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NOISE IMPROVEMENT AND STOCHASTIC RESONANCE IN PARALLEL ARRAYS OF SENSORS WITH SATURATION

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ABSTRACT

Linear sensors very commonly display saturation at large inputs. Large information-carrying signals are thus distorted in their transduction by such sensors. We show that purposeful addition of noise, in various conditions, can be used as a means of reducing the distortion experienced by large signals in their transduction. This can occur when the noise plays the role of a random bias to isolated sensors. This can occur also through a distinct mechanism of improvement by noise, arising when the sensors are replicated in parallel arrays providing enriched representation capability. These two distinct mechanisms of improvement by noise, demonstrated here with saturating sensors, can be viewed as extensions of the phenomenon of stochastic resonance, both in isolated nonlinearities and in arrays of nonlinearities.

1. INTRODUCTION

When signal and noise interact nonlinearly, there exists a possibility for the noise to cooperate constructively with the signal. This can give way to an improvement of the performance of some processing done on the signal thanks to the action of the noise. This possibility is generically known under the name of stochastic resonance [1, 2, 3]. Since its introduction some twenty years ago in the context of geophysics [4, 5], stochastic resonance has gradually been observed in a still-increasing variety of processes, including electronic circuits [6, 7], optical devices [8, 9], chemical reactions [10, 11], neurons [12, 13].

Thus far, stochastic resonance has mainly been reported with nonlinear systems incorporating thresholds or potential barriers [14, 2, 15, 3, 16]. In these circumstances, the mechanism of improvement, qualitatively, is that the noise assists small signals in overcoming the thresholds or barriers. Recently, another form of stochastic resonance was proposed in [17, 18], with parallel arrays of threshold devices, under the name of suprathreshold stochastic resonance. This form in [17, 18] applies to signals of arbitrary amplitude, which do not need to be small and subthreshold, whence the name. This suprathreshold stochastic resonance operates through a distinct mechanism of improvement, based on independent noises injected onto the devices, to induce more variability and a richer representation capability in the individual responses collected over the array. Several extensions of this form of suprathreshold stochastic resonance have been proposed in [19, 20, 21, 22]. The name "suprathreshold" stochastic resonance might suggest that the threshold is an essential ingredient. In the present report, we will show that a comparable effect of noise improvement in parallel arrays of sensors, can be obtained with threshold-free devices representing sensors linear for small to moderate inputs and saturating at large inputs. The present results extend both on stochastic resonance concerning isolated saturating nonlinearities [23] and concerning arrays of threshold nonlinearities [17, 22].

2. SENSORS WITH SATURATION

We consider a generic sensor whose input–output characteristic is modeled by the memoryless function $g(.)$. A central concern in sensors design is often to realize a characteristic $g(.)$ which is as linear as possible, at least for inputs which are not too large. At the same time, at large inputs, linear sensors very commonly will display saturation. We will model such sensors, alternatively, by the soft saturation

$$g(u) = \tanh(\beta u) \; , \tag{1}$$

or the hard-limiting saturation

$$g(u) = \begin{cases} -1 & \text{for} \quad \beta u \leq -1 \\ \beta u & \text{for} \quad -1 < \beta u < 1 \\ 1 & \text{for} \quad \beta u \geq 1 \; , \end{cases} \tag{2}$$

with the sensitivity parameter $\beta > 0$.

These saturating sensors $g(.)$ are used here for the transduction of an input signal $x(t)$. This input $x(t)$ is considered of sufficiently large amplitude, so as to drive the sensors in their saturating regions, at least on some occasions, in such a way that $x(t)$ experiences a strong distortion in its transduction. We will show that this distortion can be reduced by addition of noise, via two distinct possible mechanisms of stochastic resonance.

For this purpose, we introduce a parallel array of N identical sensors $g(.)$ receiving the input signal $x(t)$, according to the configuration of Fig. 1, similar to the architecture also considered in [24, 25, 17]. For the stochastic resonance, we arrange for the possibility of a noise $\eta_i(t)$, independent of $x(t)$, to be added to $x(t)$ at each sensor i. Accordingly, each sensor i produces the output signal

$$y_i(t) = g[x(t) + \eta_i(t)] \; , \qquad i = 1, 2, \dots N \; . \tag{3}$$

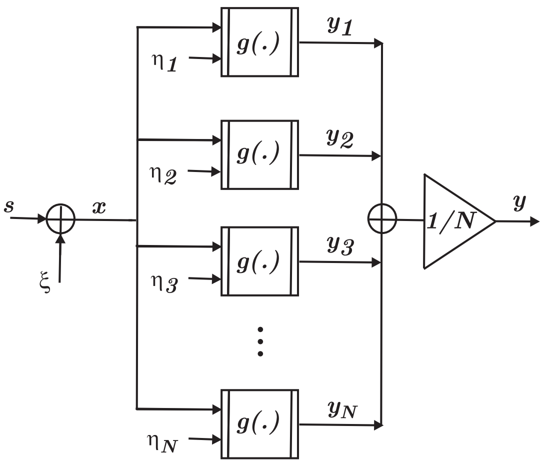

Fig. 1. Parallel array of N identical sensors $g(.)$.

The N sensor noises $\eta_i(t)$ are white, mutually independent and identically distributed with cumulative distribution function $F_\eta(u)$ and probability density function $f_\eta(u) = dF_\eta(u)/du$. The response $y(t)$ of the array is obtained by averaging the outputs of all the sensors, as

$$y(t) = \frac{1}{N}\sum_{i=1}^{N} y_i(t) \; . \tag{4}$$

3. TRANSMISSION OF A PERIODIC SIGNAL

To illustrate the possibility of a noise-improved transmission with the saturating sensors, we consider as our information-carrying signal a deterministic periodic component $s(t)$ with period T_s and "large" amplitude. This signal $s(t)$ is corrupted by $\xi(t)$ a stationary white noise, independent of both $s(t)$ and the $\eta_i(t)$, and with probability density function $f_\xi(u)$. The sensors thus receive at their input the signal-plus-noise mixture $x(t) = s(t) + \xi(t)$.

An appropriate measure to assess the transmission of the periodic signal $s(t)$ by the array, is a signal-to-noise ratio (SNR), defined in the frequency domain, and which measures, in the output signal $y(t)$, the part contributed by the periodic input and the part contributed by the noise [26, 2]. When $s(t)$ is deterministic periodic with period T_s, the output signal $y(t)$ generally is a cyclostationary random signal, endowed with a power spectrum containing spectral lines at integer multiples of $1/T_s$, emerging out of a continuous noise background [26]. The SNR $\mathcal{R}_{\rm out}$ is defined as the power contained in the output spectral line at the fundamental $1/T_s$ divided by the power contained in the noise background in a small frequency band ΔB around $1/T_s$.

For the output signal $y(t)$ of Eq. (4), the power contained in the output spectral line at the frequency $1/T_s$ is given [26] by $|\overline{Y}_1|^2$, where $\overline{Y}_1$ is the Fourier coefficient at the fundamental of the T_s-periodic nonstationary output expectation $\mathrm{E}[y(t)]$, i.e.

$$\overline{Y}_1 = \left\langle \mathrm{E}[y(t)] \exp\Big(-\imath\frac{2\pi}{T_s}t\Big)\right\rangle \; , \tag{5}$$

with the time average defined as

$$\langle ... \rangle = \frac{1}{T_s}\int_0^{T_s} ... \, dt \; . \tag{6}$$

The magnitude of the continuous noise background in the output spectrum is measured [26] by the stationarized output variance $\langle \mathrm{var}[y(t)]\rangle$, with the nonstationary variance given by $\mathrm{var}[y(t)] = \mathrm{E}[y^2(t)] - \mathrm{E}[y(t)]^2$ at a fixed time t.

A signal-to-noise ratio $\mathcal{R}_{\rm out}$, at the fundamental frequency $1/T_s$ in the ouput $y(t)$, follows as

$$\mathcal{R}_{\rm out} = \frac{|\overline{Y}_1|^2}{\langle \mathrm{var}[y(t)]\rangle \, \Delta t \, \Delta B} \; , \tag{7}$$

where Δt is the time resolution of the measurement (i.e. the signal sampling period in a discrete-time implementation), throughout this study we take $\Delta t \Delta B = 10^{-3}$.

At time t, for a fixed given value x of the input $x(t)$, one has, according to Eq. (4), the conditional expectations

$$\mathrm{E}[y(t)|x] = \mathrm{E}[y_i(t)|x] \tag{8}$$

and

$$\mathrm{E}[y^2(t)|x] = \frac{1}{N}\,\mathrm{E}[y_i^2(t)|x] + \frac{N-1}{N}\,\mathrm{E}^2[y_i(t)|x] \tag{9}$$

which are independent of i since the $\eta_i(t)$ are i.i.d. Furthermore, because of Eq. (3), one has for any i,

$$\mathrm{E}[y_i(t)|x] = \int_{-\infty}^{+\infty} g(x+u) f_\eta(u) du \tag{10}$$

and

$$\mathrm{E}[y_i^2(t)|x] = \int_{-\infty}^{+\infty} g^2(x+u) f_\eta(u) du \; . \tag{11}$$

Also, since $x(t) = s(t) + \xi(t)$, the probability density for the value x is $f_\xi\big(x - s(t)\big)$, yielding

$$\mathrm{E}[y(t)] = \int_{-\infty}^{+\infty} \mathrm{E}[y(t)|x] f_\xi\big(x - s(t)\big) dx \; , \tag{12}$$

and

$$\mathrm{E}[y^2(t)] = \int_{-\infty}^{+\infty} \mathrm{E}[y^2(t)|x] f_\xi\big(x - s(t)\big) dx \; . \tag{13}$$

Now, from these Eqs. (8)–(13), the output SNR $\mathcal{R}_{\rm out}$ of Eq. (7) can be computed for the transmission of $s(t)$ by the array. Especially, the SNR $\mathcal{R}_{\rm out}$ is obtained as a function of the properties of the input noise $\xi(t)$ and of the sensor noises $\eta_i(t)$ conveyed by the probability densities $f_\xi(u)$ and $f_\eta(u)$, and for arbitrary choices concerning the waveform of the periodic component $s(t)$, the sensor characteristic $g(.)$, and the array size N.

4. NOISE-IMPROVED TRANSMISSION

4.1. Enriched representation by noise

For the saturating sensors of Eqs. (1)–(2), we demonstrate with the SNR $\mathcal{R}_{\rm out}$ in various representative conditions, that the transmission of large input signals $s(t)$ that saturate the sensors, can be improved by addition of a nonzero amount of the sensor noises $\eta_i(t)$.

Whereas the input noise $\xi(t)$ is considered as a noise imposed by the external environment (and chosen Gaussian throughout), the sensor noises $\eta_i(t)$ are considered as purposely added noises applied to influence the operation of the array. When the noises $\eta_i(t)$ are distributed with a density $f_\eta(u)$ uniform over $[-\sqrt{3}\sigma_\eta, \sqrt{3}\sigma_\eta]$, associated with the soft saturation of Eq. (1), the integrals of Eqs. (10)–(11) give

$$\mathrm{E}[y_i(t)|x] = \frac{1}{2\sqrt{3}\beta\sigma_\eta}\ \ln\left(\frac{\cosh\left[\beta(x+\sqrt{3}\sigma_\eta)\right]}{\cosh\left[\beta(x-\sqrt{3}\sigma_\eta)\right]}\right) \quad (14)$$

and

$$\mathrm{E}[y_i^2(t)|x] = \frac{1}{2\sqrt{3}\beta\sigma_\eta}\Big(\tanh\left[\beta(x-\sqrt{3}\sigma_\eta)\right] + 2\sqrt{3}\beta\sigma_\eta - \tanh\left[\beta(x+\sqrt{3}\sigma_\eta)\right] \Big) . \quad (15)$$

For the transmission of a sinusoidal input signal

$$s(t) = A\cos(2\pi t/T_s) \ , \quad (16)$$

the SNR $\mathcal{R}_{\rm out}$ resulting from Eq. (7) is represented in Fig. 2.

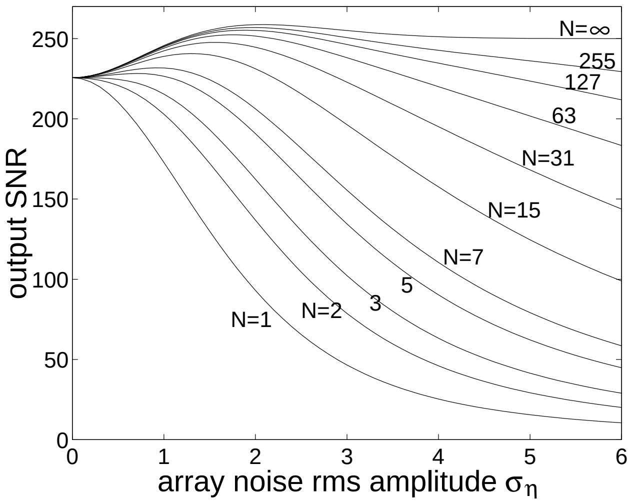

Fig. 2. Signal-to-noise ratio $\mathcal{R}_{\rm out}$ of Eq. (7) at the output of the array of sensors, as a function of the rms amplitude σ_η of the sensor noises $\eta_i(t)$ chosen zero-mean uniform. The periodic input is $s(t) = 2\cos(2\pi t/T_s)$ buried in a zero-mean Gaussian noise $\xi(t)$ with rms amplitude $\sigma_\xi = 2$. The array is made of N identical sensors $g(.)$ with $\beta = 1$ and the soft saturation of Eq. (1).

When the noises $\eta_i(t)$ are distributed with a zero-mean Gaussian density $f_\eta(u)$ with standard deviation σ_η, associated with the hard saturation of Eq. (2), the integrals of Eqs. (10)–(11) give

$$\mathrm{E}[y_i(t)|x] = 1 - (1+\beta x)\ F_\eta\Big(-\frac{1}{\beta}-x\Big) - (1-\beta x)\ F_\eta\Big(\frac{1}{\beta}-x\Big) + \beta\sigma_\eta^2\left[f_\eta\Big(-\frac{1}{\beta}-x\Big) - f_\eta\Big(\frac{1}{\beta}-x\Big)\right] \quad (17)$$

and

$$\mathrm{E}[y_i^2(t)|x] = 1 + \Big(1-\beta^2x^2-\beta^2\sigma_\eta^2\Big)\left[F_\eta\Big(-\frac{1}{\beta}-x\Big) - F_\eta\Big(\frac{1}{\beta}-x\Big)\right] - \beta^2\sigma_\eta^2\left[\Big(x+\frac{1}{\beta}\Big)\ f_\eta\Big(\frac{1}{\beta}-x\Big) - \Big(x-\frac{1}{\beta}\Big)\ f_\eta\Big(-\frac{1}{\beta}-x\Big)\right] . \quad (18)$$

For the transmission of the sinusoidal signal $s(t)$ of Eq. (16), the SNR $\mathcal{R}_{\rm out}$ resulting from Eq. (7) is represented in Fig. 3.

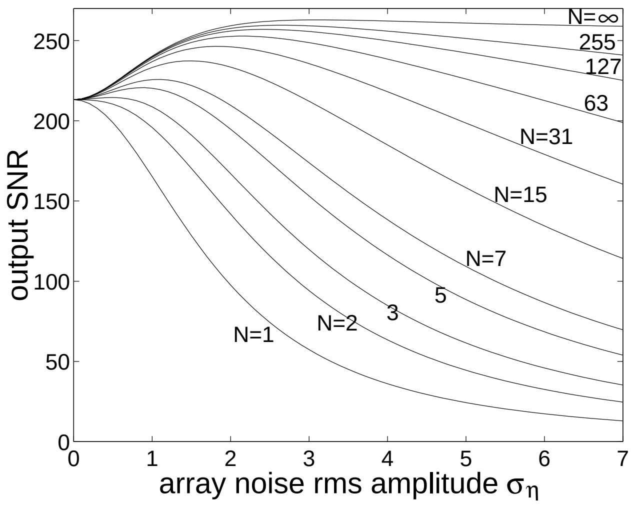

Fig. 3. Signal-to-noise ratio $\mathcal{R}_{\rm out}$ of Eq. (7) at the output of the array of sensors, as a function of the rms amplitude σ_η of the sensor noises $\eta_i(t)$ chosen zero-mean Gaussian. The periodic input is $s(t) = 2\cos(2\pi t/T_s)$ buried in a zero-mean Gaussian noise $\xi(t)$ with rms amplitude $\sigma_\xi = 2$. The array is made of N identical sensors $g(.)$ with $\beta = 1$ and the hard saturation of Eq. (2).

Figures 2–3 reveal similar features concerning the transmission by the saturating sensors. The sinusoidal signal $s(t)$ of Eq. (16), apart from its large amplitude A that saturates the sensors, is optimally centered in relation to the characteristics of Eqs. (1)–(2). As a result, in Figs. 2–3, with a single sensor ($N = 1$) no improvement of the output SNR $\mathcal{R}_{\rm out}$ occurs as the level σ_η of the sensor noises $\eta_i(t)$ is increased, but a monotonic degradation of $\mathcal{R}_{\rm out}$ as σ_η grows. By contrast, when the sensors are replicated in arrays of moderate size ($N \gtrsim 5$), the added noises $\eta_i(t)$ start to play a constructive role, and an improvement of the output SNR $\mathcal{R}_{\rm out}$ occurs. Thanks to the addded noises $\eta_i(t)$, each sensor in the array responds differently to the same input signal $x(t)$, instead of responding in unison. This allows a richer representation of the input $x(t)$ when all the individual sensor outputs $y_i(t)$ are collected over the array, and this translates into an enhancement of the SNR $\mathcal{R}_{\rm out}$ which culminates at a maximimum for an optimal nonzero amount of the sensor noises. Thanks to this action of the noise enabling a richer represenation capability, the array with added noises $\eta_i(t)$ performs better than a single sensor with no added noise. Figures 2–3 also show that the efficacy of enhancement of the output SNR $\mathcal{R}_{\rm out}$ gets more pronounced as the array size N increases.

4.2. Biasing by noise

For the signal $s(t)$ of Eq. (16), which is optimally centered in relation to the sensor characteristics of Eqs. (1)–(2), the mechanism of improvement by addition of noise is, qualitatively, an enriched representation capability afforded by the sensor noises $\eta_i(t)$ when the sensors are replicated in an array. As a result, this mechanism does not operate with a single sensor, as shown by the case $N = 1$ in Figs. 2–3 where no improvement of the SNR $\mathcal{R}_{\rm out}$ takes place when the noise level σ_η increases.

Another distinct mechanism is possible for improvement by addition of noise. This mechanism will operate in the presence of a signal $s(t)$ not optimally centered in relation to the sensor characteristics of Eqs. (1)–(2). On the contrary, the signal $s(t)$ will be strongly offset towards the saturating regions of the sensors, for instance according to the model

$$s(t) = S_0 + A\cos(2\pi t/T_s) , \qquad (19)$$

where S_0 is a "large" DC component of the sinusoidal input $s(t)$.

In such condition, Fig. 4 shows the resulting SNR $\mathcal{R}_{\rm out}$ from Eq. (7) at the output of an array of hard-saturation sensors as in Eq. (2).

In the condition of Fig. 4, because of the strong offset $S_0 = 3$, the input signal $s(t) = 3 + 2\cos(2\pi t/T_s)$ evolves permanently above the saturation level $1/\beta = 1$ of the sensors with the hard characteristic of Eq. (2). Consequently, $s(t)$ alone would only elicit a constant response $y_i(t) = 1$ from the sensors, and any information concerning the temporal variation of $s(t)$ would be lost at the output. This would translate into a zero SNR $\mathcal{R}_{\rm out}$ at the output. The presence of the input noise $\xi(t)$ induces a cooperation with $s(t)$, so as to bring, on some occasions, the input signal $s(t) + \xi(t)$ back into the linear part of the characteristic of the sensors of Eq. (2). This produces a response $y_i(t)$ at the sensor output, which bears some correlation with the coherent input $s(t)$. This translates into a nonzero output SNR $\mathcal{R}_{\rm out}$ thanks to the action of the native input noise $\xi(t)$, which is visible in Fig. 4 at $\sigma_\eta = 0$ when no sensor noises $\eta_i(t)$ are added. This action of the noise can be viewed as a form of biasing by noise: addition of noise moves the operating zone of a nonlinearity towards a region more favorable, on average, to the processing of the coherent signal.

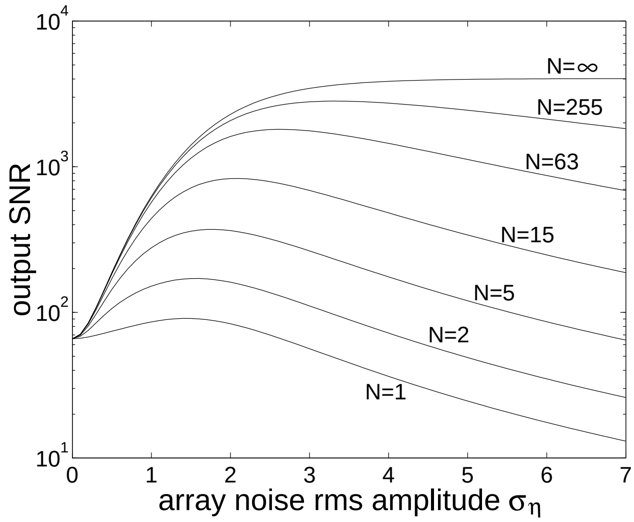

Fig. 4. Signal-to-noise ratio $\mathcal{R}_{\rm out}$ of Eq. (7) at the output of the array of sensors, as a function of the rms amplitude σ_η of the sensor noises $\eta_i(t)$ chosen zero-mean Gaussian. The periodic input is $s(t) = 3 + 2\cos(2\pi t/T_s)$ buried in a zero-mean Gaussian noise $\xi(t)$ with rms amplitude $\sigma_\xi = 0.5$. The array is made of N identical sensors $g(.)$ with $\beta = 1$ and the hard saturation of Eq. (2).

This beneficial action of the noise persists when more noise is added through injection of the sensor noises $\eta_i(t)$. This is specifically observed at $N = 1$ in Fig. 4, with a single sensor, where the SNR $\mathcal{R}_{\rm out}$ can be increased by purposeful application of the noise $\eta_1(t)$. This injected noise $\eta_1(t)$ adds up to the native noise $\xi(t)$, and reinforces the favorable effect of the biasing by noise, the efficacy of which culminates for an optimal nonzero rms level of $\eta_1(t)$ maximizing $\mathcal{R}_{\rm out}$. At $N = 1$ with a single sensor, the array effect of Section 4.1 does not take place, and it is the distinct mechanism of biasing by noise which is responsible for the enhancement of the output SNR $\mathcal{R}_{\rm out}$ observed in Fig. 4 when the noise level σ_η is raised above zero.

At $N > 1$ in Fig. 4, in genuine arrays, both mechanisms of improvement by noise can operate: the array effect of enriched representation as in Section 4.1, and the effect of biasing by noise already occuring at $N = 1$.

Both mechanisms also apply, in a similar way, with arrays of soft-saturation sensors as in Eq. (1), as illustrated by Fig. 5.

5. CONCLUSION

The present results illustrate the possibility of two distinct mechanisms by which the noise can play a part in improving the transduction of an information-carrying signal by saturating sensors. Both mechanisms can also operate, in similar

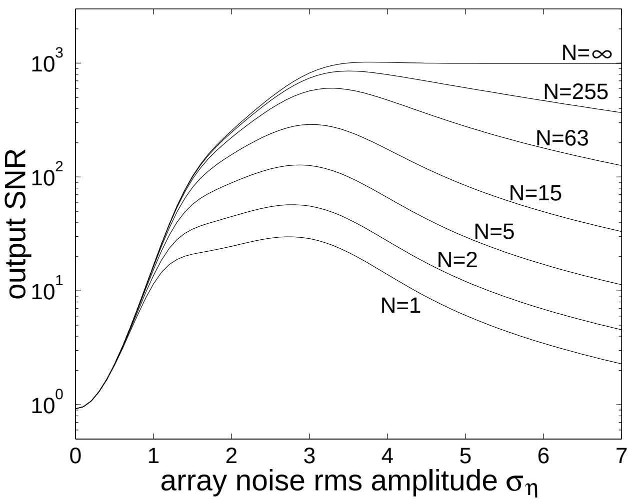

Fig. 5. Signal-to-noise ratio $\mathcal{R}_{\rm out}$ of Eq. (7) at the output of the array of sensors, as a function of the rms amplitude σ_η of the sensor noises $\eta_i(t)$ chosen zero-mean uniform. The periodic input is $s(t) = 5 + \cos(2\pi t/T_s)$ buried in a zero-mean Gaussian noise $\xi(t)$ with rms amplitude $\sigma_\xi = 0.5$. The array is made of N identical sensors $g(.)$ with $\beta = 1$ and the soft saturation of Eq. (1).

ways, in other types of nonlinearities like for instance threshold nonlinearities.

The first mechanism is an effect of biasing by noise. It can operate in isolated nonlinearities, where added noise somehow shifts the operating zone towards a region more favorable to the signal. This mechanism is also known as a form of stochastic resonance, especially reported and analyzed on the occasion of threshold nonlinearities [27, 26, 3]. The present results illustrate that this mechanism also applies with saturating nonlinearities.

The second mechanism of improvement by noise is an effect of enriched representation in nonlinear arrays. Added noises force devices replicated in a parallel array to respond differently to a common input signal, henceforth producing a richer signal representation at the global level of the array. When applied to threshold nonlinearities, this mechanism was presented under the name of suprathreshold of stochastic resonance in [17]. Our results here demonstrate that this mechanism is not restricted to threshold nonlinearities. It is essentially a collective effect in nonlinear arrays, which does not require a threshold, but can in fact occur in many types of smooth, threshold-free, nonlinearities.

Also, the two mechanisms of improvement by noise in saturating sensors were shown here in the transduction of a periodic signal measured by the SNR $\mathcal{R}_{\rm out}$ of Eq. (7), but based on previous studies of stochastic resonance, it is likely that similar effects will carry over to the transmission or processing of other (nonperiodic) signals with other measures of performance.

Beyond the case of the saturating nonlinearities that we have tested here, it is important to bear in mind that these two distinct mechanisms of improvement by noise, can possibly apply to improve the performance of many different types of nonlinear sensors. This will be specially relevant when no full control is available on a nonlinear sensor characteristic, and when henceforth purposeful addition of noise can be envisaged as a means of improving the performance.

Also, the reported mechanisms of improvement by noise are relevant to a class of natural processes involved in complex information processing tasks, namely, neuronal processes. These processes incorporate in a prevalent way the basic ingredients lying at the root of the reported effects (noise, arrays, nonlinearities, saturation, threshold, ...), from which, and by which, very efficient information processing ensues. This can suggest the development of new generations of nonlinear sensing arrays, with neural inspiration, for intelligent information processing benefiting from their ability to exploit the noise.

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