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NOISE-ASSISTED IMAGE TRANSMISSION WITH SPECKLE NOISE

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1. ABSTRACT

A simple model of coherent imaging system is shown to exhibit noise-assisted transmission with increasing level of a multiplicative speckle noise. Such an effect is related to the stochastic resonance phenomena where noise can assist signal in its processing or transmission by nonlinear systems.

2. INTRODUCTION

Stochastic resonance (SR) is a nonlinear effect whereby the transmission or processing of an information-carrying signal can be improved by means of an increase of the noise level. Stochastic resonance has been reported in various types of physical systems, including electronic circuits, lasers, magnetic superconducting devices, or neural systems (for reviews see [1] in physics, [2] in electrical engineering, and [3] in signal processing). In all the above systems, stochastic resonance is observed with a temporal (monodimensional) information signal. Up to now, only a few studies have addressed stochastic resonance with spatial (bidimensional) signals or images. This type of stochastic resonance has been observed in optical devices [4], in image perception by the visual system (see [5] for the initial psychophysical experiment and see [6] for a recent review), in super-resolution techniques for imaging sensors [8, 7], and recently in image restoration [9].

We demonstrate here a new instance of stochastic resonance applied, to our knowledge for the first time, to coherent imaging (SONAR, SAR, or LASER). This takes the form of a noise-assisted image transmission by a nonlinear sensor in presence of a speckle noise. As we will recall here, speckle noise can be viewed as a multiplicative noise. This feature is, by itself, challenging in the framework of stochastic resonance since most of the studies demonstrating noise-assisted signal processing have considered additive coupling between signal and noise. By contrast with the present report, the possibility of stochastic resonance with a multiplicative signal–noise coupling has been demonstrated in [10, 11, 12, 13, 14, 15] for temporal signals exclusively.

3. A TRANSMISSION PROBLEM

Let us consider a gray-level image $S(u,v)$ where the pixels are indexed by integer coordinates (u,v) and have intensity $S(u,v) \in [0,1]$. We assume that image $S(u,v)$ is large enough so that a statistical description of the distribution of intensities on the image is meaningful: image $S(u,v)$ possesses an empirical histogram of intensities, the normalized version of which defining the probability density $p_S(j)$ for the intensity on image $S(u,v)$. A noise $N(u,v)$, statistically independent of $S(u,v)$, corrupts each pixel of image $S(u,v)$. The noise values are independent from pixel to pixel, and are distributed according to the probability density $p_N(j)$. The noise N is ergodic and $p_N(j)$ is the density, equivalently, at a given pixel over successive realizations of $N(u,v)$ or over the ensemble of pixels of image $N(u,v)$ in any given realization. The input image $S(u,v)$ and the noise $N(u,v)$ are coupled to produce an intermediate image $X(u,v)$ which impinges onto a nonlinear imaging detector producing the output image $Y(u,v)$ according to:

$$Y(u,v) = g[X(u,v)] \;, \tag{1}$$

the input–output characteristic $g(.)$ of the imaging system and the image–noise coupling being, at this stage, arbitrary functions.

4. QUANTIFICATION OF SR WITH IMAGES

We now introduce similarity measures between the information-carrying input image $S(u,v)$ and output image $Y(u,v)$. One possibility is provided by the normalized cross-covariance

$$C_{SY} = \frac{\langle (S - \langle S \rangle)(Y - \langle Y \rangle) \rangle}{\sqrt{\langle (S - \langle S \rangle)^2 \rangle \, \langle (Y - \langle Y \rangle)^2 \rangle}} \;, \tag{2}$$

where $\langle . \rangle$ denotes an average over the images. C_{SY} is close to one when images S and Y carry strongly similar structures, and close to zero when the images are unrelated. Another possibility is to define a mutual information I_{SY} between the pixels of images $S(u,v)$ and $Y(u,v)$, as

$$I_{SY} = H(Y) - H(Y|S) \;, \tag{3}$$

with standard definitions [18] for the entropies $H(.)$ [1].

$$H(Y) = \int_j -dj \, p_Y(j) \log_2[p_Y(j)] \;, \tag{4}$$

[1] Obvious alternative standard formulas written with probabilities instead of densities will replace Eqs. (4)–(5) if Y happens to take discrete rather than continuously distributed values.

and

$$H(Y|S) = \int_s ds\, p_S(s) \int_j -dj\, p_{Y|s}(j) \log_2[p_{Y|s}(j)] \quad (5)$$

with the conditional density defined by $p_{Y|s}(j)dj = \Pr\{Y \in [j, j+dj) \,|\, S = s\}$, and the marginal density which is $p_Y(j) = \int_s ds\, p_S(s)\, p_{Y|s}(j)$.

In principle, when $p_S(j)$, $p_N(j)$ and $g(.)$ are all three given, it is possible to theoretically predict the input–output similarity measures C_{SY} and I_{SY}. By such means, one has thus in principle access to measures C_{SY} and I_{SY}. Once in their possession, one can then check whether these measures experience nonmonotonic evolutions culminating at a maximum when the level of the noise N is raised, this identifying a stochastic resonance effect. For illustration of the capability of measures C_{SY} and I_{SY} to quantify a stochastic resonance effect in image detection, we shall consider a simple case where C_{SY} and I_{SY}, in addition to their experimental evaluation through pixels counting on the images, can also be explicitly predicted theoretically, thus providing a basis for comparison and assessment of the method.

To have an easy description of the statistical properties of input image $S(u, v)$ conveyed by the gray-level distribution $p_S(u)$, we shall consider in the sequel a binary image with intensities $S(u, v) \in \{R_0, R_1\}$. For illustration we have considered in Figs. 1 and 2 a 300×273 image consisting of the shape of an airplane, for which the probability of the bright pixels is $\Pr\{S = R_1\} = p_1 = 0.27$ and $\Pr\{S = R_0\} = 1 - p_1$ for the dark pixels. The imaging detector $g(.)$ is taken as a hard limiter with threshold θ, i.e.

$$g[X(u, v)] = \begin{cases} 0 & \text{for} \quad X(u, v) \le \theta \\ 1 & \text{for} \quad X(u, v) > \theta \,. \end{cases} \quad (6)$$

This hard limiter constitutes a very basic model for imaging systems when they operate, in the low flux domain, close to their threshold. In addition, this nonlinear model presents the advantage of being completely tractable analytically. Alternatively, the hard limiter in Eq. (6) can also be viewed as a single step in a multilevel quantizer or a threshold in a high level image processing task like segmentation or detection.

5. APPLICATION TO COHERENT IMAGING

In coherent imaging systems, a coherent illumination of a scene with inherent irregularities induces scattered waves having their phases which interfere very rapidly over the returned wave front. On an imaging detector, this produces images with very irregular variations of intensity with a noise-like grainy appearance called speckle. The effect can be modeled [17] as a multiplicative noise $N(u, v)$, that when acting on the input image $S(u, v)$, produces on the detector of Eq. (6) the nonlinear multiplicative mixture

$$X(u, v) = S(u, v) \times N(u, v) \,, \quad (7)$$

with a probability density $p_N(j)$ of the speckle noise $N(u, v)$ given by

$$p_N(j) = \frac{1}{\sigma_N} \exp\left(-\frac{j}{\sigma_N}\right) \,, \qquad j \ge 0 \quad (8)$$

with mean and standard deviation σ_N and rms amplitude $\sqrt{2}\sigma_N$. $S(u, v)$ corresponds to an image of the reflectivity of the scene with two homogeneous regions object and background characterized by $\{R_0, R_1\}$. Equations (7) and (8), constitute the simplest model of speckle noise which is valid if the detector pixel size is smaller than the speckle grain size [17].

We have a possibility of a theoretical description of the imaging detector of Eq. (6) through an explicit theoretical derivation of C_{SY} and I_{SY}. We introduce the conditional probability $p_{1k} = \Pr\{Y = 1 \,|\, S = R_k\}$ which amounts to $\Pr\{N > \theta/R_k\} = 1 - F_N(\theta/R_k)$, with $k \in \{0, 1\}$, where $F_N(j) = \int_{-\infty}^{j} p_N(j')dj'$ is the cumulative distribution of the noise N. When the probability density $p_N(j)$ of the speckle noise is given by Eq. (8), we have

$$F_N(j) = 1 - \exp\left(-\frac{j}{\sigma_N}\right), \qquad j \ge 0 \,. \quad (9)$$

Similarly, we define $\Pr\{Y = 1\} = q_1$ with probability q_1 expressable as $q_1 = p_1 p_{11} + (1 - p_1)p_{10}$. With Eqs. (2)–(3), we shall quantify the similarity between the output image $Y(u, v)$ and a binary reference image $S'(u, v)$ similar to $S(u, v)$ but with $R_0 = 0$ (background) and $R_1 = 1$ (object).

Theoretically, for a binary image $S'(u, v)$, we have the average $\langle S'\rangle = 1\times\Pr\{S' = 1\}+0\times\Pr\{S' = 0\}$, thus $\langle S'\rangle$ is simply $p_1 = \Pr\{S' = 1\}$. In the same way we have $\langle S'Y\rangle = 1 \times 1 \times \Pr\{Y = 1\,;\, S' = 1\} = p_1 p_{11}$. The numerator of Eq. (2) is expressable as $\langle S'Y\rangle - \langle S'\rangle\langle Y\rangle$. The denominator of Eq. (2) is nothing else than the product of the standard deviations $\mathrm{sd}(S') \times \mathrm{sd}(Y)$, with $[\mathrm{sd}(S')]^2 = \langle S'^2\rangle - \langle S'\rangle^2$ and $\langle S'^2\rangle = 1^2 \times \Pr\{S' = 1\} + 0^2 \times \Pr\{S' = 0\} = p_1$. Also $[\mathrm{sd}(Y)]^2 = \langle Y^2\rangle - \langle Y\rangle^2$ with $\langle Y^2\rangle = q_1$. Collecting these results, we have for the cross-covariance of Eq. (2)

$$C_{S'Y} = \frac{p_1 p_{11} - p_1 q_1}{\sqrt{(p_1 - p_1^2)(q_1 - q_1^2)}} \,. \quad (10)$$

Using the function $h(u) = -u \log_2(u)$, the entropies are

$$\begin{aligned} H(Y) \quad &= \quad h[p_1 p_{11} + (1 - p_1)p_{10}] \\ &\quad + h[p_1(1 - p_{11}) + (1 - p_1)(1 - p_{10})] \,, \quad (11) \end{aligned}$$

and

$$\begin{aligned} H(Y|S') \quad &= \quad (1 - p_1)[h(p_{10}) + h(1 - p_{10})] \\ &\quad + p_1[h(p_{11}) + h(1 - p_{11})] \,, \quad (12) \end{aligned}$$

and $I_{S'Y}$ of Eq. (3) follows.

As visible in Fig. 1, the two similarity measures $C_{S'Y}$ and $I_{S'Y}$ undergo a resonant evolution as the noise level is raised, and culminate at a maximum for an optimal noise level. The theoretical predictions for measures $C_{S'Y}$ and $I_{S'Y}$ shown in

Fig. 1 closely match their experimental evaluations and reproduce the nonmonotonic characteristics of stochastic resonance. The cooperative effect quantitatively illustrated in Fig. 1 can also be visually appreciated in Fig. 2. Another important feature to notice in Fig. 1 is that the cooperative effect of the noise is robustly preserved for all values of the hard limiter threshold θ. This is in contrast with the standard stochastic resonance mechanism acting with additive noise in nonlinear systems with thresholds, for which the information carrying signal has to be subthreshold to benefit from an injection of noise.

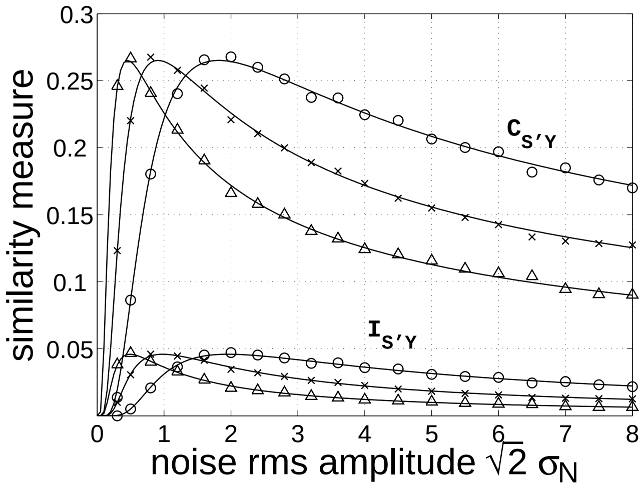

Fig. 1. Input–output similarity measures $C_{S'Y}$ of Eq. (2), $I_{S'Y}$ of Eq. (3) versus the rms amplitude of speckle noise N. The solid lines are theoretical predictions. The discrete data points are experimental evaluations through pixels counting on images for various values of the hard limiter threshold θ with $\triangle, \times, \circ$ respectively for $\theta = 0.4, 0.8, 1.6$ for image $S(u,v)$ intensities $\{R_0 = 1/2, R_1 = 1\}$.

Fig. 2. Output image $Y(u,v)$ of the hard limiter for increasing rms amplitude $\sqrt{2}\sigma_N$ of the speckle noise $N(u,v)$. From left to right $\sqrt{2}\sigma_N = 0.6, 1.8$ (optimal value), 6 with threshold $\theta = 1.6$ and $\{R_0 = 1/2, R_1 = 1\}$ as in Fig. 1.

In practice, real scenes may be more complex than binary images. We now consider the case where the reflectivity of the background and the object in $S(u,v)$, instead of being represented by two fixed values $\{R_0, R_1\}$, are both distributed over their own ranges of possible values. If the object or background reflectivity is not constant, the probability density function of the noise in these regions will no longer correspond to Eq. (8). Consider $\mathcal{R}$ the random variable representing the distribution of reflectivity in each region, the probability density function of the speckle in the intermediate image $X(u,v)$ is then given by

$$p_X(j) = \int \frac{1}{r} p_N(j/r) p_{\mathcal{R}}(r) dr \ , \tag{13}$$

with $p_N(.)$ the probability density function of Eq. (8) [17]. For illustration, we have considered, in Figs. 3 and 4, $\mathcal{R}$ uniformally distributed around minimum and maximum values (R_m, R_M) with $R_M - R_m = \Delta_R$. In this case, it is possible to have an exact analytical expression for $p_X(.)$ with

$$p_X(j) = \frac{1}{\sigma_N \Delta_R} \int_{j/(\sigma_N R_M)}^{j/(\sigma_N R_m)} \frac{1}{r} \times \exp(-r) dr \ , \tag{14}$$

and using a special function which is the exponential integral function $\mathrm{E}_1(j) = \int_1^{+\infty} \frac{1}{j'} \times \exp(-jj')dj'$, we have

$$p_X(j) = \frac{1}{\sigma_N \Delta_R} \left[\mathrm{E}_1\left(\frac{j}{\sigma_N R_M}\right) - \mathrm{E}_1\left(\frac{j}{\sigma_N R_m}\right) \right] \ . \tag{15}$$

The primitive function $\Phi(j)$ of function $\mathrm{E}_1(j)$ is expressable as $\Phi(j) = j \times \mathrm{E}_1(j) - \exp(-j)$ and the cumulative distribution function $F_X(.)$ of $p_X(.)$ is then following as

$$F_X(j) = \frac{1}{\Delta_R} \Bigg\{ \frac{j}{\sigma_N} \left[\mathrm{E}_1\left(\frac{j}{\sigma_N R_M}\right) - \mathrm{E}_1\left(\frac{j}{\sigma_N R_m}\right) \right] - R_M \left[\exp\left(-\frac{j}{\sigma_N R_M}\right) - 1 \right] + R_m \left[\exp\left(-\frac{j}{\sigma_N R_m}\right) - 1 \right] \Bigg\} \ . \tag{16}$$

As illustrated in Figs. 3 and 4, the nonmonotonic evolution of the similarity measures, observed in Fig. 1, is preserved when the reflectivity of the background and the object in $S(u,v)$ are distributed. This noise-assisted image transmission with speckle noise occurs in Figs. 3 and 4, even when the distributions in the two regions are overlapping.

6. DISCUSSION

We have demonstrated the possibility of a noise-assisted image transmission with multiplicative speckle noise in a coherent imaging system with a hard limiter. The evolution of input–output similarity measures in Figs. 1, 2, 3, 4 as a function of the level of the speckle noise shows the signature of stochastic resonance. In this study the information carrying signal does not have to be subthreshold to benefit from the noise. This is an important difference with stochastic resonance in nonlinear systems with additive signal–noise coupling. Another difference with usual stochastic resonance studies is the speckle noise standard deviation which is equal to its mean value. Therefore, when the standard deviation of the speckle noise is increased, the mean value of this noise is increased in the same way. This is not the case in usual

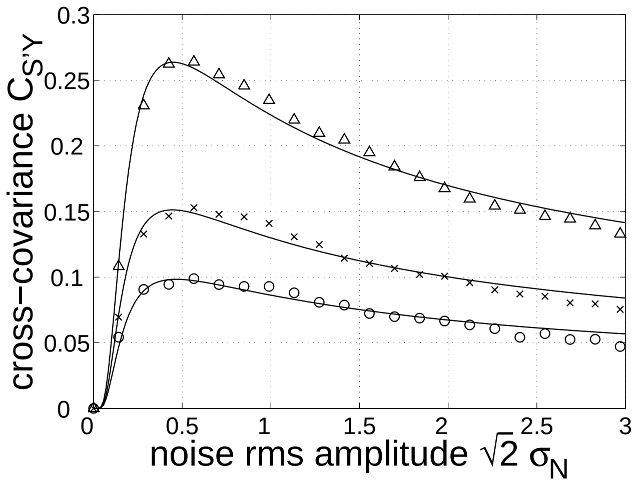

Fig. 3. Input–output normalized cross-covariance $C_{S'Y}$ of Eq. (2) versus the rms amplitude of speckle noise N. The solid lines are theoretical predictions. The discrete data points are experimental evaluations for various values of the reflectivity distribution with $\triangle, \times, \circ$ respectively for $\Delta_R = 0, 0.2, 0.3$ being identical in the background and the object regions. We take $R_m = 1/2$ in the background and $R_M = 1$ in the object (the distributions in the two regions are not overlapping when $\Delta_R = 0.2$ and overlapping when $\Delta_R = 0.3$). The hard limiter threshold θ is fixed to 0.4.

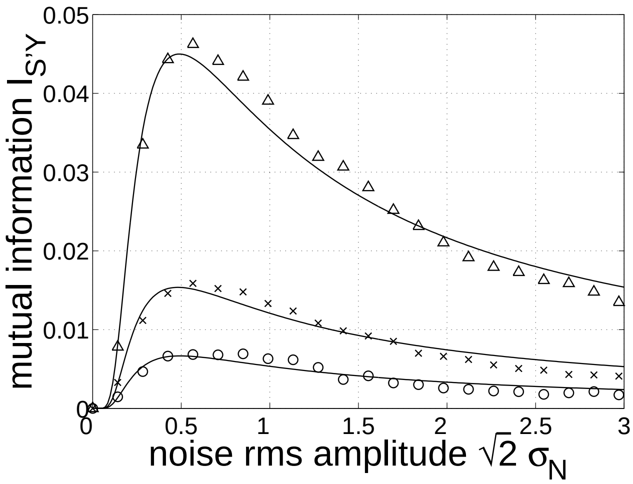

Fig. 4. Same as Fig. 3 with the input–output mutual information $I_{S'Y}$ of Eq. (3).

stochastic resonance studies involving additive noise with a mean kept constant when the standard deviation of the noise is raised.

A question arising, when a novel noise–assisted signal processing effect is uncovered, is how to tune the noise level. Here, the standard deviation of the speckle noise in Eq. (8) is related to the reflected intensity of the coherent wave [17] with

$$\sigma_N = I_i \times \mathcal{R} \,, \tag{17}$$

where I_i is the intensity of the incident wave and $\mathcal{R}$ the reflection coefficient on the scattering surface [2]. The reflection coefficient $\mathcal{R}$ often depends on the wavelength of the incident wave. It is therefore possible to adjust the level of the speckle noise by tuning two macroscopic deterministic parameters: intensity or wavelength of the incident wave.

The noise model chosen here can be considered as the simplest model of speckle and it could be interesting to pursue this work with more sophisticated models like general Gamma law distribution with a realistic model for reflectivity [19], or like speckle models taking into account the Poisson distribution at low flux [17]. The simple threshold detector chosen here could be replaced by a multilevel quantizer to assess the influence of speckle noise level on the quantization distorsion. Other image processing tasks like segmentation or detection could be studied in this framework. Finally, other imaging systems involving nonlinear signal–noise coupling also constitute perspectives of this work.

[2] $\mathcal{R}$ takes different values in the background and in the object (this is at the root of the contrast between the two regions). As a consequence, the standard deviation of the speckle noise takes different values in these two regions. The noise level in Fig. 1, 2, 3 and 4 corresponds to the common reference of $\mathcal{R} = 1$ before action of the multiplicative coupling by the object or background.

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