Noise-enhanced SNR gain in parallel array of bistable oscillators

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Introduction: Stochastic resonance (SR) is a well-known noise-induced nonlinear phenomenon existing in a variety of systems, and by which detection of a periodic or aperiodic signal can be enhanced by the addition of noise. The measure most frequently employed for conventional (periodic) SR is the signal-to-noise ratio (SNR). The SNR gain defined as the ratio of the output SNR over the input SNR, also attracts much interest in exploring situations where it can exceed unity. The SNR gain has been studied in the less stringent condition of narrowband noise [1]. Here, we address the more stringent condition of parallel array with size \(N\). The internal state \(x_i(t)\) of each oscillator is described as

\[
\frac{dx_i(t)}{dt} = F(x_i(t)) = x_i(t) - \frac{1}{N} \sum_{j=1}^{N} x_j(t) + \xi_i(t),
\]

where \(\xi_i(t) = \xi_i(t + 1)\). At time \(t\) and \(N \to \infty\), we have

\[
\lim_{N \to \infty} R_{s}(t) = \lim_{N \to \infty} \frac{\langle E[X_i(t)]E[X_i(t + 1)] \rangle}{N}
\]

and

\[
C_{y_i}(t) = \lim_{N \to \infty} C_{y_i}(t) - \lim_{N \to \infty} \frac{\langle E[X_i(t)]E[X_i(t + 1)] \rangle}{N^2}
\]

for \(i \neq j\) and \(i, j = 1, 2, \ldots, N\). Note that \(E[X_i(t)] = E[X_i(t + 1)]\). The output SNR is the power contained in the output spectral line 1/\(T_s\) divided by the power contained in the noise background in a small frequency bin \(\Delta f = 1/T_s\) around 1/\(T_s\), i.e.

\[
R_{\text{out}}(1/T_s) = \frac{\left| \tilde{Y}_s \right|^2}{\left( \text{var}[y(t)]H(1/T_s)\Delta f \right)}
\]

where \(C_{y_i}(0) = \text{var}[y(t)]\) is the stationary variance of \(y(t)\), \(C_{y_i}(t) = \text{var}[y(t)]b(t)\) and the correlation coefficient \(b(t)\) has a Fourier transform \(F[b(t)] = H(t)\) [2]. In the same way, the mixture of the state \(s(t) + \xi(t)\) has an input SNR as

\[
R_{\text{in}}(1/T_s) = \sigma_s^2 = \frac{A^2}{\Delta f} \left( \frac{\Delta f}{\sigma_s^2} \right)
\]

where \(\sigma_s\) is the rms amplitude of the discrete implementation of Gaussian noise \(z_i(t)\). Thus, the SR gain follows as

\[
G(1/T_s) = \frac{R_{\text{out}}(1/T_s)}{R_{\text{in}}(1/T_s)} = \frac{\left| \tilde{Y}_s \right|^2}{\text{var}[y(t)]H(1/T_s)A^2/4}
\]

Since the indices \(i\) and \(j\) are different, but arbitrary in (5) and (6), we can adopt two bistable oscillators, each embedded with independent noise, to evaluate the SNR gain of a parallel array with size \(N \to \infty\). This model is tractable and effective. Numerical evolution of SNR gains for generic arrays will be presented in future studies in detail.

\[\text{Fig. 1 SNR gain } G(1/T_s)\text{ against rms amplitude } \sigma_s \text{ of array noise } \eta(t)\]

Results: Fig. 1 shows that, for a given noisy input, the SNR gain

\[\text{SNR gain } G(1/T_s)\text{ against } \sigma_s\]

\[a \quad \text{SNR gain } G(1/T_s)\text{ against } \sigma_s\]

\[b \quad \text{SNR gain } G(1/T_s)\text{ against } \sigma_s\]
G(1/T_s) for \( N \geq 2 \) behaves as an SR-type function of the rms amplitude \( \sigma_n \) of the array noise \( \eta(t) \). It is more a collective effect of the nonlinear array, and appears for not only suprathreshold inputs but also subthreshold signals, as illustrated in Fig. 1. Importantly, Fig. 1 reveals that the regions of SNR gains \( G(1/T_s) \) rising above unity, via increasing the rms amplitude \( \sigma_n \), are possible for moderately large array \( N \). As the amplitude \( A \) increases, \( G(1/T_s) \) reaches a larger maximal value for the same array size \( N \). As \( N \to \infty \), the maximal \( G(1/T_s) \) is around 1.4 for \( A = 1.0 \), as seen in Fig. 1a, whereas the maximal \( G(1/T_s) \) is about 1.2 for \( A = 0.38 \), as shown in Fig. 1b. We note that \( \eta(t) \) are more controllable than the original input noise \( \xi(t) \). Thus, this nonlinear collective characteristic of dynamical arrays provides a preferable strategy for processing periodic signals than linear systems.

Conclusions: We have studied the SNR gain of a parallel uncoupled array of bistable oscillators operating in a fixed mixture of sinusoidal signal and Gaussian white noise. Owing to the added array noise, the regions of SNR gains exceeding unity are observed for both supra-threshold and subthreshold inputs. We have shown that the performance of an infinite array can be closely approached by a finite array of moderate size, indicating a promising application in array signal processing. Interesting open questions are, for instance, at a given input SNR, to find the optimal array parameters and optimal added noise level, to maximise the output SNR gain.

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References