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Benefits from a speckle noise family on a coherent imaging transmission

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ABSTRACT

We analyze image transmission in a coherent imaging system, in the presence of speckle noise modeled with the family of Gamma probability density functions of varying order. It is demonstrated that speckle noise can improve the transmission of a coherent image. Exact analytical expressions are obtained for both the best achievable performance and the optimal amount of speckle noise maximizing the transmission efficacy. These expressions allow us to analyze and control the conditions under which the coherent imaging system can take advantage of the speckle noise. The influence of the contrast in the coherent image, and of the order of the Gamma probability density describing the statistical fluctuations of the speckle, are given special attention. These results make a contribution to the understanding of the mechanisms of improvement by noise in nonlinear information processing.

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1. Introduction

Speckle noise [1,2] is encountered in most coherent imaging systems. Examples are active imaging systems with laser (light amplification by simulated emission of radiation) or synthetic-aperture radar (SAR) systems [3–5], or digital holography [6], or satellite image processing [7], or sonar and echographic imagery with acoustic waves [8,9]. It has been recently shown [10,11] that speckle noise is able, in some cases, to assist the transmission of a coherent image. Comparable constructive effects of noise have also been reported in other imaging systems, in the visual system [12–15] as well as in other optical processes [16–20]. These demonstrations of the possibility of mechanisms of improvement by noise open new perspectives for image and information processing. The results of the present paper seek to contribute in this direction.

A specificity which occurs with speckle noise is the multiplicative action of the noise. This type of multiplicative signal–noise coupling has not been much investigated in the context of the effects of improvement by noise [21–25]. Yet, multiplicative coupling reveals new properties of the effect, especially relevant for coherent imaging. This has been recently illustrated by Ref. [11] which presented for the first time experimental evidence of a noise-aided transmission in the case of a simple model of speckle noise. In the present paper, we also consider the transmission of an image by a coherent imaging system as in [11]. We extend the investigation to a family of speckle noises, modeled by Gamma probability density functions of varying order. For a simple coherent imaging system as in [11], we here work out a detailed theoretical study analyzing the constructive action of speckle noise. We derive exact analytical expressions for both the best achievable performance and the optimal amount of speckle noise maximizing the transmission efficacy. These expressions describe the influence of the relevant parameters of the system, especially the order of the Gamma probability density modeling the fluctuations of the speckle, and the contrast in the coherent image. We establish that a constructive role of the speckle noise is accessible for every value of the order of the Gamma probability density, with a more pronounced efficacy as the order increases. The analysis allows us to predict how much benefit the coherent imaging system can get from speckle noise, and also to control the conditions on the system parameters under which the transmission can benefit from the speckle.

2. Coherent imaging model

In coherent imaging a physical scene, modeled by an input image $S(u,v)$ where (u,v) are the spatial coordinates of the pixels, is illuminated by a coherent wave. This scene consists in objects with irregularities at the wavelength scale. As a consequence, the phases of the transmitted or backscattered wavefront interfere with one another, in either a constructive or destructive way. On the imaging sensor, the acquisition with these interferences results in an intermediate image $X(u,v)$ with intensity fluctuations, superimposed on the intensity from the imaged scene [26]. The fluctuations observed on image $X(u,v)$ have a grainy appearance and are called speckle noise. Image $X(u,v)$ can be modeled by a mixture between the physical scene $S(u,v)$ and a speckle noise $N(u,v)$ according to the multiplicative relation [27]

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$$X(u,v) = S(u,v) \times N(u,v). \tag{1}$$

Different statistical models exist to represent speckle noise in coherent imaging. A frequently chosen model, valid if the detector pixel size is smaller than the speckle grain size, is given by the exponential probability density function [26]

$$p_N^1(r) = \frac{1}{\sigma_N} \exp\left(\frac{-r}{\sigma_N}\right), \quad \text{for } r \geqslant 0. \tag{2}$$

In the probability density function of Eq. (2), r is a dummy variable representing the values accessible to the intensity of the speckle, and σ_N is at the same time the mean and the standard deviation of the speckle.

The statistical properties of the speckle can be further affected in several ways. For example, it is not unusual in SAR imaging for several acquisitions of the same scene to be combined. Also, elements of the coherent imaging system, like for example the source of the incident wave, may move faster than the time exposure of the sensor. This movement affects the statistical properties of the speckle noise contained in the acquired image. These different situations are equivalent to the situation where the speckle noise contained in image $X(u,v)$ comes from an average, on intensity basis, of L images. In such cases of an average of L independent speckle realizations, the probability density function results as an L-fold convolution of the probability density function of Eq. (2). This L-fold convolution leads, after the averaging process, to a speckle image $N(u,v)$ with speckle intensities following a Gamma probability density function of order L [26]

$$p_N^L(r) = \left(\frac{L}{\sigma_N}\right)^L \frac{r^{L-1}}{\Gamma(L)} \exp\left(\frac{-Lr}{\sigma_N}\right), \quad \text{for } r \geqslant 0. \tag{3}$$

In the probability density function model of Eq. (3), the noise level σ_N controls both the mean $E[N(u,v)] = \sigma_N$ and the standard deviation $\sigma_N/\sqrt{L}$ of the speckle noise $N(u,v)$. In Eq. (3) the parameter L, called the speckle order, is an integer and the Gamma function can be written $\Gamma(L) = (L-1)!$. For example, a non polarized incident wave, as it contains two independent polarizations, will lead to a speckle noise that can be modeled with Eq. (3) for $L = 2$ [28]. The probability density function of Eq. (3) can also model situations where speckle noise arises with an unknown order L like in the case of a fast movement of the elements as described before.

The intermediate image $X(u,v)$ resulting from the coupling modeled by Eq. (1) is then acquired by the sensor of the imaging system described by the characteristic function $g(\cdot)$, producing an output image $Y(u,v)$ so that

$$Y(u,v) = g[X(u,v)]. \tag{4}$$

We shall study the impact of speckle noise on the efficacy of the transmission. We assess this impact with a measure of performance as explained in the following section.

3. Assessment of the transmission

A meaningful input–output measure of similarity in image processing is the root mean square error (rms error) [29]. We use here the rms error to measure the similarity between the input image $S(u,v)$ and the output image $Y(u,v)$, which is defined by

$$Q_{SY} = \sqrt{\langle (Y-S)^2 \rangle} = \sqrt{\langle Y^2 \rangle + \langle S^2 \rangle - 2\langle SY \rangle}, \tag{5}$$

where $\langle \cdots \rangle$ denotes an average over the images. The effect of speckle noise $N(u,v)$ on the transmission of the physical scene $S(u,v)$, through this measure, will be studied in terms of statistical properties of the images. We consider the case of a binary image for $S(u,v)$, consisting of an object of interest characterized by its intensity I_1, on a background of intensity I_0. Without loss of generality, we assume that $0 \leqslant I_0/I_1 < 1$. We take for the imaging sensor described by $g(\cdot)$ in Eq. (4) a hard limiter function with a threshold θ. This function $g(\cdot)$ implements a rectangular function under the form

$$g[X(u,v)] = \begin{cases} 1 & \text{for} \quad X(u,v) > \theta \\ 0 & \text{for} \quad X(u,v) \leqslant \theta. \end{cases} \tag{6}$$

This model permits to carry out a complete analytical treatment. This type of sensor captures the basic characteristics of more sophisticated systems. Multilevel quantizers or high-level processing tasks like for example binary decision tasks are among these systems. Furthermore, neurons building the visual system are often modeled with a hard limiter. For these reasons, the hard limiter of Eq. (6) is a useful model for imaging systems when they operate at low flux domains. With this imaging sensor, the output image $Y(u,v)$ takes its values in the binary set $\{0,1\}$. We compare image $Y(u,v)$ with a binary reference pattern $S'(u,v)$ which equals $S(u,v)$ when $I_0 = 0$ and $I_1 = 1$. The rms error between $S'(u,v)$ and $Y(u,v)$ is obtained by replacing $S(u,v)$ by $S'(u,v)$ in Eq. (5). We assume that $S(u,v)$ (as well as $S'(u,v)$) is large enough to consider that a statistical approach has sense and we define the probabilities

$$p_1 = \Pr\{S = I_1\} = 1 - \Pr\{S = I_0\} = \Pr\{S' = 1\} \tag{7}$$

and

$$q_1 = \Pr\{Y = 1\}. \tag{8}$$

We also introduce the conditional probabilities

$$p_{10} = \Pr\{Y = 1 | S = I_0\} = \Pr\{Y = 1 | S' = 0\} \tag{9}$$

and

$$p_{11} = \Pr\{Y = 1 | S = I_1\} = \Pr\{Y = 1 | S' = 1\}. \tag{10}$$

With these definitions, the average $\langle S'^2 \rangle = \Pr\{S' = 1\} = p_1$ and the average $\langle Y^2 \rangle = \Pr\{Y = 1\} = q_1$. Besides, this probability is expressible as

$$q_1 = p_1 p_{11} + (1 - p_1) p_{10}. \tag{11}$$

The computation of $Q_{S'Y}$ also requires the average

$$\begin{aligned} \langle S'Y \rangle &= \Pr\{Y = 1, S' = 1\} = \Pr\{Y = 1 | S' = 1\} \Pr\{S' = 1\} \\ &= p_1 p_{11}. \end{aligned} \tag{12}$$

The output image $Y(u,v)$ depends on the relation between the input mixture $S(u,v) \times N(u,v)$ and the threshold θ of the imaging sensor. Thus, the conditional probabilities described before can be written

$$p_{1k} = \Pr\{S \times N > \theta | S = I_k\} = \Pr\{N > \theta / I_k\} \quad \text{with } k \in \{0,1\}. \tag{13}$$

We have then

$$p_{1k} = 1 - F_N^L(\theta / I_k) \quad \text{with } k \in \{0,1\}, \tag{14}$$

where $F_N^L(\cdot)$ is the cumulative distribution function of the speckle noise defined by $F_N^L(u) = \int_{-\infty}^{u} p_N^L(x) \mathrm{d}x$. It is possible to get an analytical expression of the cumulative distribution function from the probability density function of Eq. (3). We have the integral

$$F_N^L(r) = \left(\frac{L}{\sigma_N}\right)^L \frac{1}{\Gamma(L)} \int_0^r x^{L-1} \exp\left(\frac{-Lx}{\sigma_N}\right) \mathrm{d}x, \tag{15}$$

and successive integrations by parts lead to the expression

$$F_N^L(r) = 1 - \exp\left(\frac{-Lr}{\sigma_N}\right) \sum_{n=0}^{L-1} \frac{1}{n!} \left(\frac{Lr}{\sigma_N}\right)^n. \tag{16}$$

All put together the rms error can be rewritten

$$Q_{S'Y} = \sqrt{p_1 + (1 - p_1) p_{10} - p_1 p_{11}}, \tag{17}$$

with conditional probabilities p_{10} and p_{11} depending on σ_N, L and the quantities $\{I_0, I_1, \theta\}$ through the cumulative distribution function. This expression gives access to a complete analytical form for $Q_{S'Y}$ through Eqs. (14) and (16), thanks to the simple choices for the imaging sensor and the input image. With the form of Eq. (17), we can study the impact of the parameters of the system and the speckle noise on the coherent imaging transmission.

4. Constructive role of speckle noise

We now demonstrate that, in some conditions, speckle noise can be useful in the transmission of the input image $S(u,v)$. Fig. 1 shows the evolution of the rms error $Q_{S'Y}$ as a function of the noise level σ_N, for different values of the speckle order L and with other parameters being fixed. We can see in Fig. 1 that the analytical predictions in solid lines are in close agreement with the numerical simulations in discrete points. Curves present a nonmonotonic evolution with a minimum value of the rms error for a nonzero value of the noise level σ_N. This means that it is possible to improve the transmission of a scene in a coherent imaging system by raising the level of speckle noise. We have the possibility to study the mechanism of improvement by noise through the influence of the speckle order L on the transmission. The effect is already visible at $L=1$, as also seen experimentally in [11]. It is shown in Fig. 1 that the effect is preserved for every order L. Besides, we can see in Fig. 1 that the minimum value of the rms error, reflecting an optimal noise-aided transmission, is decreased when L is raised. In the limit case $L\to\infty$, the minimal rms error tends to zero. An example of this beneficial impact of the speckle noise on the transmission is shown in Fig. 2. The shape of the object in the illustrated output images $Y(u,v)$ can be compared to the one in the binary input image $S(u,v)$ as a visual appreciation of the efficacy of the transmission. The upper panel of the figure illustrates that the best transmission is obtained at the intermediate level $\sigma_N=\sigma_{\rm opt}$ of the speckle noise. The influence of the order L is shown in the lower panel, where increasing L from Fig. 2d–f leads to an improved transmission.

We now propose a qualitative explanation of the constructive action of speckle noise as a function of the order L. Introducing the probability $p_0=\Pr\{S=I_0\}=1-p_1$, intermediate image $X(u,v)$ can be described by the conditional probability density functions

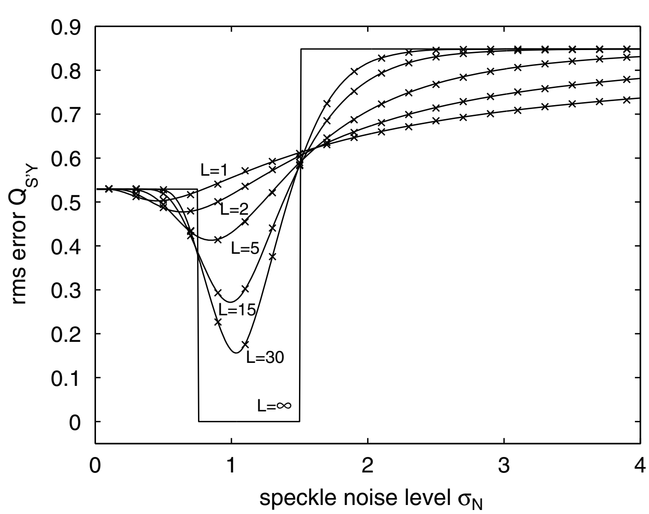

Fig. 1. rms error of Eq. (17) as a function of the level σ_N of the speckle noise, for different values of the speckle order L at threshold $\theta=0.75$. The characteristics of the input image $S(u,v)$ are $I_0=0.5$, $I_1=1$ and $p_1=0.28$. Solid lines correspond to the analytical predictions of Eq. (17). Discrete points correspond to numerical simulations, averaging L independent realizations of a speckle noise with probability density function of Eq. (2).

$$p_{X|S=I_k}(r)=\frac{p_k}{I_k}p_N^L\left(\frac{r}{I_k}\right)\quad\text{with } k\in\{0,1\}. \tag{18}$$

Fig. 3 shows the conditional probability density functions $p_{X|S=I_0}(r)$ and $p_{X|S=I_1}(r)$ for distinct values of the speckle order L, at a fixed value of σ_N. Threshold θ is fixed and represented by the vertical dashed line. The black zone, located under the curve $p_{X|S=I_0}(r)$, represents the probability of error to transmit a pixel value of I_1 instead of the correct value I_0 for the background. In the same way the gray zone, located under the curve $p_{X|S=I_1}(r)$, represents the probability of error to transmit a pixel value of I_0 instead of I_1 in the region of the object. Thus, a reduction of both zones means an improved transmission of the input image $S(u,v)$. This is obtained by raising the speckle order L as illustrated in Fig. 3. As L increases, the variance σ_N^2/L of the speckle noise decreases. Therefore, when $L\to\infty$, the conditional probability density functions $p_{X|S=I_0}(r)$ and $p_{X|S=I_1}(r)$ evolve to Dirac delta functions, creating the particular shape of the rms error in Fig. 1 at this limit case. When a constructive role of speckle noise is possible, it is of interest to know exactly how to get the best benefit. This point is described in the following section.

4.1. Optimal level of noise

It is possible to theoretically predict the optimal level of noise that minimizes the error, i.e. the value of σ_N for which the derivative of $Q_{S'Y}$ of Eq. (17) is zero. This is expressed by the equation

$$(1-p_1)\frac{\partial p_{10}}{\partial\sigma_N}-p_1\frac{\partial p_{11}}{\partial\sigma_N}=0. \tag{19}$$

Using a change of variable, we introduce the standardized probability density function $p_{\rm stand}(u)=\sigma_N p_N^L(\sigma_N u)$, with cumulative distribution function $F_{\rm stand}(u)=F_N(\sigma_N u)$. The conditional probabilities of Eq. (14) become

$$p_{1k}=1-F_{\rm stand}\left(\frac{\theta}{\sigma_N I_k}\right)\quad\text{with } k\in\{0,1\}. \tag{20}$$

We obtain the derivatives

$$\frac{\partial p_{1k}}{\partial\sigma_N}=\frac{\theta}{I_k\sigma_N^2}p_{\rm stand}\left(\frac{\theta}{\sigma_N I_k}\right)\quad\text{with } k\in\{0,1\}, \tag{21}$$

and Eq. (19) leads to

$$\frac{p_{\rm stand}\left(\frac{\theta}{I_0\sigma_N}\right)}{p_{\rm stand}\left(\frac{\theta}{I_1\sigma_N}\right)}=\frac{I_0}{I_1}\,\frac{p_1}{1-p_1}. \tag{22}$$

Considering the hypothesis $\theta\neq0$ and $p_1\neq1$, one can find that the optimal value for σ_N which minimizes the input–output rms error when using a speckle noise of order L is

$$\sigma_{\rm opt}=\frac{I_0-I_1}{I_0I_1}\,\frac{L\theta}{\ln(K)}\quad\text{with } K=\left(\frac{I_0}{I_1}\right)^L\frac{p_1}{1-p_1}. \tag{23}$$

Expression of Eq. (23) can be rewritten

$$\sigma_{\rm opt}=\frac{\theta(1/I_1-1/I_0)}{\frac{1}{L}\ln\left(\frac{p_1}{1-p_1}\right)+\ln\left(\frac{I_0}{I_1}\right)}, \tag{24}$$

illustrating how this quantity depends on the order L. This is depicted in Fig. 4 showing the values taken by the optimal level of speckle noise $\sigma_{\rm opt}$ as a function of L. Raising L implies a better contrast in the intermediate image $X(u,v)$ so that, in order to keep the transmission in optimal conditions, the value of $\sigma_{\rm opt}$ needs to be increased to a lesser extent for high values of the order L. This increase of $\sigma_{\rm opt}$ leads to a maximum value $\sigma_{\rm lim}$ obtained when $L\to\infty$, that can be expressed by

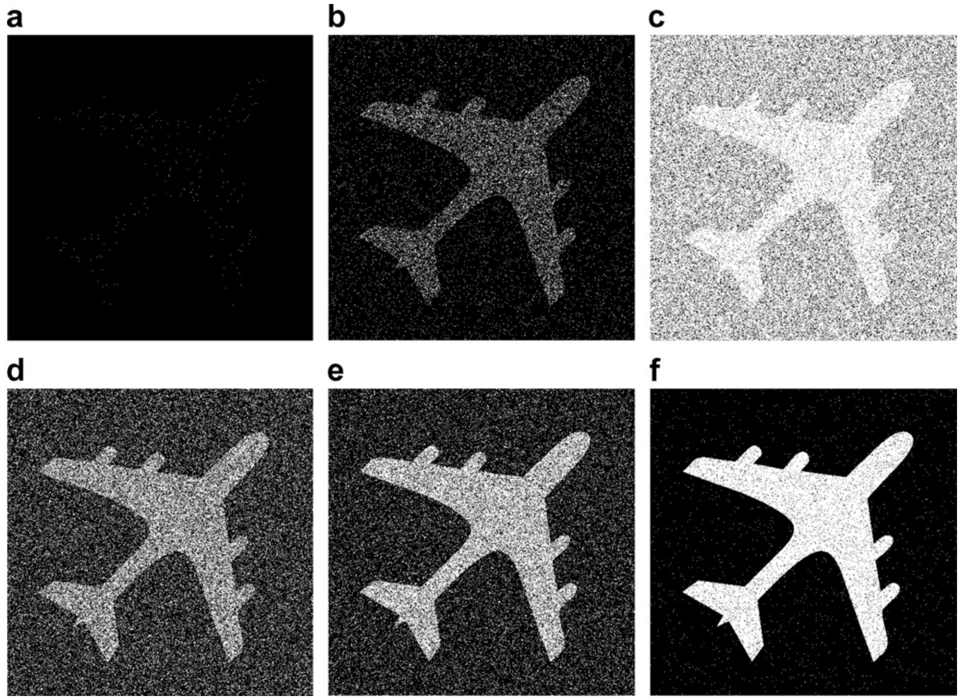

Fig. 2. Examples of simulated images for $Y(u,v)$ of Eq. (4) at the output of the sensor, varying the level σ_N of the speckle noise (upper panel) and the order L (lower panel). In the upper panel, the order is fixed to $L = 2$ and the level of the speckle is (a) $\sigma_N = 0.2$ (b) $\sigma_N = \sigma_{\rm opt} = 0.55$ and (c) $\sigma_N = 3$. The rms error is minimized at the intermediate noise level $\sigma_N = \sigma_{\rm opt}$ in (b). In the lower panel, the level of the speckle noise is $\sigma_N = 1$ and its order is (d) $L = 2$, (e) $L = 5$ and (f) $L = 30$. The rms error decreases monotonically as L is raised from (d) to (f). For all these images, $S(u,v)$ is a 500×500 image with probability $p_1 = 0.2$, with intensities $I_0 = 0.4$ and $I_1 = 0.8$. The threshold of the sensor is $\theta = 0.6$.

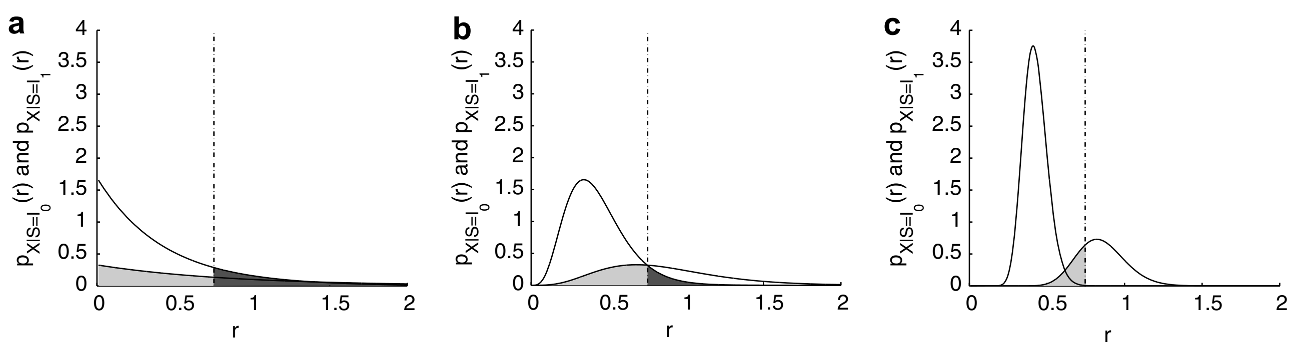

Fig. 3. Probability density functions $p_{X|S=I_0}(r)$ and $p_{X|S=I_1}(r)$ of Eq. (18) at threshold $\theta = 0.75$ and $\sigma_N = 0.85$, for (a) $L = 1$ (b) $L = 5$ and (c) $L = 30$. Same input image conditions as in Fig. 1. Black and gray zones correspond to the probability of wrong transmission, respectively in the object region and in the background region, of input image $S(u,v)$.

$$\sigma_{\rm lim} = \theta \frac{(I_0 - I_1)}{I_0 I_1 \ln(I_0/I_1)}. \tag{25}$$

The opportunity to find an analytical expression for the optimal level of noise is quite rare in the literature of useful noise effects, where studies are usually limited to explore the phenomenon numerically. The expression of Eq. (23) also points out the influence of the different parameters of the system, providing a better understanding of the mechanism of transmission improvement by noise.

4.2. Conditions for a constructive role of speckle noise

It is interesting to know for a given scene whether speckle noise can have or not a constructive role on the transmission. This is characterized by a nonmonotone evolution of a performance measure, with a determined set of parameters. This particular set of parameters is usually chosen in order to illustrate the possibility of an improvement by noise. We now define constraints on this set that one must verify in order to get the effect. These constraints are given by the analytical expression of $\sigma_{\rm opt}$. The optimal level of speckle noise $\sigma_{\rm opt}$ must be positive, therefore Eq. (23) must verify the condition $K < 1$. This leads to the inequality

$$\left(\frac{I_0}{I_1}\right)^L < \frac{1 - p_1}{p_1}, \tag{26}$$

which gives a condition of a noise-aided transmission as a function of the parameters of the coherent imaging system. Condition of Eq. (26) can also be written

$$\frac{I_0}{I_1} < h(p_1) \quad \text{with } h(p_1) = \left(\frac{1 - p_1}{p_1}\right)^{1/L}. \tag{27}$$

As we have supposed $0 \leqslant I_0/I_1 < 1$, the condition of Eq. (26) is always verified when $1 < \frac{1-p_1}{p_1}$ which is the same as $p_1 < 1 - p_1$ or $2p_1 < 1$. In other words, an input image with $p_1 < 1/2$ can always get benefit from speckle noise. This is depicted in Fig. 5a showing

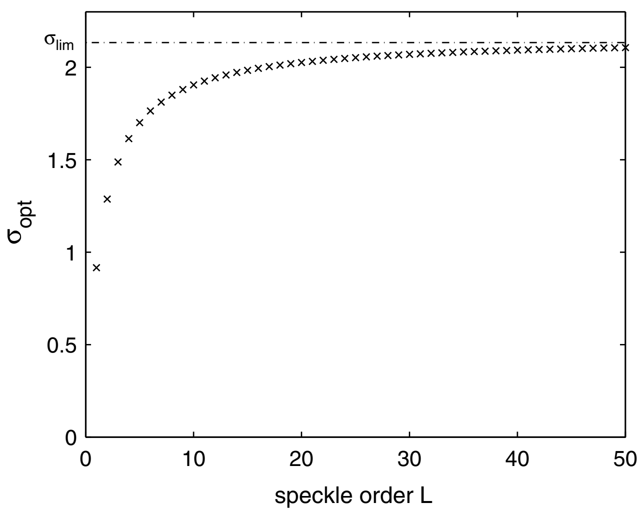

Fig. 4. Optimal value σ_{opt} of the speckle noise as a function of the speckle order L (integer values) at threshold $\theta = 1.5$ with σ_{lim} given in Eq. (25). Same input image conditions as in Fig. 1.

the function $h(\cdot)$ as a function of the parameter p_1, for distinct values of the speckle order L. The hypothesis $0 \leqslant I_0/I_1 < 1$ corresponds to the lower part of the figure, under the dashed line. This ratio must also verify the condition of Eq. (27), thus the value I_0/I_1 must be located under the curve at given L. It is the case for all values of L when $0 < p_1 < 1/2$. For higher values of p_1, the domain of the contrast ratio I_0/I_1 where speckle noise can improve the transmission is constrained by the curves, and this condition is less restrictive as L is increased. This corroborates the idea that improvement by speckle noise appears more easily for high values of the order L. The limit of function $h(\cdot)$ when $L \to \infty$ equals 1, reflecting that there is no condition for the effect to occur in the limit case. To summarize, the set of conditions $0 \leqslant I_0/I_1 < 1$, $0 < p_1 < 1$ and Eq. (27) give the domain where the transmission can get benefit from speckle noise, represented for the case $L = 1$ by the gray area in Fig. 5a. On the contrary, it is not possible to get benefit from speckle noise with parameters corresponding to outside the gray area. As pointed out by Eq. (27), the contrast in the input image I_0/I_1 plays an important role in the effect. Fig. 5b shows the rms error $Q_{S'Y}$ as a function of the noise level σ_N, at $p_1 = 0.7$ and $L = 2$. The situation where $L = 2$ corresponds to an intermediate curve of Fig. 5a. Fig. 5b illustrates the influence of the contrast ratio I_0/I_1 on the noise-aided transmission. For strong contrasts between the object and the background, i.e. when I_0/I_1 is small, speckle noise has a more constructive effect on the transmission. When I_0/I_1 takes high values, $h(p_1)$ must be sufficiently high to verify Eq. (27). For high values of $h(p_1)$, the range of the corresponding values of p_1 is smaller than for small values of $h(p_1)$. Thus, when the contrast ratio I_0/I_1 increases, p_1 must be reduced in order to get the effect, corresponding to a small object of interest in relation to the input image size.

4.3. Best achievable performance

The analytical form of the optimal level of noise permits to have direct access to the best performance that is possible to obtain, which is the minimal rms error Q_{min} between input image $S(u,v)$ and output image $Y(u,v)$. It is enough to inject σ_{opt} of Eq. (23) in the expression of $Q_{S'Y}$ of Eq. (17). This leads to the theoretical prediction

$$Q^2_{min} = p_1 + (1-p_1)K^{\frac{1}{1-I_0/I_1}} \sum_{n=0}^{L-1} \frac{1}{n!}\left[\frac{\ln(K)}{\frac{I_0}{I_1}-1}\right]^n - p_1 K^{\frac{I_0/I_1}{1-I_0/I_1}} \sum_{n=0}^{L-1} \frac{1}{n!}\left[\frac{\frac{I_0}{I_1}\ln(K)}{\frac{I_0}{I_1}-1}\right]^n, \qquad (28)$$

with $K = (\frac{I_0}{I_1})^L \frac{p_1}{1-p_1}$. With the expression of Eq. (28), one is able to know exactly the best performance that is possible to obtain with speckle noise, as a function of the order L of its probability density function and the input image characteristics. Fig. 6a shows the evolution of the minimal value Q_{min} for the rms error as a function of the speckle order L. As already seen in Fig. 1, speckle noise has a more important effect on the transmission when L is increased, leading to a lower value of the minimal rms error. When the order tends to the limit case $L \to \infty$, the speckle noise probability density function tends to the Dirac delta function at the mean value σ_N which is

$$p_D(r) = \delta(r - \sigma_N). \qquad (29)$$

Then the conditional probabilities become

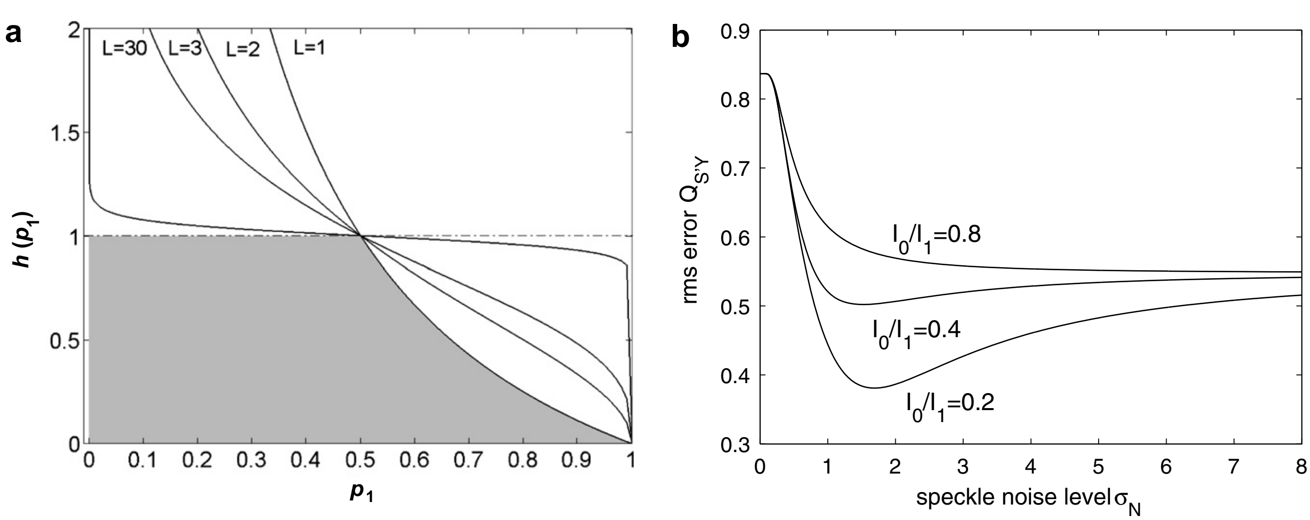

Fig. 5. (a) Evolution of the function $h(\cdot)$ of Eq. (27) as a function of the probability p_1, for different values of the speckle order L. As an illustration, the gray zone represents at $L = 1$ the domain where speckle noise has a constructive role on the transmission. (b) rms error $Q_{S'Y}$ of Eq. (17) as a function of the speckle noise level σ_N, for different values of the contrast ratio I_0/I_1. The other parameters are fixed to $p_1 = 0.7$, $L = 2$ and $\theta = 0.5$.

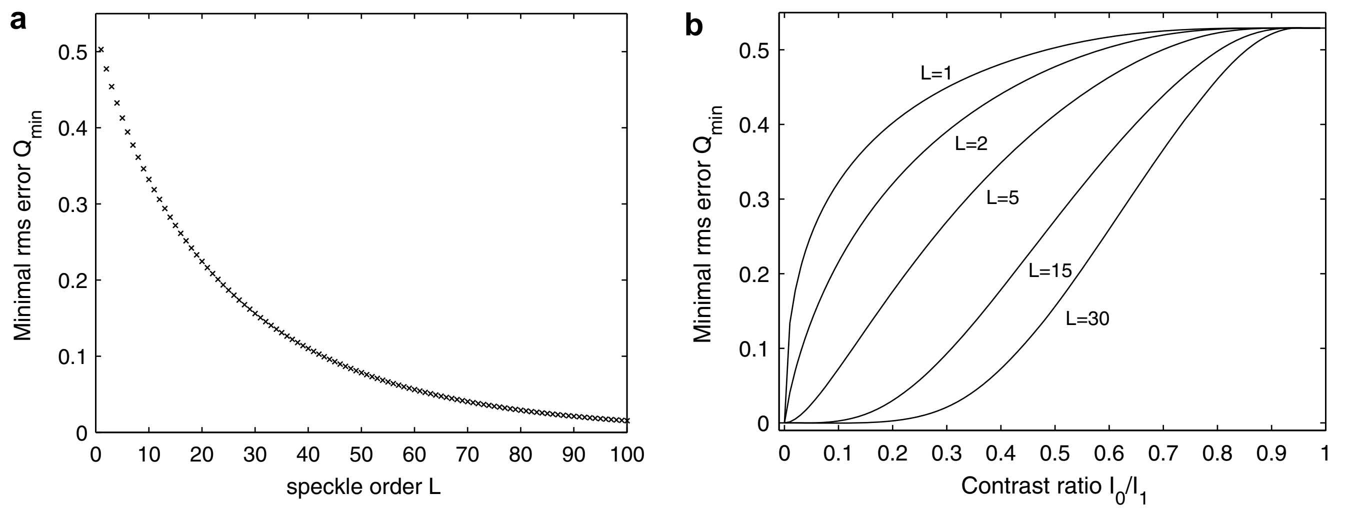

Fig. 6. Minimal rms error Q_{min} of Eq. (28) as a function of (a) the speckle order L (integer values) at threshold $\theta = 1$ and (b) the contrast ratio I_0/I_1, for different values of the speckle order L. Same input image conditions as in Fig. 1.

$$p_{1k} = \begin{cases} 1 & \text{if } \frac{\theta}{I_k} < \sigma_{\lim} \\ 0 & \text{otherwise} \end{cases} \quad \text{with } k \in \{0, 1\}. \tag{30}$$

Together with the expression of the rms error of Eq. (17), one can find that the minimal rms error tends to zero corresponding to a perfect transmission, here in the simple case of binary images. The contrast I_0/I_1 appears in the best achievable performance of Eq. (28). This corroborates this idea that this ratio has an important effect on the noise-aided transmission. The minimal rms error as a function of I_0/I_1 follows a monotone evolution, as depicted in Fig. 6b for distinct values of L. When input image has a low contrast between the object and the background, i.e. when I_0/I_1 is close to 1, the minimal rms error Q_{min} is high and it can be found from Eq. (27) that Q_{min} tends to $\sqrt{\overline{p}_1}$, the same value for every order L of the speckle noise. This means that when I_0/I_1 is close to 1 (low contrasts), the optimal transmission depends on the image but not on the order L. When $I_0/I_1 \rightarrow 0$ (strong contrasts) the best rms error Q_{min} which is possible to obtain tends to zero for every speckle order L.

5. Discussion

We studied the possibility of a constructive action of a family of speckle noises of varying order L on a coherent imaging transmission. This constructive action of speckle noise has been experimentally observed at the order $L = 1$ in [11]. We have demonstrated here that an improvement by speckle noise is achievable for every value of the order L, illustrating the sturdiness of the effect. We have shown that best performances are obtained for high values of the speckle order L. Thus, it could be interesting to work in coherent imaging conditions with high values of L, but this may have to be traded off with a possible loss of spatial resolution [30]. An analytical expression of the optimal level of noise maximizing the transmission efficacy is obtained, as a function of the system parameters which include the speckle order. Whatever the value of the speckle order L, we are thus able to know how to get the best benefit from speckle noise. The different parameters of the system can be controlled experimentally. The level σ_N of the speckle noise is directly linked to the intensity A of the incident wave through the relation $\sigma_N = A \times R$, where R is the reflection coefficient on the scattering surface. The level σ_N of the speckle noise can thus be tuned by the intensity of the incident wave. This possibility to control the level of the speckle has been experimentally implemented in [11]. The order L of the speckle noise can be controlled, for instance, by moving the diffuser within the time exposure of the camera [2].

The exact analytical expression of the best achievable performance is also obtained. For a simple model of coherent imaging system, this expression allows to know how much benefit this system can gain from speckle noise, by means of noise-aided transmission. Besides, the theoretical analysis allows one to precisely define the characteristics of the coherent image for which speckle noise can have a constructive action on the transmission. This is obtained in the form of constraints on the characteristics of the coherent image, like its contrast. The present study shows that the object/background contrast in the coherent image is an important parameter for the feasibility of the improvement by speckle noise. For further analysis of the effect, more complex image models could be investigated, with distributed values of gray levels over the object and over the background. Also, we used here a hard limiter to represent the imaging system, implementing a binary object/background segmentation. More sophisticated models for the imaging system could also be investigated, in order to appreciate how the benefits from the speckle noise evolve in such conditions.

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