individual responses $y_i(t)$ over the array, which otherwise would act in unison. This diversity translates into a richer representation of the input $s(t)$, whence the improved transmission performance revealed in Fig. 8.

A similar behavior is observed in Fig. 9, for an array of N identical saturating nonlinearities as in Fig. 2(c), transmitting the periodic input $s(t) = 3\cos(2\pi t/T_s)$. In this case, the input $s(t)$ is also exactly centered, in the linear part of the nonlinearity of Fig. 2(c), but since $s(t)$ is large it saturates the nonlinearity and it is thus strongly distorted in its transmission. Figure 9 reveals that, thanks to the added array noises $\eta_i(t)$, moderately large arrays ($N \gtrsim 5$) perform better than a single nonlinearity with no added noise. An optimal nonzero amount of the array noises $\eta_i(t)$ maximizes the output SNR $\mathcal{R}_1$, and the improvement by noise gets more pronounced as the array size N increases. This form of stochastic resonance in array of saturating nonlinearities in order to reduce the distortion of a strong signal, is a novel feature, not previously reported. It can be verified that it is also observed, in a comparable way, with the smooth nonlinearities of Fig. 2(d).

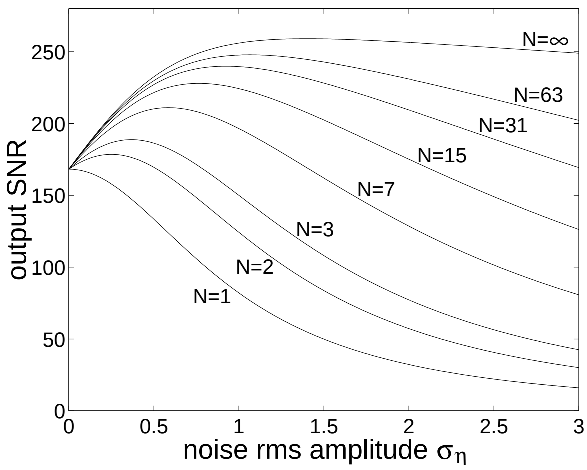

Figure 8: Output signal-to-noise ratio $\mathcal{R}_1$ from Eq. (6) as a function of the rms amplitude σ_η of the array noises $\eta_i(t)$ chosen zero-mean Gaussian. The periodic input is $s(t) = 1 + \cos(2\pi t/T_s)$ buried in a zero-mean Gaussian noise $\xi(t)$ with rms amplitude $\sigma_\xi = 1$. The array is made of N identical two-state quantizers as in Fig. 2(a).

5 Conclusion

We have shown various possible forms of stochastic resonance, or improvement by noise, in nonlinear systems. Two distinct mechanisms apply: In single nonlinearities, noise can play the role of a "random bias", displacing the operating zone into a region more favorable to the transmission of a signal not optimally positioned. In arrays, noise can bring variability and hence further enhanced representation capability of a signal otherwise optimally positioned. Improvement by noise have been shown here in the transmission of a periodic signal assessed by a signal-to-noise ratio; yet it is known that a similar effect can occur with other types of signals involved in other types of processing, with other measures of performance improvable by the noise [18, 19, 14, 3, 20, 21, 15]. Stochastic resonance stands as an emergent phenomenon in the realm of complex systems, and the details of its various forms and properties largely remain to be explored, especially for the novel form in arrays addressed here. Also, the nonlinear situations that are involved are quite reminiscent of those encountered in neuronal systems, where stochastic resonance could play a part to contribute to the performance in information processing, under detailed modalities that also remain to be elucidated.

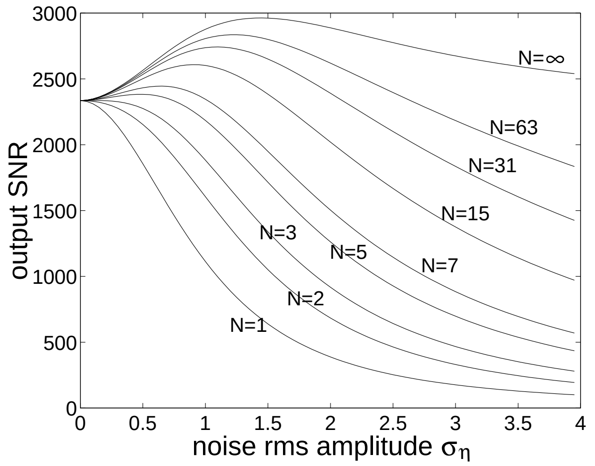

Figure 9: Output signal-to-noise ratio $\mathcal{R}_1$ from Eq. (6) as a function of the rms amplitude σ_η of the array noises $\eta_i(t)$ chosen zero-mean Gaussian. The periodic input is $s(t) = 3\cos(2\pi t/T_s)$ buried in a zero-mean Gaussian noise $\xi(t)$ with rms amplitude $\sigma_\xi = 1$. The array is made of N identical saturating nonlinearities as in Fig. 2(c).

References

[1] L. Gammaitoni, P. Hänggi, P. Jung, and F. Marchesoni, "Stochastic resonance", *Reviews of Modern Physics*, vol. 70, pp. 223–287, 1998.

[2] F. Chapeau-Blondeau, "Stochastic resonance and the benefit of noise in nonlinear systems", in *Noise, Oscillators and Algebraic Randomness: From Noise in Communication Systems to Number Theory*, M. Planat, Ed., pp. 137–155. Springer, Berlin, 2000.

[3] G. P. Harmer, B. R. Davis, and D. Abbott, "A review of stochastic resonance: Circuits and measurement", *IEEE Transactions on Instrumentation and Measurement*, vol. 51, pp. 299–309, 2002.

[4] R. Benzi, A. Sutera, and A. Vulpiani, "The mechanism of stochastic resonance", *Journal of Physics A*, vol. 14, pp. L453–L458, 1981.

[5] X. Godivier and F. Chapeau-Blondeau, "Noise-assisted signal transmission in a nonlinear electronic comparator: Experiment and theory", *Signal Processing*, vol. 56, pp. 293–303, 1997.

[6] D. G. Luchinsky, R. Mannella, P. V. E. McClintock, and N. G. Stocks, "Stochastic resonance in electrical circuits – I: Conventional stochastic resonance", *IEEE Transactions on Circuits and Systems – II: Analog and Digital Signal Processing*, vol. 46, pp. 1205–1214, 1999.

[7] M. I. Dykman, G. P. Golubev, I. K. Kaufman, D. G. Luchinsky, P. V. E. McClintock, and E. A. Zhukov, "Noise-enhanced optical heterodyning in an all-optical bistable system", *Applied Physics Letters*, vol. 67, pp. 308–310, 1995.

[8] F. Vaudelle, J. Gazengel, G. Rivoire, X. Godivier, and F. Chapeau-Blondeau, "Stochastic resonance and noise-enhanced transmission of spatial signals in optics: The case of scattering", *Journal of the Optical Society of America B*, vol. 15, pp. 2674–2680, 1998.

[9] D. S. Leonard and L. E. Reichl, "Stochastic resonance in a chemical reaction", *Physical Review E*, vol. 49, pp. 1734–1739, 1994.

[10] M. I. Dykman, T. Horita, and J. Ross, "Statistical distribution and stochastic resonance in a periodically driven chemical system", *Journal of Chemical Physics*, vol. 103, pp. 966–972, 1995.

[11] A. Bulsara, E. W. Jacobs, T. Zhou, F. Moss, and L. Kiss, "Stochastic resonance in a single neuron model: Theory and analog simulation", *Journal of Theoretical Biology*, vol. 152, pp. 531–555, 1991.

[12] F. Chapeau-Blondeau, X. Godivier, and N. Chambet, "Stochastic resonance in a neuron model that transmits spike trains", *Physical Review E*, vol. 53, pp. 1273–1275, 1996.

[13] D. Rousseau, J. Rojas Varela, and F. Chapeau-Blondeau, "Stochastic resonance for nonlinear sensors with saturation", *Physical Review E*, vol. 67, pp. 021102,1–6, 2003.

[14] N. G. Stocks, "Information transmission in parallel threshold arrays: Suprathreshold stochastic resonance", *Physical Review E*, vol. 63, pp. 041114,1–9, 2001.

[15] D. Rousseau, F. Duan, and F. Chapeau-Blondeau, "Suprathreshold stochastic resonance and noise-enhanced Fisher information in arrays of threshold devices", *Physical Review E*, vol. 68, pp. 031107,1–10, 2003.

[16] F. Chapeau-Blondeau and X. Godivier, "Theory of stochastic resonance in signal transmission by static nonlinear systems", *Physical Review E*, vol. 55, pp. 1478–1495, 1997.

[17] D. Rousseau and F. Chapeau-Blondeau, "Suprathreshold stochastic resonance and signal-to-noise ratio improvement in arrays of comparators", *Physics Letters A*, vol. 321, pp. 280–290, 2004.

[18] S. Mitaim and B. Kosko, "Adaptive stochastic resonance", *Proceedings of the IEEE*, vol. 86, pp. 2152–2183, 1998.

[19] F. Chapeau-Blondeau, "Noise-assisted propagation over a nonlinear line of threshold elements", *Electronics Letters*, vol. 35, pp. 1055–1056, 1999.

[20] S. Zozor and P. O. Amblard, "On the use of stochastic resonance in sine detection", *Signal Processing*, vol. 82, pp. 353–367, 2002.

[21] F. Chapeau-Blondeau, "Stochastic resonance for an optimal detector with phase noise", *Signal Processing*, vol. 83, pp. 665–670, 2003.