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FRACTAL IMAGES WITH ITERATED FUNCTION SYSTEMS : REACHING AESTHETIC THROUGH PARAMETERS EXPLORATION

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Abstract: Fractal patterns contain rich potentialities to be explored in the context of fashion design. In particular, automatic generation of fractal images based on iterated function systems (IFS) could stimulate creativity and allow fast generation of solutions. However, this relies on efficient user interfaces and interaction techniques. In this paper, we describe and analyze a basic model of IFS and address the issue of controlling the fractal images they generate. We specially focus on fast exploration of the IFS parameters space and aesthetic properties of the generated images through both 2D visualization and 3D mapping on virtual characters.

1 INTRODUCTION

There is a multitude of domains in which aesthetic component plays an important role. For example to estimate the interest of solutions proposed in architecture, product design and decoration, or in the field of fashion.

Automatic generation of solutions is a promising domain. However, we are confronted with the difficult problem of the formalization of aesthetic criteria. Moreover, the aesthetic interest of a solution generated by a computer lies in the way the other human beings are going to perceive generated solutions. Some works tend to prove that regularity (symmetry, regularity in scale, etc.) is one of the criteria of beauty.

This leads us to envisage the use of fractals images and in particular their automatic generation by Iterated Function Systems (IFS).

IFS have been introduced in the context of fractal geometry. For image processing, they have found applications for image synthesis and image compression (Peitgen and Richter, 1986),(Barnsley, 1993; Lu, 1997; Turner and Blackledge, 1998). IFS contain rich potentialities to be explored, particularly in the context of fashion design. Indeed, automatic generation of fractal image could stimulate creativity and allow fast exploration of solutions space of a given IFS. However, efficient user interfaces and interaction techniques have to be developed and evaluated.

Here, we describe and analyze a basic model of IFS and address the issue of fast generation of the fractal images they generate. We specially focus on fast exploration of the IFS parameters space and on aesthetic properties of the generated images through both 2D visualization and 3D mapping on virtual characters. We developed a virtual reality module that can load and visualize any 3D model with which a user can interact (Richard, 2004).

2 IFS MODEL

We consider the set I of two-dimensional signals or images $s(x,y) \in \mathbb{R}$ with spatial coordinates (x,y) defined over the support $[0,1[\times[0,1[= \mathcal{S}$ (Portefaix, et al. 2002; Portefaix, et al. 2003). A transformation T is introduced which maps an initial image of I into another (final) image of I. The final image is obtained as the union of 4 sub-images defined over the 4 quarters of support $\mathcal{S}$, i.e. $[0,1/2[\times[0,1/2[= \mathcal{S}_1$, $[1/2,1[\times[0,1/2[= \mathcal{S}_2$, $[0,1/2[\times[1/2,1[= \mathcal{S}_3$, and $[1/2,1[\times[1/2,1[= \mathcal{S}_4$, over which each sub-image is a contracted version of the initial image with affinely transformed gray levels. Explicitly, transformation T is defined by the union of the four subtransformations:

$$\left| \begin{array}{ccl} \mathcal{S} \times \mathrm{I\!R} & \longrightarrow & \mathcal{S}_1 \times \mathrm{I\!R} \\ \big((x,y),\, s(x,y)\big) & \longmapsto & \big((\frac{x}{2}, \frac{y}{2}),\, a_1 s(x,y) + b_1 \\ & & + c_1 x + d_1 y\big) \end{array} \right. \tag{1}$$

$$\left| \begin{array}{ccl} \mathcal{S} \times \mathrm{I\!R} & \longrightarrow & \mathcal{S}_2 \times \mathrm{I\!R} \\ \big((x,y),\, s(x,y)\big) & \longmapsto & \big((\frac{1}{2} + \frac{x}{2}, \frac{y}{2}),\, a_2 s(x,y) + b_2 \\ & & + c_2 x + d_2 y\big) \end{array} \right. \tag{2}$$

$$\left| \begin{array}{ccl} \mathcal{S} \times \mathrm{I\!R} & \longrightarrow & \mathcal{S}_3 \times \mathrm{I\!R} \\ \big((x,y),\, s(x,y)\big) & \longmapsto & \big((\frac{x}{2}, \frac{1}{2} + \frac{y}{2}),\, a_3 s(x,y) + b_3 \\ & & + c_3 x + d_3 y\big) \end{array} \right. \tag{3}$$

and

$$\left| \begin{array}{ccl} \mathcal{S} \times \mathrm{I\!R} & \longrightarrow & \mathcal{S}_4 \times \mathrm{I\!R} \\ \big((x,y),\, s(x,y)\big) & \longmapsto & \big((\frac{1}{2} + \frac{x}{2}, \frac{1}{2} + \frac{y}{2}),\, a_4 s(x,y) \\ & & + b_4 + c_4 x + d_4 y\big) \end{array} \right. \tag{4}$$

with real coefficients a_j, b_j, c_j and d_j verifying $0 < |a_j| < 1$, for $j = 1$ to 4, so as to have contractive mappings.

The transformation T defined by Eqs. (1)–(4) implements on both the spatial coordinates (x,y) and the gray level $s(x,y)$, contractive affine transforms. Consequently, the mapping $s(x,y) \longmapsto T[s(x,y)]$ is also a contractive affine transform. It results (Barnsley, 1993) that $s(x,y) \longmapsto T[s(x,y)]$ admits one single fixed point, i.e. an image $\sigma(x,y)$ verifying $T[\sigma(x,y)] = \sigma(x,y)$ also called the attractor of transformation T. Starting from any initial image $s_0(x,y) \in I$, iterative application of the transformation T defined by Eqs. (1)–(4) realizes an IFS. The process converges to a unique attractor $\sigma(x,y)$ that is completely determined by the set of 16 parameters $\{(a_j, b_j, c_j, d_j), j = 1 \ldots 4\}$. An important property of this correspondence (Barnsley, 1993) is that small smooth changes in $\{(a_j, b_j, c_j, d_j), j = 1 \ldots 4\}$ are associated to small smooth changes in $\sigma(x,y)$.

The attractor $\sigma(x,y)$ is defined as the solution to the fixed-point equation (Portefaix, et al. 2003).

$$\begin{cases} \sigma(x,y) = a_1 \sigma(2x, 2y) + b_1 + c_1 \times (2x) \\ \quad + d_1 \times (2y), \forall (x,y) \in \mathcal{S}_1 \\ \sigma(x,y) = a_2 \sigma(2x-1, 2y) + b_2 + c_2 \times (2x-1) \\ \quad + d_2 \times (2y), \forall (x,y) \in \mathcal{S}_2 \\ \sigma(x,y) = a_3 \sigma(2x, 2y-1) + b_3 + c_3 \times (2x) \\ \quad + d_3 \times (2y-1), \forall (x,y) \in \mathcal{S}_3 \\ \sigma(x,y) = a_4 \sigma(2x-1, 2y-1) + b_4 + c_4 \times (2x-1) \\ \quad + d_4 \times (2y-1), \forall (x,y) \in \mathcal{S}_4 \,. \end{cases} \tag{5}$$

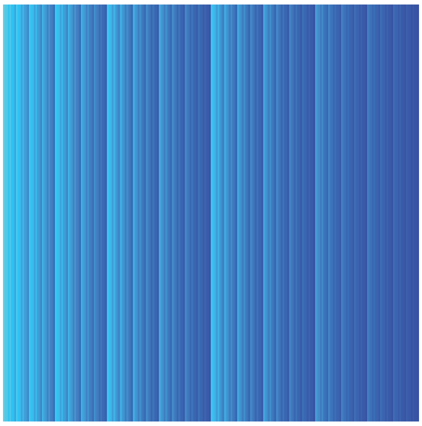

Figure 1: An example of the attractor $\sigma(x,y)$ of the IFS of Eqs. (1)–(4) obtained with a given set of parameters $\{(a_j, b_j, c_j, d_j), j = 1 \ldots 4\}$.

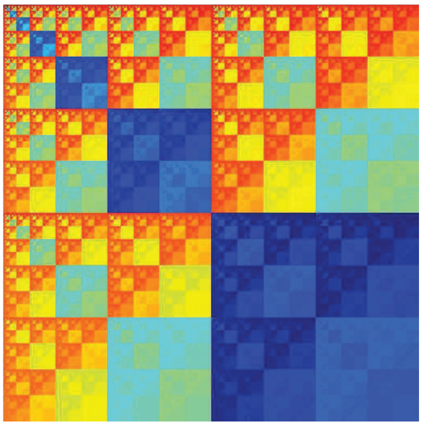

Figure 2: An example of the attractor $\sigma(x,y)$ of the IFS of Eqs. (1)–(4) obtained with another set of parameters $\{(a_j, b_j, c_j, d_j), j = 1 \ldots 4\}$.

Such a functional equation expresses a self-transformability property of attractor $\sigma(x,y)$, which confers to it a self-affine or fractal character. This translates into complicated shapes for $\sigma(x,y)$, with structures or details occurring at all scales, as visible on the image of $\sigma(x,y)$ shown in Fig. 2.

Depending on the parameters, the generated images can display more or less prominently their inherent fractal structure. For instance, in Fig. 1, the apparent homogeneity of the image produces a relatively poor visual traduction of the fractal structure. By contrast, Fig. 2 provides a very vivid traduction of this fractal structure. This shows that certain sets of parameters are more suited than others for a prominent traduction of the fractal structure. The achievement with fractals of aesthetic criteria and the exploration of the solutions thus rest on the capacity to control in a precise way the parameters of the IFS.

Determining how to choose the parameters $\{(a_j,b_j,c_j,d_j)\}$ of the IFS in order to impose prescribed properties onto its attractor, is a key issue for image modeling from IFS. Yet, it is usually not possible to analytically solve Eq. (5) so as to obtain an explicit expression of $\sigma(x,y)$ as a function of the parameters $\{(a_j,b_j,c_j,d_j), j=1\ldots4\}$. The ability of IFS of generating attractors with rich structures is exploited for fractal image compression (Barnsley, 1993; Lu, 1997), where the parameters of the IFS (similar to $\{(a_j,b_j,c_j,d_j)\}$) are usually determined by minimizing a mean-squared difference between the attractor and the target image to be coded.

By contrast our approach rests currently on a manual exploration of the parameters space, via a user interface. Our objective is to try to progressively increase the aesthetic performances of the generated fractals images. In close future, we plan to automatically explore parameters space by seeking to optimize the value of quantitative criteria measuring the aesthetic qualities of the images.

3 PARAMETERS EXPLORATION

As we described it previously, each fractal image generated by Eqs. (1)–(4) is completely defined by 16 parameters $\{(a_j,b_j,c_j,d_j), j=1\ldots4\}$. In order to be able to control these parameters, we designed an developed a user interface composed of three windows. The monitor screen was symmetrically divided in two parts :(1) a left part containing the 3D layout and a (2) a right part containing both the generated fractal image (top window) and a control interface (bottom window). The control interface, illustrated in Fig. 3 allows the user to select one out of four predefined sets of parameters by clicking on given "motif" buttons. A button provides an easy way to configure any color pallet. These pallets may be made from two, three of four different colors.

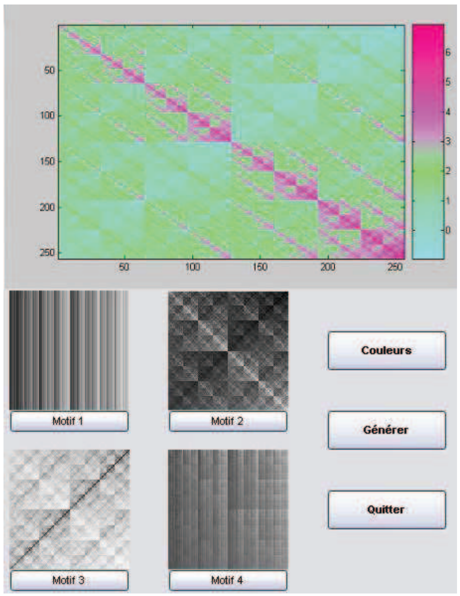

Figure 3: Snapshot of both the control interface (bottom) and the generated fractal images (top).

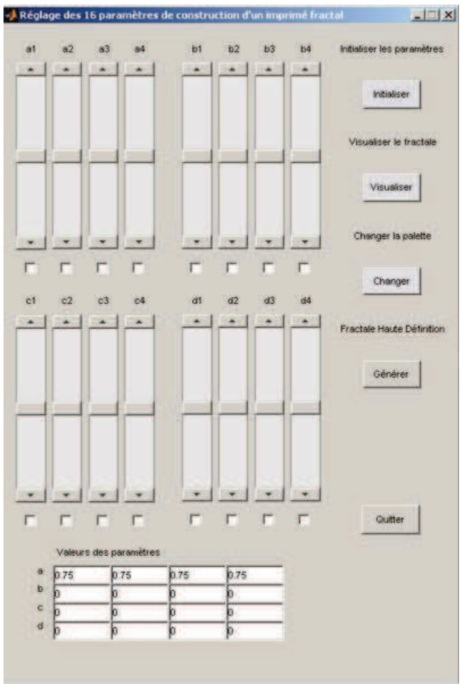

Figure 4: Snapshot of the window allowing a fine tuning of the IFS parameters.

A color configuration interface can be launched from the main control interface. The user may also launch another window, illustrated in Fig. 3, that allows individual tuning of the 16 IFS parameters through scrolling buttons. This window also displays the numerical values of these parameters.

4 3D MAPPING

We developed 3D mapping of the generated fractal images. A virtual character wearing a swimming suit is used for 3D visualization of the results (Fig. 4). The user is able to rotate or zoom into the character using mouse based techniques. In this way, he/she could get better estimation of the esthetic properties of the generated images. Generation of corresponding high resolution images could be launched via the main interface. The associated file is required is required for fabric printing using a digital printing.

Figure 5: Snapshot of the window containing the 3D layout.

5 CONCLUSION AND FUTURE WORK

This paper describes and analyzes a basic model of IFS and addresses the issue of controlling the fractal images they generate. We specially focus on fast exploration of the IFS parameters space and aesthetic properties of the generated images through both 2D visualization and 3D mapping on virtual characters. Further development will be on automatic strategies for controlling the parameters selection through quantitative criteria devised to convey aesthetic properties.

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