

## INTERFERER REJECTION IMPROVED BY NOISE IN ULTRA-WIDEBAND TELECOMMUNICATIONS

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Ultra-wideband technology uses baseband transmission of low-power ultra-short information-bearing impulses, and represents a promising approach for very-high speed wireless communications with multiple access, as well as for low-rate high-accuracy positioning systems. The first demonstrations of the huge potential of ultra-wideband were based on hypotheses of perfect power control and multiuser interferences modeled as white Gaussian noise. Here, by explicitly modeling the interference with an external impulse signal, we demonstrate the possibility of improving the rejection of the interferer thanks to a constructive action of the noise. This is interpreted as a novel instance of the phenomenon of stochastic resonance or improvement by noise in signal processing.

*Keywords:* Ultra-wideband communications; interferer; noise; stochastic resonance.

### 1. Introduction

Ultra-wide band (UWB) telecommunications [1, 2] use low-power ultra-short impulses to convey information. This technology stands as a promising approach to provide very-high speed communications with multiple access, usable for new generations of wireless networks. In addition, the principle of UWB is also applicable to radar, imaging, and positioning systems. The basic proposal of UWB communications was introduced in [3] and described under the name of Time Hopping Pulse Position Modulation. With several users simultaneously, the multiple access to UWB systems was studied in [4, 5] with a single-user matched filter (SUMF) receiver. The SUMF is a simple correlation detector which has the advantage of

being a relatively easily implementable and low-cost receiver. In order to calculate the performance of such a UWB system [4, 5], the multiuser interference was supposed to be a zero-mean Gaussian random process, the channel was in free space propagation, and multipath was not considered. Later, further studies have followed, addressing complex propagation conditions [6], multipath channels [7], various receivers types [8, 9], and the Gaussian approximation for the multiple user interferences was worked out in [10, 11].

In the present article, we will consider the multiuser interference caused by only one external UWB signal (called interferer) on a UWB receiver receiving a principal UWB signal. We shall assume free space propagation, perfect power control and no synchronisation between our receiver and the interferer. Through explicit modeling of the interference, we will demonstrate the possibility of a novel instance of the phenomenon of stochastic resonance, under the form of a noise-improved interferer rejection in the UWB communication.

Stochastic resonance is a phenomenon which was introduced some twenty five years ago in the context of nonlinear physics [12, 13]. This (paradoxical) phenomenon describes situations where the processing of a signal or information-bearing quantity can be improved by the action of noise. In such circumstances, a sufficient amount of noise does not necessarily act as a nuisance but can play a beneficial role. Since its introduction, stochastic resonance has gradually been observed, under various forms, in a still-increasing variety of processes, including electronic circuits [14, 15], optical devices [16, 17], neurons [18–20]. Forms of stochastic resonance have also been reported for classical tasks of signal processing, like signal detection [21–23] or signal estimation [24, 25], where in each case a conventional measure of performance can be improved by the noise. The inventory of the various possible forms of stochastic resonance, or improvement by noise, is not yet completed; and novel forms continue to be uncovered and analyzed. In addition to its important conceptual significance, stochastic resonance presents interesting potentialities to investigate for innovative nonconventional techniques for information processing able to constructively exploit the noise. In the present paper, we will establish the possibility of a novel form of this effect of improvement by noise, at the occasion of a UWB communication process where we show that the rejection of an external interferer impulse signal can be enhanced by the action of noise.

## 2. A UWB Communication Model

### 2.1. *Signal model*

In Time Hopping Pulse Position Modulation UWB, time is divided in frames of duration  $T_f$ . Each frame is divided in  $N_s$  slots of duration  $T_s$ . A user sends one impulse per frame, in a particular slot. The slot used by a user can change from frame to frame, following a time hopping code. This allows to smooth the spectrum of the UWB signal. The position of the impulse in a slot may be slightly moved with a fixed time shift  $\delta$  in order to code the binary information carried by the signal. It is possible to use more than one impulse, it is to say more than one frame, to code one bit. But we focus in this article on a UWB system with one impulse per bit. Perfect power control is assumed, i.e. each UWB impulse sent by one of the  $N_u$  users is received with the same amplitude. In our particular study  $N_u = 2$ .

A sketch of a typical UWB signal is shown in Fig. 1, with reference to Eq. (4) for the notations.

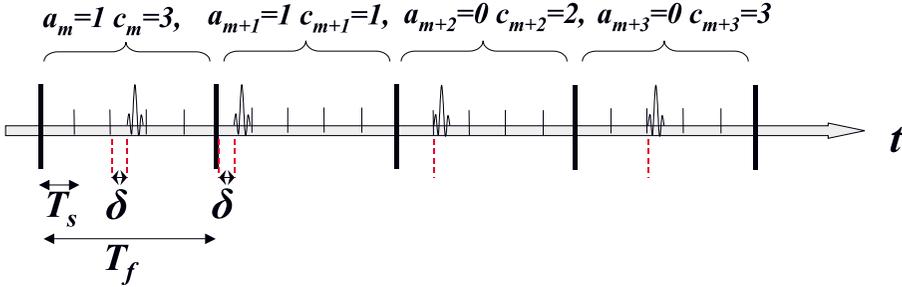


Fig. 1. A typical UWB signal, with reference to Eq. (4) for the notations.

We use classic UWB impulses having the shape of the second derivative of a Gaussian pulse [5], as used in most UWB studies, according to the analytical model

$$w(t) = A_w \left[ 1 - 4\pi \left( \frac{t}{T_w} \right)^2 \right] \exp \left[ -2\pi \left( \frac{t}{T_w} \right)^2 \right], \tag{1}$$

with the constant  $A_w = 2\sqrt{2}/\sqrt{3T_w} \approx 1.633/\sqrt{T_w}$  ensuring for  $w(t)$  the unit-energy condition

$$\int_{-\infty}^{+\infty} w^2(t)dt = 1 \tag{2}$$

for any  $T_w$ . The parameter  $T_w$  fixes the time scale of the impulse  $w(t)$ . Since we have  $|w(T_w)/w(0)| \approx 0.02$  and  $|w(2T_w)/w(0)| \approx 6 \times 10^{-10}$ , it can be assumed in practice that the effective duration of  $w(t)$  is no longer than  $\sim 2T_w$ . A plot of  $w(t)$  is shown in Fig. 2.

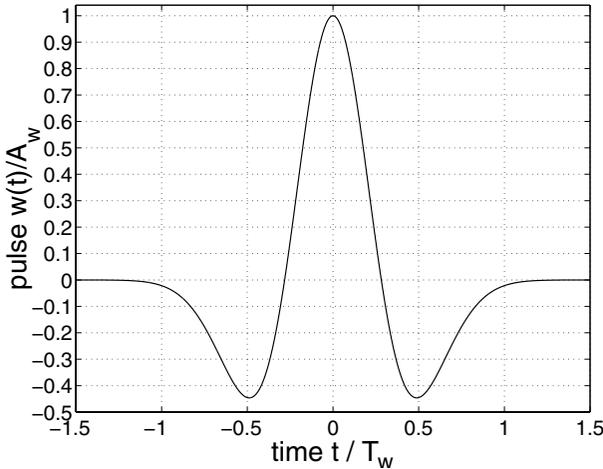


Fig. 2. UWB impulse  $w(t)$  according to Eq. (1).

The autocorrelation function of  $w(t)$  is given by

$$\gamma(\tau) = \int_{-\infty}^{+\infty} w(t)w(t + \tau)dt = \left[ 1 - 4\pi\left(\frac{\tau}{T_w}\right)^2 + \frac{4\pi^2}{3}\left(\frac{\tau}{T_w}\right)^4 \right] \exp\left[-\pi\left(\frac{\tau}{T_w}\right)^2\right], \tag{3}$$

and is plotted in Fig. 3.

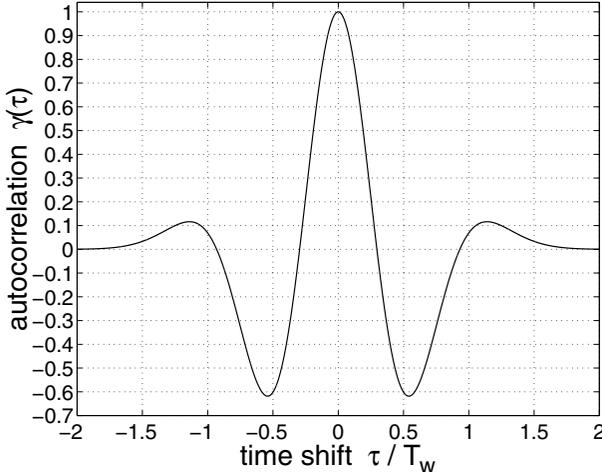


Fig. 3. Autocorrelation function  $\gamma(\tau)$  from Eq. (3) for the UWB impulse  $w(t)$  of Eq. (1).

Various values are possible for the time shift  $\delta$ . We will consider in the following  $\delta = \delta_{\text{opt}} = 0.5144T_w$  as defined in [26]. In some studies  $\delta$  is taken as  $\delta = \delta_{\text{min}} = 0.5422T_w$  with  $\gamma(\delta_{\text{min}}) = \min(\gamma)$ . But here  $\delta_{\text{opt}}$  does not verify this property, as it is chosen to maximize the throughput as described in [26]. It is important to notice that we have chosen the time shift  $\delta$  in such a way that  $\gamma(\delta) \neq \min(\gamma)$ . We can notice also that  $\gamma(0) = \max(\gamma)$ . Then  $\gamma(0) - \gamma(\delta)$  is not the maximum of the function  $t \mapsto \gamma(0 + t) - \gamma(\delta + t)$ .

We are interested in determining the data sent by user 1. We assume the receiver and user 1 perfectly synchronized. Considering only user 1, the received signal is

$$s(t) = \sum_m w(t - mT_f - c_mT_s - a_m\delta) + \eta(t). \tag{4}$$

In Eq. (4), the transmitted bits  $\{a_m\}$  have their values in  $\{0, 1\}$ ,  $\{c_m\}$  is the sequence representing the time hopping code, and  $\eta(t)$  is a white Gaussian noise.

The receiver considered here is the SUMF [27]. The output of the SUMF while receiving the bit  $a_0$  is

$$x(t) = \int_{-\infty}^t s(t')v(t' - c_0T_s)dt', \tag{5}$$

with

$$v(t) = w(t) - w(t - \delta). \tag{6}$$

And the decided received bit, when the output of the SUMF is read at a reading time  $t_r$ , depends on the sign of  $x(t_r)$  according to

$$\hat{a}_0 = \frac{1 - \text{sign}[x(t_r)]}{2}. \tag{7}$$

**2.2. Interferer model**

The interferer signal uses the same impulse as our user signal, but it is not synchronous. It means that in a frame of our user signal, the probability of receiving an interfering impulse is uniformly distributed over the whole duration  $T_f$  of the frame. In order to simplify the notations, we assume that the origin of time  $t = 0$  is, in the slot of user 1, the location where an impulse  $w(t)$  is centered when a bit 0 is received. This allows us to express the contribution  $s_u(t)$  of user 1 to the received signal carrying a bit  $a$  as

$$s_u(t) = w(t - a\delta). \tag{8}$$

There is an interfering impulse, and as the interferer is not synchronous with the receiver, the interfering impulse is received at time  $\tau_i$ , with  $\tau_i$  uniformly distributed over the whole duration  $T_f$  of the frame. The received interfering impulse, noted  $s_i(t)$ , is

$$s_i(t) = w(t - \tau_i). \tag{9}$$

Then the received signal is

$$s(t) = w(t - a\delta) + w(t - \tau_i) + \eta(t). \tag{10}$$

The output of the SUMF receiver may be noted

$$x(t) = x_u(t) + x_i(t) + b(t), \tag{11}$$

with

$$x_u(t) = \int_{-\infty}^t w(t' - a\delta)v(t')dt', \tag{12}$$

$$x_i(t) = \int_{-\infty}^t w(t' - \tau_i)v(t')dt', \tag{13}$$

$$b(t) = \int_{-\infty}^t \eta(t')v(t')dt'. \tag{14}$$

The decided received bit at a reading time  $t_r$  depends on the sign of  $x(t_r)$  according to Eq. (7). When  $t_r \rightarrow \infty$ , Eqs. (12) and (13) are expressable, respectively, as

$$x_u(t_r \rightarrow \infty) = \gamma(a\delta) - \gamma(a\delta - \delta), \tag{15}$$

and

$$x_i(t_r \rightarrow \infty) = \gamma(\tau_i) - \gamma(\tau_i - \delta). \tag{16}$$

As we mentioned earlier, the impulse  $w(t)$  of Eq. (1) has an effective duration no longer than  $\sim 2T_w$ . As a result, Eqs. (15) and (16) will provide a very good approximation of  $x_u(t_r) + x_i(t_r)$  as soon as the reading time  $t_r$  equals a few  $T_w$ .

The precise value chosen for  $t_r$  will fix the actual duration of the slot. Figure 4 shows  $x_u(t_r) + x_i(t_r)$ , when  $t_r \rightarrow \infty$  as it results from Eqs. (15)–(16), as a function of  $\tau_i$  the arrival time of the interfering impulse, and when user 1 sends a bit  $a = 0$ . We can notice in Figs. 4 and 5 that the minimum of  $x_u(t_r) + x_i(t_r)$  is not zero but is slightly negative.

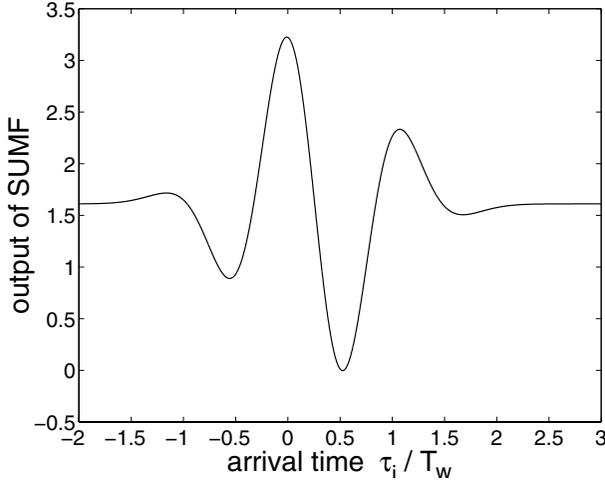


Fig. 4. Output  $x_u(t_r) + x_i(t_r)$  of the single-user matched filter from Eqs. (15)–(16), as a function of the arrival time  $\tau_i$  of the interfering impulse.

### 3. Noise-Improved Interferer Rejection

#### 3.1. Artificial noise increasing the performance

When  $b(t)$  is negligible, an interfering impulse causes an error when the sign of the output  $x_u(t_r) + x_i(t_r)$  at the receiver is the opposite of the sign of  $x_u(t_r)$ . Figure 5 shows a zoom of Fig. 4 in the inversion region where  $x_u(t_r) + x_i(t_r)$  changes sign.

From Fig. 5, and considering the case of a bit 0 sent by user 1, the probability to have an interference causing an error is  $D_0/T_f$ . The domain  $D_0$  is the duration where  $x_u(t_r) + x_i(t_r)$  is negative as shown in Fig. 5 (the case where the bit sent by user 1 is equal to 1 is symmetrical). The probability of error is therefore

$$P_{e1} = \frac{D_0}{T_f}. \tag{17}$$

Let us suppose that we add to  $x_u(t_r) + x_i(t_r)$  an artificial noise  $\xi$ , taking the value  $+|V_{\min}|$  and  $-|V_{\min}|$  randomly with mean 0, where  $-|V_{\min}|$  is the (negative) absolute minimum of  $x_u(t_r) + x_i(t_r)$  shown in Fig. 5. There is an error when  $x_u(t_r) + x_i(t_r) + \xi$  is negative. The interference causing an error without  $\xi$ , that is to say when the interfering impulse is received in  $D_0$ , will be compensated by the noise  $\xi$  with a probability 1/2. On the other hand, an interfering impulse received in  $D_1$  or  $D_2$  will also cause an error with a probability 1/2 because of  $\xi$ . Hence the

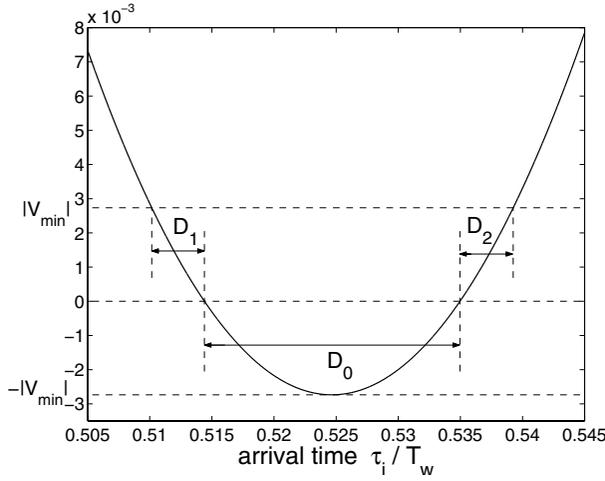


Fig. 5. Inversion area of  $x_u(t_r) + x_i(t_r)$  of Fig. 4.

new probability of error is

$$P_{e2} = \frac{1}{2} \frac{D_0}{T_f} + \frac{1}{2} \frac{D_1 + D_2}{T_f}. \tag{18}$$

We can note that  $D_1 + D_2 < D_0$ . This inequality can be demonstrated by using the fact that the second derivative of the function  $x_u(t_r) + x_i(t_r)$  is positive in  $D_1$ ,  $D_0$  and  $D_2$ . Then  $P_{e2} < P_{e1}$ . The probability of error has decreased thanks to  $\xi$ . By adding the noise  $\xi$  we have improved the performance of the receiver.

### 3.2. Gaussian noise increasing the performance

In a similar way to Sec. 3.1, by adding to  $x_u(t_r) + x_i(t_r)$  the Gaussian noise  $b(t)$  with an appropriate rms amplitude, we may expect to improve the performance. Let us call  $\sigma$  the rms amplitude of the Gaussian noise  $b(t)$  of Eq. (11) that results from the filtering by the SUMF in Eq. (14) of the white Gaussian noise  $\eta(t)$  of Eq. (10). Based on the standard theory of detection in Gaussian noise from the output of the SUMF [27], the overall probability of error  $P_e$  in the presence of the interferer, can be deduced as

$$P_e = \frac{1}{T_f} \int_{-T_f/2}^{T_f/2} Q\left(\frac{x_u(t_r) + x_i(t_r, \tau_i)}{\sigma}\right) d\tau_i, \tag{19}$$

where

$$Q(u) = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left(\frac{u}{\sqrt{2}}\right) \tag{20}$$

represents the complementary cumulative distribution function of the standardized Gaussian law.

Figure 6 represents the probability of error  $P_e$  of Eq. (19), as a function of the rms amplitude  $\sigma$  of the Gaussian noise  $b(t)$ , for different typical values of the frame duration  $T_f$ .

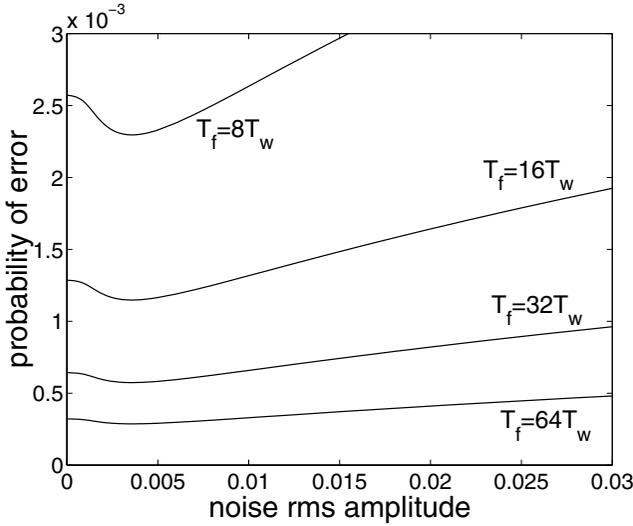


Fig. 6. Probability of error  $P_e$  of Eq. (19), as a function of the rms amplitude  $\sigma$  of the Gaussian noise at the output of the SUMF, for different frame durations  $T_f$ .

Figure 6 reveals that the minimum of the probability of error  $P_e$  is obtained for an optimal nonzero value  $\sigma_{opt}$  of the noise level  $\sigma$ . This is a constructive action of the noise, which is able to improve the transmission performance in the presence of an interferer. This constructive action of the noise is observed in Fig. 6 for all the configurations of  $T_f$ . Moreover, the optimal noise level  $\sigma_{opt}$  minimizing  $P_e$  is found almost exactly the same for every  $T_f$ . This is because the integrand  $Q$  of the right-hand side of Eq. (19), varies significantly only over a region of a few  $T_w$  around  $\tau_i = 0$  as it results from Fig. 4, i.e. over the slot of user 1. Extending the domain of integration  $[-T_f/2, T_f/2]$  of Eq. (19), essentially changes (decreases) the probability that the interfering impulse falls in the slot of user 1, but does not affect the constructive action of the noise that essentially takes place when the interfering impulse falls in the slot of user 1. The constructive action of the noise can be seen as an assistance to reject an interfering impulse falling in the slot of user 1, through a mechanism qualitatively described in Section 3.1 based on Fig. 5. The rejection in Fig. 6 is done most efficiently when the Gaussian noise  $b(t)$  is at an optimal rms amplitude  $\sigma_{opt}$  whose precise value depends on the configuration of interaction over the slot of user 1, of the received impulse and of the interfering impulse. Increasing the frame duration  $T_f$  does not affect this mechanism of interaction over the slot of user 1, but only changes the probability with which this interaction takes place, whence a  $\sigma_{opt}$  insensitive to  $T_f$ . Also, for a similar reason in Fig. 6, as the values of  $T_f$  are within ratios of 2, it can be verified that the corresponding  $P_e$  are also in similar ratios of 2. Therefore, for any  $T_f$  in Fig. 6, a similar reduction of about 11% is observed at the optimal noise level  $\sigma_{opt}$  for the probability of error  $P_e(\sigma_{opt})$  compared to its value  $P_e(\sigma = 0)$  in the absence of noise.

In Fig. 6, thanks to the favorable action of the noise, there is a range  $(0, \sigma_{max} > \sigma_{opt})$  for the noise level  $\sigma$ , where the probability of error  $P_e$  remains below its value

$P_e(\sigma = 0)$  in the absence of noise. At  $\sigma = \sigma_{\max}$  the probability of error  $P_e$  recovers its value  $P_e(\sigma = 0)$  at  $\sigma = 0$ . As long as  $0 < \sigma < \sigma_{\max}$ , the presence of the noise improves the performance.

Numerical simulation matches the theoretical analysis. This is verified by the results of Fig. 7 presenting a Monte Carlo simulation of the complete UWB transmission. Figure 7 again illustrates the central property of a nonmonotonic evolution of the performance  $P_e$  as the noise level  $\sigma$  is raised, with the possibility of improving  $P_e$  by increasing  $\sigma$ .

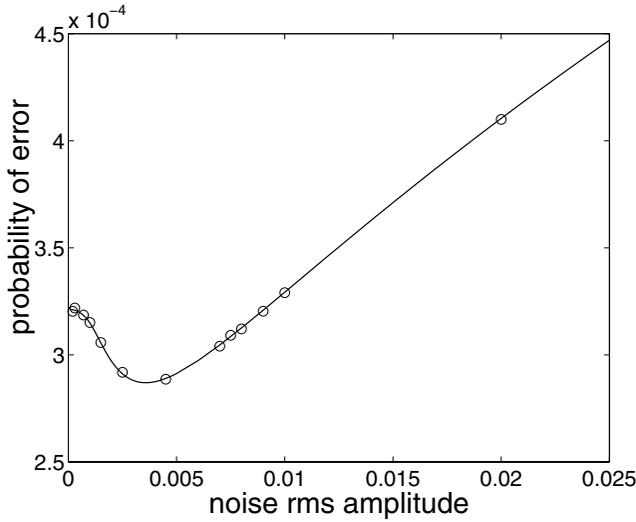


Fig. 7. Probability of error  $P_e$ , as a function of the rms amplitude  $\sigma$  of the Gaussian noise at the output of the SUMF, for  $T_f = 64T_w$ . The solid line is the theoretical  $P_e$  of Eq. (19), the discrete points (o) result from a Monte Carlo simulation of the complete UWB transmission, with  $10^9$  trials for each value tested for  $\sigma$ .

In the present setting, the existence of a nonmonotonic evolution of  $P_e$  in Figs. 6–7, critically depends on the possibility of the SUMF output  $x_u(t_r) + x_i(t_r)$  of Fig. 4 to become negative for some values of the interferer arrival time  $\tau_i$ . This is necessary for the mechanism of Sec. 3.1 and Fig. 5 to take place. In turn, the possibility of a negative  $x_u(t_r) + x_i(t_r)$  depends on the choice of the time shift  $\delta$ . This entails that a nonmonotonic  $P_e$  does not occur if  $\delta = \delta_{\min} = 0.5422T_w$  as defined in Sec. 2.1, since in this case  $x_u(t_r) + x_i(t_r)$  can never go negative. A shorter  $\delta < \delta_{\min}$ , like  $\delta = \delta_{\text{opt}} = 0.5144T_w$  as chosen in Figs. 6–7, authorizes  $x_u(t_r) + x_i(t_r)$  negative and hence a nonmonotonic  $P_e$ . An even shorter  $\delta < \delta_{\text{opt}}$  preserves the possibility of a nonmonotonic  $P_e$  as we have verified, but with a larger overall  $P_e$ . Short  $\delta$ 's, although associated to a larger  $P_e$  also provide a higher transmission rate. Altogether, as explained in Sec. 2.1, it is the choice  $\delta = \delta_{\text{opt}}$  that maximizes the throughput in bits/s [26], when no interferer is present. In addition,  $\delta = \delta_{\text{opt}}$  is associated to a nonmonotonic  $P_e$  improvable by noise when an interferer is present, as revealed by Figs. 6–7.

When the mechanism of improvement occurs, it takes the form as in Sec. 3.1, of a noise directly acting on a test statistic of a detector so as to improve its performance

in the decision. Apparently, this is a specific mechanism for the constructive action of the noise, not common to other forms of stochastic resonance. This setting of UWB communications thus offers an extension of the applicability of the effect of improvement by noise.

#### 4. Conclusion

The present results establish that a form of stochastic resonance, or improvement by noise, can be obtained in a UWB communication process, as the rejection enhanced by noise of an interferer impulse signal. This effect can be interpreted as a novel instance of stochastic resonance. To our knowledge, stochastic resonance under this form of interferer-impulse rejection improved by noise in UWB communications, has never been reported before. This contributes to the on-going inventory of the various possible forms of the phenomenon of stochastic resonance as improvement by noise. It is to note that a recent study [28] used a classic stochastic resonator, under the form of a bistable dynamic system, to implement a stroboscopic scheme for the detection of a sinusoidal signal, with rejection capability observed in the presence of a nearby sinusoid. Yet, as explained in [28], the rejection property in [28] is essentially due to the spectral filtering by the strobed dynamical system, but not due specifically to the beneficial action of noise in the process, by contrast to our present UWB rejection.

The feasibility of a form of improvement by noise of interferer rejection, was established here with a simple model of UWB communications (perfect power control, single interferer). We are now in the process of testing the robustness of the effect when more general conditions of UWB communications are considered. We have taken into account multipath propagation with attenuation, and multiuser interferences. In these more elaborate conditions, numerical simulations have shown that the stochastic resonance or improvement by noise of the performance is essentially preserved, under various detailed modalities which remain open for further investigation. Also, in practice, UWB signals coexist with other sinusoidal or narrowband radio signals [29]. Interference with such narrowband signals was not taken into account in the present study. The possibility of a stochastic resonance effect with both types of interferer signal (impulse and narrowband) constitutes an open perspective for the future. Such investigations will be useful to appreciate, beyond the present proof of feasibility, the practical impact of the effect for UWB communications.

At a broader level, it can be argued that UWB communication bears some similarities with neuronal communication. Both processes rely on short stereotyped impulses (action potentials for neurons [30]), and encode information in the temporal sequencing of the impulses that are propagated. Such a scheme for coding and transmission is associated to very high efficacy in neuronal systems (inherently noisy) for information processing. The stochastic resonance reported here under the form of a noise-improved interferer-impulse rejection, might also be relevant to communication in interconnected neuronal networks, and could play a part in their high efficacy. Reciprocally, inspiration from neurons could suggest improved schemes for information communication (and processing) based on impulse coding and endowed with capabilities to exploit the unavoidable ambient noise.

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