A theoretical model of acoustoelectric transducer with a nonuniform distribution of piezoelectric coefficient: Application to transducer optimization

François Chapeau-Blondeau and James F. Greenleaf
Biodynamics Research Unit, Department of Physiology and Biophysics, Mayo Clinic/Foundation, Rochester, Minnesota 55905

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A general equation is derived that describes the behavior of a piezoelectric transducer with a nonuniform distribution of piezoelectric coefficient within its bulk, when submitted to an arbitrary distribution of acoustic pressure. Based on this equation, an expression for the receiving transfer function of the transducer is calculated. The results demonstrate the dependence of the transfer function on the distribution of piezoelectric coefficient, and that it is possible to benefit from a nonuniform distribution to optimize the transfer function. The general equation also describes the influence of the external electric circuit loading the transducer, which leads to another independent means of optimizing the transfer function. The proposed model combines effects of piezoelectric material characteristics, acoustic backing, and electric loading, without resorting to Mason or other equivalent circuits for the transducer.

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INTRODUCTION

Acoustoelectric transducers, which are devices to convert acoustic energy into electric energy and vice versa, present a very large variety of applications, as for example ultrasonic biomedical imaging. Different physical principles can be employed to implement these transducers. The piezoelectric effect is one of the most widely used in the low ultrasonic range. A piezoelectric transducer consists of a piece of piezoelectric material mounted with appropriate electrode patterns. Very often it has the shape of a flat piezoelectric plate electroded on both faces. Usually, as mechanical resonators, piezoelectric transducers exhibit high-quality factors, meaning that they can vibrate easily only in the vicinity of specified frequencies. This leads to an acoustoelectric transfer function of the transducer that is mainly defined in a limited range of frequency around a center frequency. For many applications, it appears desirable to be able to optimize this transfer function, in order for instance to broaden the bandwidth of the transducer, or increase its sensitivity. Among the few techniques available for this purpose are the use of acoustic backing materials or front matching layers.[1,2] We propose in this article a new technique for optimizing the transfer function of a piezoelectric transducer, which is based on the use of a nonuniform distribution of piezoelectric coefficient within the bulk of the transducer.

We start first by deriving a general equation describing the behavior of such a nonuniform transducer when submitted to an arbitrary distribution of acoustic pressure. Then, based on this equation, a receiving transfer function is defined for the transducer, and we examine how the nonuniform distribution of piezoelectric coefficient enables its optimization. The general equation also suggests that it is possible to benefit from the external electric circuit loading the transducer to further optimize its transfer function. The proposed model includes effects of acoustic backing, electric loading, as well as material characteristics. To describe the behavior of the transducer, we do not resort, as it is done very often, to Mason[3] or other equivalent circuits.[4,5] These equivalent circuit approaches are based on analogies between mechanical quantities and transmission line theory quantities, instead we propose a different approach based essentially on Maxwell's equations and their consequences when applied to a piezoelectric material.

I. GENERAL EQUATION OF THE TRANSDUCER

Let us consider a transducer, represented in Fig. 1, made of a plate of piezoelectric material electroded on both faces. The lateral dimensions of the plate are taken very large compared to its thickness e, and it is assumed that all the quantities in the bulk of the transducer are uniform in each plane parallel to the plane of the plate and can vary spatially only in the direction of the Oz axis perpendicular to the plane of the transducer.

The material parameters that play a role in the acoustoelectric transduction process, namely, the piezoelectric coefficients d_{33} and g_{33} and the dielectric permittivity ϵ_{33}, will be allowed to be nonuniform in the Oz direction, and will be

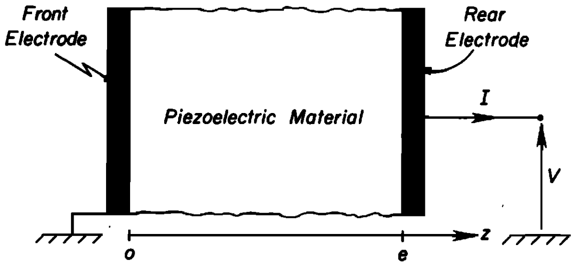

FIG. 1. Configuration of the piezoelectric transducer.

represented respectively by the functions $d_{33}(z)$, $g_{33}(z)$, and $\epsilon_{33}(z)$. These three functions need not be continuous and can exhibit discontinuities.

We now suppose that the transducer is submitted to a distribution of acoustic pressure, uniform in each plane perpendicular to Oz, and represented by the function $p(z,t)$, which stands for the value of the acoustic pressure at abscissa z at time t in the piezoelectric material.

We take the front electrode of the transducer, located at $z=0$, as the reference for electric potential. As a response to the pressure distribution $p(z,t)$, depending on the electric boundary conditions between the two electrodes, a variation of the potential V of the rear electrode located at $z=e$ can occur, or an electric current I can flow through this rear electrode.

The question we shall address now is how V and I relate to the distribution of pressure and piezoelectric material parameters. To answer this question, we start with the basic laws defining the expressions of the current I and the potential V. If the current I is counted positively when it flows out of the rear electrode, we have

$$I = -S\frac{d}{dt}\sigma(e), \tag{1}$$

where $\sigma(e)$ is the superficial density of charge induced on the rear electrode of surface S.

We can express $\sigma(e)$ in terms of the electric field $E(z)$ and the electric polarization $P(z)$ in the piezoelectric material. It then follows that

$$I = S\frac{d}{dt}[\epsilon_0 E(e) + P(e)], \tag{2}$$

where ϵ_0 is the dielectric permittivity of vacuum.

Also, for the potential V of the rear electrode, we have

$$V = -\int_0^e E(z)dz. \tag{3}$$

Now that I and V have been expressed in terms of $E(z)$ and $P(z)$, let us see how $E(z)$ and $P(z)$ relate to the pressure distribution $p(z,t)$ and the piezoelectric material parameters.

First, in this one-dimensional model, the Maxwell–Poisson equation can be written as

$$\frac{d}{dt}(\epsilon_0 E + P) = 0, \tag{4}$$

which yields

$$\epsilon_0 E(z) + P(z) = C_1, \tag{5}$$

where C_1 is a constant in space.

The total electric polarization $P(z)$ at abscissa z in the material can be expressed as

$$P(z) = [\epsilon_{33}(z) - \epsilon_0]E(z) + d_{33}(z)p(z,t). \tag{6}$$

In the right-hand side of expression (6), the first term represents, as a definition of the dielectric permittivity $\epsilon_{33}(z)$, the dielectric polarization at abscissa z; the second term is the piezoelectric polarization generated by the acoustic pressure at abscissa z.

This way of expressing electrical and mechanical quantities by means of scalar parameters as done in Eq. (6), is consistent with the assumption of rotational symmetry of the transducer around the Oz axis, as stated in the beginning of this section. Such a situation is met most of the time with piezoelectric transducers, and permits, as we shall see, to eventually reach a simple equation describing the behavior of the system. A more general description would require a tensor form of Eq. (6). We note that Eqs. (4) and (6) still hold in the case where discontinuities in the functions $\epsilon_{33}(z)$ and $d_{33}(z)$ produce discontinuities in the electric field $E(z)$ and polarization $P(z)$.

By combining (5) and (6), we get

$$E(z) = [d_{33}(z)/\epsilon_{33}(z)]p(z,t) + C_1/\epsilon_{33}(z). \tag{7}$$

Substituting this last expression in (3), and using the relationship between the piezoelectric coefficients $d_{33}(z)/\epsilon_{33}(z) = g_{33}(z)$, one obtains

$$C_1 = \left(\int_0^e \frac{dz}{\epsilon_{33}(z)}\right)^{-1}\left(-V + \int_0^e g_{33}(z)p(z,t)dz\right). \tag{8}$$

According to Eq. (5), the current I defined by (2) can be written as

$$I = S\frac{d}{dt}C_1. \tag{9}$$

Using in (9) the expression of C_1 given by (8), and making the suitable arrangements, we end up with a relation governing the time evolution of the current $I(t)$ and the potential $V(t)$:

$$I(t) + C\frac{d}{dt}V(t) = C\int_0^e g_{33}(z)\frac{\partial}{\partial t}p(z,t)dz, \tag{10}$$

where C, the capacitance of the transducer, is defined as

$$C = \bar{\epsilon}_{33}(S/e),$$

with

$$\frac{1}{\bar{\epsilon}_{33}} = \frac{1}{e}\int_0^e \frac{dz}{\epsilon_{33}(z)}. \tag{11}$$

Equation (10) is a general equation relating the electric signals $I(t)$ and $V(t)$, which can be obtained from a piezoelectric transducer, to the acoustic pressure distribution within the transducer, and to the parameters of the piezoelectric material, which are allowed to be nonuniform throughout the bulk of the material. In practice, the transducer is terminated in an external electrical circuit, which gives another relationship connecting $I(t)$ and $V(t)$, and thus a fully determined system.

II. APPLICATION TO SIMPLE ELECTRICAL BOUNDARY CONDITIONS

To gain some insight into what can be deduced from Eq. (10), we shall see now how it applies in some particular situations where the transducer is used with different simple electrical boundary conditions between its two electrodes.

First, let us assume that the transducer is kept in short circuit. We have then $V=0$, and Eq. (10) gives an expression for the short circuit current $I_{sc}(t)$ flowing between the two electrodes of the transducer. We find

$$I_{sc}(t) = C\int_0^e g_{33}(z)\frac{\partial}{\partial t}p(z,t)dz. \tag{12}$$

If now the transducer is kept in open circuit conditions, we have $I = 0$, and (10) gives an expression for the open circuit voltage $V_{oc}(t)$ across the transducer. We obtain, assuming that when the pressure is zero throughout the transducer its output voltage is also zero:

$$V_{oc}(t) = \int_0^e g_{33}(z) p(z,t) dz. \tag{13}$$

We note that

$$I_{sc}(t) = C \frac{d}{dt} V_{oc}(t). \tag{14}$$

Thus, in general, the short and open circuit signals that are generated by a piezoelectric transducer relate differently to the pressure distribution and material parameters.

In practice, to enable the measurement of an electrical signal, the transducer is often terminated in a resistive load R, yielding $V = RI$. Using Eq. (10) with the expressions given by (12) and (13), one finds that the current I flowing through the resistance, or the voltage V across the resistance, are governed by

$$I(t) + RC \frac{d}{dt} I(t) = I_{sc}(t), \tag{15}$$

$$V(t) + RC \frac{d}{dt} V(t) = RC \frac{d}{dt} V_{oc}(t). \tag{16}$$

If we write these two last equations in the frequency domain, noting the Fourier transform of each function with an accentuated letter, we obtain

$$\hat{I}(\omega) + iRC\omega \hat{I}(\omega) = \hat{I}_{sc}(\omega), \tag{17}$$

$$\hat{V}(\omega) + iRC\omega \hat{V}(\omega) = IRC\omega \hat{V}_{oc}(\omega). \tag{18}$$

For the angular frequencies ω that satisfy the condition $RC\omega \ll 1$, it is possible to neglect the second term in the left-hand side of (17) and thus obtain $\hat{I}(\omega) = \hat{I}_{sc}(\omega)$, which shows that in these conditions the current flowing through the external resistance can be equated to the short circuit current.

On the other hand, for the angular frequencies such that $RC\omega \gg 1$, the first term of the left-hand side of Eq. (18) can be neglected leading to $\hat{V}(\omega) = \hat{V}_{oc}(\omega)$, which indicates that in these conditions the voltage across the external resistance can be equated to the open circuit voltage.

So, let us suppose we consider a pressure distribution with a given frequency spectrum, and a piezoelectric transducer of capacitance C terminated in a resistance R. If the value of R is large enough so that the condition $RC\omega \gg 1$ is met throughout the spectrum of the pressure, then the voltage across R is the open circuit voltage and relates to the pressure distribution through Eq. (13). If the value of R is small enough so that the condition $RC\omega \ll 1$ is met throughout the spectrum of the pressure, then the current flowing across R is the short circuit current and relates to the pressure distribution through Eq. (12). However, in these conditions, the voltage RI_{sc} obtained across the resistance verifies according to (14) $|R\hat{I}_{sc}| = RC\omega |\hat{V}_{oc}|$, so, as $RC\omega \ll 1$, this voltage RI_{sc} measured in short circuit conditions will always be much smaller than the voltage measured in open circuit conditions.

In many practical cases, the signal $V_{oc}(t)$ constitutes the response of the transducer being actually measured. Taking the Fourier transform of Eq. (13) leads to an explicit expression of the spectrum $\hat{V}_{oc}(\omega)$ of the electrical response of the transducer, as a function of the spectrum $\hat{p}(z,\omega)$ of the pressure distribution within the piezoelectric material, such that

$$\hat{V}_{oc}(\omega) = \int_0^e g_{33}(z) \hat{p}(z,\omega) dz. \tag{19}$$

So, the frequency response of the transducer depends on the spectrum of the pressure, and also on the distribution of piezoelectric coefficient $g_{33}(z)$. We shall now derive a transfer function that will be a characteristic of the transducer only, and see how $g_{33}(z)$ enables us to optimize this transfer function.

III. DERIVATION OF A TRANSFER FUNCTION

Thus far, we have not considered any particular form for the pressure distribution $p(z,t)$ within the transducer. In many situations where a piezoelectric transducer is utilized, the pressure $p(z,t)$ in the bulk of the piezoelectric material is produced by an acoustic wave impinging on the front face of the transducer.

So, let us now consider an incident acoustic wave constituted by a linear superposition of plane waves traveling in the Oz direction toward the positive z's. This incident wave can be represented by the function $p_1(z,t)$ defined in the region $z \leqslant 0$. At abscissa $z = 0$, where the front face of the transducer is located, one can write for the incident wave before it penetrates the transducer

$$p_1(0,t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{p}_1(0,\omega) \exp(i\omega t) d\omega, \tag{20}$$

$\hat{p}_1(0,\omega)$ being the frequency spectrum of the incident wave on the front face of the transducer defined as

$$\hat{p}_1(0,\omega) = \int_{-\infty}^{+\infty} p_1(0,t) \exp(-i\omega t) dt. \tag{21}$$

At abscissa $z = 0$, part of the incident wave $p_1(z,t)$ is reflected by the interface, and part penetrates the transducer. At the other interface at $z = e$, part of the wave is reflected into the transducer, and part is transmitted to the backing medium.

In order to deduce in these conditions the electrical response of the transducer given by (19), we first have to compute an expression for the resultant distribution of pressure $p(z,t)$, or its Fourier transform $\hat{p}(z,\omega)$, within the transducer. Thus, we have to make assumptions on the acoustic properties of the different media in contact to know how the incident pressure will get distributed.

So, let us assume that the propagating medium of the incident wave has an acoustic impedance Z_1, and the backing medium of the transducer an acoustic impedance Z_3. The piezoelectric material of the transducer will have an acoustic impedance Z_2 and a propagation constant γ that will be a complex function of the angular frequency ω defined as

$$\gamma(\omega) = \beta(\omega) - i\alpha(\omega), \tag{22}$$

where $\alpha(\omega)$ is a real function representing the attenuation of acoustic waves in the transducer, and $\beta(\omega) = \omega/c$, c being the phase velocity of acoustic plane waves in the transducer. The velocity c, as well as the acoustic impedances Z_1, Z_2, and Z_3, can also be functions of the angular frequency ω. However, the acoustic parameters of the transducer are assumed to be spatially uniform, while its piezoelectric parameters are not. Strictly speaking, this situation might not be rigorously possible, since the acoustic and piezoelectric parameters of a material are closely related. Nevertheless, as the so-called stiffening of the acoustic parameters due to the piezoelectric effect remains small in most materials (6), it appears as a reasonable approximation to consider the influence of such nonuniform stiffening as negligible. The configuration of the system is represented in Fig. 2.

The thickness of the electrodes of the transducer is supposed to be sufficiently small compared to the acoustic wavelengths involved, so that the perturbation of the pressure distribution caused by these electrodes can be neglected.

All the waves traveling to the right and to the left are plane waves. In particular, in the transducer, the component of frequency ω of the spectrum $\hat{p}(z,\omega)$ of the total acoustic pressure, is composed of the sum of a plane wave traveling to the right and a plane wave traveling to the left; so, we can write

$$\hat{p}(z,\omega)\exp(i\omega t) = P_3(\omega)\exp i(\omega t - \gamma z) + P_4(\omega)\exp i(\omega t + \gamma z). \tag{23}$$

By expressing the acoustic boundary conditions at interfaces in $z = 0$ and $z = e$, i.e., the continuity of pressure and displacement, it is possible to relate the amplitudes $P_3(\omega)$ and $P_4(\omega)$ to $\hat{p}_1(0,\omega)$ and to the acoustic properties of the media in contact. We finally get

$$\hat{p}(z,\omega) = \tau_{12}A(x,\omega)\hat{p}_1(0,\omega), \tag{24}$$

where $A(z,\omega)$ is a complex function defined as

$$A(z,\omega) = \frac{\exp[i\gamma(e-z)] + r_{23}\exp[-i\gamma(e-z)]}{\exp(i\gamma e) + r_{12}r_{23}\exp(-i\gamma e)}, \tag{25}$$

with

$$\tau_{12} = \frac{2Z_2}{(Z_2 + Z_1)}, \quad r_{12} = \frac{(Z_2 - Z_1)}{(Z_2 + Z_1)},$$

and

$$r_{23} = \frac{(Z_3 - Z_2)}{(Z_3 + Z_2)}.$$

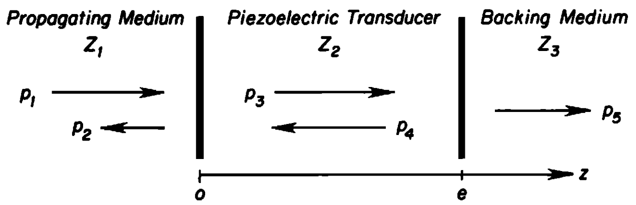

FIG. 2. Distribution of the incident acoustic pressure p_1 in the three media in contact.

Now, by replacing expression (24) in Eq. (19), the frequency response of the transducer $\hat{V}_{oc}(\omega)$ is expressed as a function of the incident pressure spectrum $\hat{p}_1(0,\omega)$, which can be considered as the excitation. It follows that

$$\hat{V}_{oc}(\omega) = \tau_{12}\hat{p}_1(0,\omega)\int_0^e g_{33}(z)A(z,\omega)dz. \tag{26}$$

It is thus possible to define a transfer function $T(\omega)$ for the transducer by

$$T(\omega) = \frac{\hat{V}_{oc}(\omega)}{\hat{p}_1(0,\omega)} = \tau_{12}\int_0^e g_{33}(z)A(z,\omega)dz. \tag{27}$$

Equation (27), together with Eq. (25), provides a theoretical model describing the transfer function of a piezoelectric transducer in the general case where the piezoelectric coefficient g_{33} of the material is nonuniform throughout the thickness. It also includes the influence of the backing medium.

Before trying different nonuniform distributions $g_{33}(z)$ to optimize $T(\omega)$, we can first see what transfer function is given by Eq. (27) when g_{33} is uniform. Figure 3 represents the modulus $|T(\omega)|$ of the transfer function plotted against frequency $f = \omega/(2\pi)$, evaluated numerically from (27) for a case representing typically a transducer made of a PVDF film of thickness $e = 200\ \mu$m, with $g_{33} = g_{33}^0 = 340\times10^{-3}$ $\mathrm{Vm^{-1}\ Pa^{-1}}$. In the calculation, we have used for PVDF $Z_2 = 2.7\times10^6$ kg m^{-2} s^{-1}, the propagating medium is water with $Z_1 = 1.48\times10^6$ kg m^{-2} s^{-1}, and the transducer has a low acoustic impedance backing medium that gives $r_{23} = -1$. We have taken for the velocity of acoustic waves in PVDF $c = 1500$ ms^{-1}. For the curve of Fig. 3, the value retained for the attenuation coefficient is $\alpha = 0$, the computation results having shown that for such a thin PVDF film the attenuation does not play an important role.

The calculated transfer function of Fig. 3, exhibiting a main peak around the frequency corresponding to the fundamental thickness mode resonance, is quite consistent with the transfer function that can be expected from a platelike PVDF transducer.

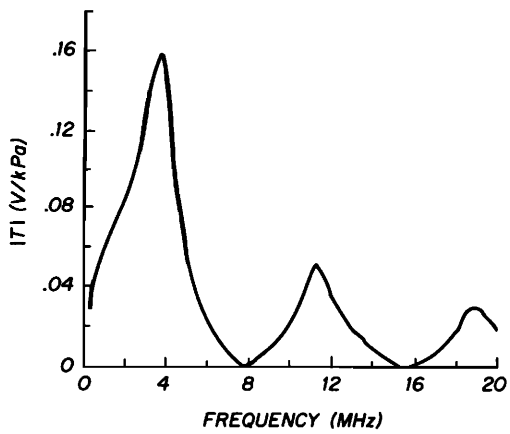

FIG. 3. Modulus of the calculated transfer function for a 200-μm-thick PVDF transducer with a uniform distribution of piezoelectric coefficient.

IV. OPTIMIZATION THROUGH THE DISTRIBUTION OF PIEZOELECTRIC COEFFICIENT

Equation (27) shows an intricate dependence of the transfer function $T(\omega)$ on the distribution of piezoelectric coefficient $g_{33}(z)$. To understand better in what manner and to what extent $T(\omega)$ can be shaped through $g_{33}(z)$, let us rewrite (27), using expression (25), in the following way:

$$T(\omega) = M(\gamma)\Big(\exp(i\gamma e)\int_0^e g_{33}(z)\exp(-i\gamma z)dz + r_{23}\exp(-i\gamma e)\int_0^e g_{33}(z)\exp(i\gamma z)dz\Big), \quad (28)$$

where

$$M(\gamma) = \tau_{12}\exp(-i\gamma e)/[1 + r_{12}r_{23}\exp(-i2\gamma e)]. \quad (29)$$

The value of $g_{33}(z)$ being zero outside the interval $[0,e]$, one can thus write

$$T(\omega) = M(\gamma)[\exp(i\gamma e)\hat{g}_{33}(\gamma) + r_{23}\exp(-i\gamma e)\hat{g}_{33}(-\gamma)], \quad (30)$$

where $\hat{g}_{33}(\gamma)$ is the Fourier transform, with a complex argument γ in the general case, of the function $g_{33}(z)$.

Thus the transfer function $T(\omega)$ has been expressed in Eq. (30) as the product of a function $M(\gamma)$, which depends on the geometry and acoustical properties of the transducer, and a term inside the brackets, which introduces the contribution of the distribution of piezoelectric coefficient through is Fourier transform.

We can proceed further if we abandon the general case for a simpler case, which is often met in practice, in which neglecting the attenuation in the transducer leads to $\gamma = \beta$ real, and a low acoustic impedance backing gives $r_{23} = -1$. Under these conditions, Eq. (30) becomes

$$T(\omega) = 2iM(\beta)\,\mathrm{Im}[\exp(i\beta e)\hat{g}_{33}(\beta)], \quad (31)$$

or

$$T(\omega) = 2iM(\beta)|\hat{g}_{33}(\beta)|\sin\{\beta e + \arg[\hat{g}_{33}(\beta)]\}, \quad (32)$$

where Im[ ] and arg[ ] stand for the imaginary part and the phase angle of a complex number, respectively.

Under the simplified conditions stated above, the modulus $|M(B)|$ is a periodic function of period π/e, oscillating between the extremum Z_2/Z_1 attained when $\beta = n\pi/e$ (n integer) and the extremum $+1$ attained when $\beta = \pi/(2e) + n\pi/e$. The peaks of $|M(\beta)|$ centered at $\beta = n\pi/e$, of height Z_2/Z_1, have a width that is a decreasing function of Z_2/Z_1. The function $|M(\beta)|$, which exhibits peaks at frequencies corresponding to the resonant frequencies of the thickness vibration modes of the transducer, with a natural peak width function of the acoustic impedance of the material, represents the spectrum of the acoustic modes of vibration of the transducer. For the transducer of Fig. 3, this acoustic vibration mode function $|M(\beta)|$ is plotted in Fig. 4, versus frequency $f = c\beta/(2\pi)$.

By taking the modulus of Eq. (32), one obtains

$$|T(\omega)| = 2|M(\beta)|\ |\hat{g}_{33}(\beta)|\ |\sin\{\beta e + \arg[\hat{g}_{33}(\beta)]\}|. \quad (33)$$

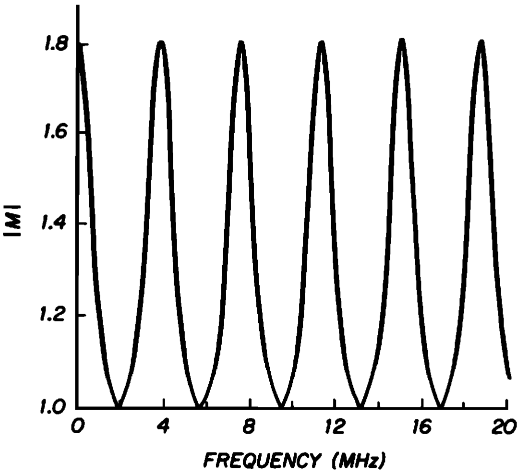

FIG. 4. Modulus of the acoustic vibration mode function for the transducer of Fig. 3.

So, Eq. (32) or (33) shows that the distribution of piezoelectric coefficient $g_{33}(z)$ plays a role in the final transfer function $T(\omega)$ by shaping the acoustic vibration mode function $M(\beta)$ through a multiplication by its Fourier transform $|\hat{g}_{33}(\beta)|$.

In the case of a uniform distribution of piezoelectric coefficient, $g_{33}(z)$ is equal to a constant in the interval $[0,e]$ and to zero elsewhere. The modulus $|\hat{g}_{33}(\beta)|$ of its Fourier transform is a sinc function having a zero at each $\beta = 2m\pi/e$ (m integer). This is why, as illustrated by Fig. 3, in the resultant transfer function of the transducer, the peaks of even-order n present in the acoustic vibration mode function $|M(\beta)|$ (see Fig. 4) have disappeared. The peak of order $n = 0$ of $|M(\beta)|$ also vanishes because $g_{33}(z)$, being real, $\arg[\hat{g}_{33}(0)]$ is zero. The peak of order $n = 1$ of $|M(\beta)|$ experiences the most important gain in the multiplicative process described by Eq. (33), $|\hat{g}_{33}(\beta)|$ having strong values in the region of this peak. Thus we end up, in the case of a transducer having a uniform distribution of piezoelectric coefficient, with a transfer function exhibiting a main peak around a frequency corresponding to the fundamental resonant thickness mode.

If we change the Fourier transform of $g_{33}(z)$, it is possible to modify the transfer function $T(\omega)$. For the transducer of Fig. 3, where the uniform distribution of piezoelectric coefficient has been replaced successively by the nonuniform distributions presented in Fig. 5, Fig. 6 shows two examples of the transfer function evaluated numerically from (27). For comparison, the mean on interval $[0,e]$ of the absolute value of these nonuniform distributions has been kept the same as that of the uniform distribution. When the results of Figs. 3 and 6 are compared, the modifications observed in the transfer function can be understood in terms of changes in the Fourier transform of $g_{33}(z)$. In situation (a), the function $g_{33}(z)$ of Fig. 5(a) possesses a Fourier transform that does not vanish in the region of $\beta = 2\pi/e$, as the Fourier transform of the uniform distribution does, but instead keeps strong values in this region. So, in the resultant transfer function of Fig. 6(a), the peak corresponding to the order $n = 2$ is present, leading to a broadening of the bandwidth of the

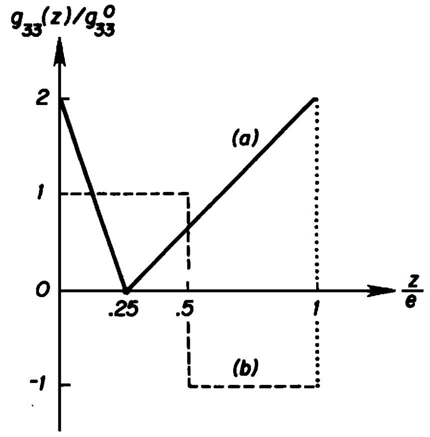

FIG. 5. Distributions of piezoelectric coefficient used in the calculation of the transfer functions of Fig. 6.

transducer. In situation (b), the Fourier transform of the distribution $g_{33}(z)$ of Fig. 5(b) is mostly concentrated in the region of $\beta = 2\pi/e$, so the resulting transfer function of Fig. 6(b) is constituted mainly by the peak corresponding to the order $n = 2$, but the sensitivity obtained for this transducer is twice as high as the sensitivity of a uniform transducer of half the thickness resonating at the same frequency.

These examples demonstrate that it is possible to modify the transfer function of the transducer by varying the distribution of piezoelectric coefficient. Modifications can be performed to transform in many different ways the transfer function of the transducer, depending on the desired "optimal" transfer function. However, to realize any arbitrarily shaped transfer function would require, as shown by Eq. (32), to specify both the modulus and phase angle of $\hat{g}_{33}(\beta)$, which may not lead after inverse Fourier transform to physically realizable distribution $g_{33}(z)$.

V. OPTIMIZATION THROUGH THE EXTERNAL ELECTRIC LOAD

We shall now present another independent way of optimizing the transfer function of the transducer, which is also suggested by the general equation (10).

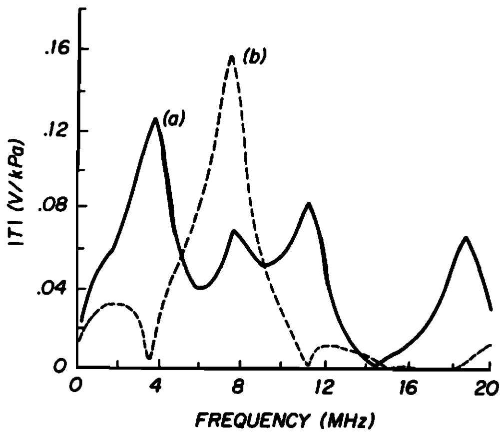

FIG. 6. Modulus of the calculated transfer functions for the transducer of Fig. 3 with the distributions of piezoelectric coefficient of Fig. 5.

As discussed in Sec. II, a very common way of measuring an electric signal out of a piezoelectric transducer is to terminate it in an external resistive load. This method, although simple, may not be optimal in all situations. Usually, the question of optimizing the electric load is addressed with the use of an equivalent circuit for the transducer.[6] We shall follow here a different approach based on Eq. (10). This equation, which relates in a general case the electric signals generated by the transducer to the pressure distribution, will now permit examination of the behavior of the transducer when terminated in an arbitrary external circuit.

First, taking the Fourier transform of Eq. (10) and using the definition of formula (19), leads to the equation in the frequency domain:

$$\hat{I}(\omega) + iC\omega\hat{V}(\omega) = iC\omega\hat{V}_{oc}(\omega). \tag{34}$$

We now assume the transducer is terminated in an arbitrary linear electric circuit, as depicted in Fig. 7, that gives the relations

$$\hat{I}(\omega) = Y_{in}(\omega)\hat{V}(\omega), \tag{35}$$

$$\hat{V}_m(\omega) = H(\omega)\hat{V}(\omega). \tag{36}$$

Here, $\hat{V}_m(\omega)$ is the voltage measured on the output of the external loading circuit, $H(\omega)$ is the transfer function of this circuit, and $Y_{in}(\omega)$ is its input admittance.

Using Eq. (34) conjointly with (35) and (36), it is possible to express the measured voltage $\hat{V}_m(\omega)$ in terms of the open circuit voltage $\hat{V}_{oc}(\omega)$, which, in turn, relates to the pressure distribution through (19). We obtain

$$\hat{V}_m(\omega) = G(\omega)\hat{V}_{oc}(\omega), \tag{37}$$

where we have defined a gain function

$$G(\omega) = iC\omega H(\omega)/[iC\omega + Y_{in}(\omega)]. \tag{38}$$

When the external load is a simple resistance, we have $Y_{in} = 1/R$ and $H = 1$, and the gain function (38) takes the form

$$G(\omega) = iRC\omega/(1 + iRC\omega). \tag{39}$$

The modulus of this gain function (39) is represented in Fig. 8(a). For the frequencies such that $RC\omega \gg 1$, we have $G(\omega) \approx 1$ and thus, as shown in Sec. II, $\hat{V}_m(\omega) = \hat{V}_{oc}(\omega)$.

If the spectrum $\hat{V}_{oc}(\omega)$ consists, as it is often the case with standard plate like piezoelectric transducers, of a main resonant peak around a center frequency ω_c, a gain function of type (39) flat in the region of ω_c might not be optimal for its measurement. A gain function $G(\omega)$ having, for instance, the form of Fig. 8(b), easily implemented with two passive resonant circuits, would lead to a broadening of the bandwidth of the response of the transducer. Following this principle, all the techniques of synthesis of linear electric circuits can be applied to define a suitable gain function for the trans-

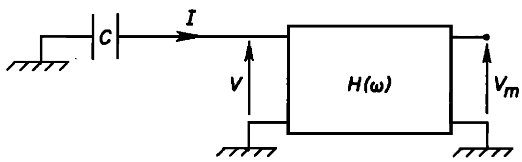

FIG. 7. Transducer loaded in an arbitrary linear electric circuit.

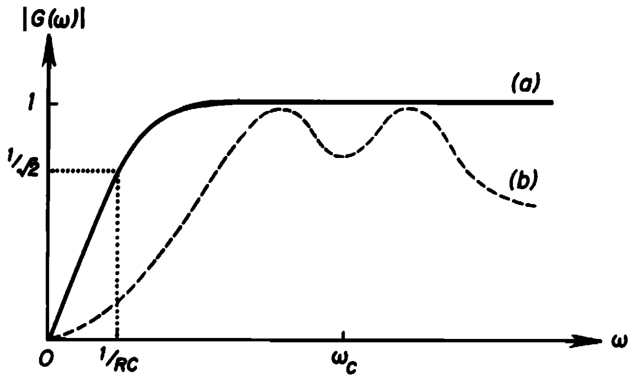

FIG. 8. Modulus of two different gain functions $G(\omega)$.

ducer, depending on what realizable frequency response one wants to achieve.

VI. TRANSDUCTION SYSTEM

If the two methods of optimization that have been described are associated, we end up with the notion of a transduction system, which is composed of a transducer with a nonuniform distribution of piezoelectric coefficient, terminated in an optimal electric load.

Using Eqs. (27) and (37), a transfer function $T_s(\omega)$ can be defined for this transduction system, which takes the form

$$T_s(\omega) = \frac{\hat{V}_m(\omega)}{\hat{p}_1(0,\omega)} = \tau_{12} G(\omega) \int_0^e g_{33}(z) A(z,\omega) dz. \quad (40)$$

The functions $g_{33}(z)$ and $G(\omega)$ provide two independent means to optimize the transfer function of the transduction system, allowing one to specify its properties to a large extent.

We emphasize, suggested by the example of Fig. 6(b), that reduction of amplitude of the transfer function occurring in the optimization process can be compensated for by adjoining to the transducer additional piezoelectric layers with appropriate profiles of a piezoelectric coefficient.

VII. CONCLUSION

We have derived a general equation [Eq. (10)] describing the behavior of a piezoelectric transducer having a nonuniform distribution of piezoelectric coefficient throughout its bulk. Based on this equation, an expression for the transfer function of the transducer has been calculated. The theoretical model predicts that it is possible to use the distribution of piezoelectric coefficient to specify to a large extent the shape of the transfer function of the transducer. Another independent way of optimizing the transfer function has been proposed which makes use of the external electric circuit loading the transducer. These techniques can lead to transfer functions which depart greatly from that of a standard platelike piezoelectric transducer, and allow one to define an optimal transfer function depending on what application is viewed.

In the model developed here, only the case of the receiving mode of the acoustoelectric transducer has been considered. The reverse case, which would be the transmitting mode, requires a somewhat different treatment. However, based on the reciprocity theorem,[7] it appears likely that the features predicted by the model in the receiving mode will have their analogs in the transmitting mode.

Transducers with a nonuniform distribution of piezoelectric coefficient can be physically implemented by various techniques. For instance, layers of a piezoelectric material can be assembled together, each layer having received a separate poling will possess a specified value for its piezoelectric coefficient, and will permit the definition of the distribution of piezoelectric coefficient by a piecewise constant function throughout the assembly. Another technique could be the use of a nonuniform electric field to carry out the poling process of a piezoelectric material. Physical implementations of transducers with a nonuniform distribution of piezoelectric coefficient have been reported[8] providing an experimental basis to the theoretical results presented here.

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