Constructive action of the speckle noise in a coherent imaging system

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A coherent imaging system with speckle noise is devised and analyzed. This demonstrates the possibility of improving the nonlinear transmission of a coherent image by increasing the level of the multiplicative speckle noise. This noise-assisted image transmission is a novel instance of stochastic resonance phenomena by which nonlinear signal processing benefits from a constructive action of noise. © 2007 Optical Society of America

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Coherent imaging is inherently associated with speckle noise. Speckle noise is a fluctuation of intensity over an image caused by very irregular spatial interference from the coherent phases. Speckle noise is often seen as a nuisance for many processing tasks in coherent imaging. Meanwhile, from other areas of information processing, it is progressively realized that noise can sometimes play a constructive role, such phenomena being known under the denomination of stochastic resonance [1,2]. *A priori* paradoxical in a linear context, stochastic resonance is a general nonlinear phenomenon that has been registered in various nonlinear physical processes, including electronic circuits, lasers (see for example [3]), magnetic superconducting devices, or neuronal systems. In all these processes, stochastic resonance was observed with a temporal (monodimensional) information signal. Up to now, only a few studies have reported stochastic resonance with spatial (bidimensional) signals or images. Stochastic resonance with images has been obtained in an optical Raman scattering experiment [4], in image perception by the visual system [5], in superresolution techniques for imaging sensors [6], and recently in image restoration [7]. Here, we demonstrate a new instance of stochastic resonance applied, to our knowledge for the first time, to coherent imaging, and taking the form of a noise-assisted image transmission by a nonlinear sensor in the presence of speckle noise. Also, as we recall here, speckle noise can be modeled as a multiplicative noise, and this feature is in itself challenging because most of the studies on stochastic resonance considered additive noise. The few that considered multiplicative noise dealt exclusively with temporal signals [8]. By contrast, we show a new form of stochastic resonance, for coherent images, with multiplicative speckle noise.

A grainylike pattern called speckle is observed when an object with roughness on a wavelength scale is illuminated by a coherent wave. On an imaging detector, the transmitted or backscattered wavefront perturbed by those irregularities produces intensity fluctuations superimposed on the macroscopic reflectivity or transparency contrast of the object. The effect on a coherent imaging system can be modeled [9] as a multiplicative noise in the following way. Let $S(u,v)$ be an input information-carrying image to be acquired, where the pixels are indexed by integer coordinates (u,v) and have intensity $S(u,v)\in[0,1]$. Let $N(u,v)$ be a multiplicative speckle noise, statistically independent of $S(u,v)$, which corrupts each pixel of image $S(u,v)$, to produce a nonlinear multiplicative mixture

$$X(u,v)=S(u,v)\times N(u,v), \tag{1}$$

where the noise values are independent from pixel to pixel, and are distributed according to the probability density $p_N(j)$ given by

$$p_N(j)=\frac{1}{\sigma_N}\exp\left(-\frac{j}{\sigma_N}\right),\quad j\geq 0, \tag{2}$$

with mean and standard deviation σ_N and root mean square (rms) amplitude $\sqrt{2}\sigma_N$. Equations (1) and (2) constitute a simple model of fully developed speckle noise that is valid if the detector pixel size is smaller than the speckle grain size [9]. The information-noise mixture $X(u,v)$ is then received by an image detector delivering the output image $Y(u,v)$ according to

$$Y(u,v)=g[X(u,v)], \tag{3}$$

the input–output characteristic $g(\cdot)$ of the imaging system being, at this stage, an arbitrary function. The coherent imaging system described in Eqs. (1)–(3) has been realized with the experimental setup of Fig. 1.

In order to assess the quality of the acquisition, we introduce an input–output measure of similarity between the information-carrying input image $S(u,v)$ [the object of the slide in Fig. 1] and output image $Y(u,v)$ [the image on the CCD matrix in Fig. 1]. We choose the input–output image rms error E_{SY}, a basic measure in the domain of image processing:

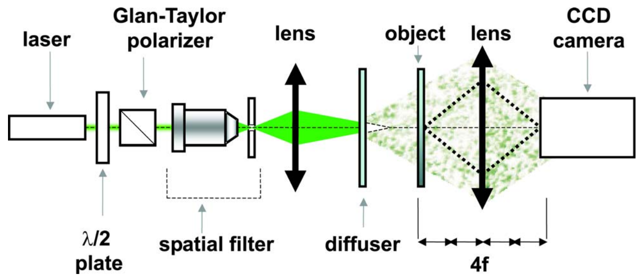

Fig. 1. (Color online) Experimental setup producing an optical version of the theoretical coherent imaging process of Eqs. (1)–(3). The λ/2 plate in association with the Glan-Taylor polarizer are used to control the intensity of the incident coherent wave coming from the second harmonic generation (532 nm, 10 mW) of a YAG:Nd compact laser. The spatial filter is used to obtain a uniform intensity on the static diffuser taken as a frosted glass. The first lens is adjusted with a micrometer-scale sensitivity linear stage to control the size of the speckle grain in the object plane. In Figs. 2 and 3 the speckle grain size has been adjusted to be much larger than the pixel size (the domain of validity of our model) and much smaller than the CCD matrix size (to diminish fluctuations from one acquisition to another). The object, a slide with calibrated transparency levels carrying the contrast of the input image $S(u,v)$, is illuminated by the speckled wave field. The second lens images the object plane on the CCD matrix of the camera. Variations of the speckle noise level in Figs. 2 and 3 are controlled by rotation of the λ/2 plate.

$$E_{SY} = \sqrt{\langle (S-Y)^2 \rangle} = \sqrt{\langle S^2 \rangle - 2\langle SY \rangle + \langle Y^2 \rangle}, \tag{4}$$

where $\langle \cdots \rangle$ denotes an average over the images. We assume that image $S(u,v)$ and speckle noise $N(u,v)$ are large enough so that a statistical description of the distribution of intensities on the image is meaningful: image $S(u,v)$ and speckle noise $N(u,v)$ possess empirical histograms of intensities, the normalized version of which is defining probability density $p_S(j)$ and $p_N(j)$ for the intensity of image $S(u,v)$ and $N(u,v)$. In principle, when $p_S(j)$, $p_N(j)$, and $g(\cdot)$ are all given, it is possible to theoretically predict the input–output image rms error E_{SY}. For instance, for $g(\cdot)$, a memoryless function on real numbers, one can use

$$\langle SY \rangle = \int_s \mathrm{d}_s\, s\, p_S(s) \int_n \mathrm{d}_n\, g(s \times n)\, p_N(n), \tag{5}$$

with similar expressions for $\langle S^2 \rangle$ and $\langle Y^2 \rangle$, and by such means one has, in principle, access to E_{SY}. We are going to show, with a specific memoryless function $g(\cdot)$, situations where an increase in the level of the speckle noise $N(u,v)$ can improve the quality of the output image $Y(u,v)$, measured by a decrease of the input-output image rms error of Eq. (4). In the following, we choose to consider, both for the experimental setup of Fig. 1 and for our theoretical coherent imaging model of Eqs. (1)–(3), a binary image, visible in Fig. 2, presenting gray level $S(u,v) \in \{R_0, R_1\}$ with $R_0 < R_1$ and 1024×1024 pixels, for which the probability of having a pixel with level R_1 is $\Pr\{S=R_1\}=p_1$ and $\Pr\{S=R_0\}=1-p_1$. For illustration, the image detector $g(\cdot)$ is taken as a memoryless hard limiter with threshold θ, i.e.,

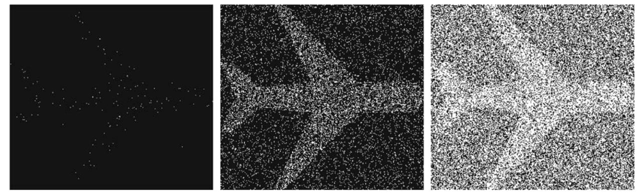

Fig. 2. Output image $Y(u,v)$ of the hard limiter of Eq. (6) for increasing rms amplitude $\sqrt{2}\sigma_N$ of the speckle noise $N(u,v)$. From left to right $\sqrt{2}\sigma_N=0.28$, 0.84 (optimal value), 2.81; with threshold $\theta=0.75$, $p_1=0.27$ and $\{R_0=1/2, R_1=1\}$.

$$g[X(u,v)] = \begin{cases} 0 & \text{for } X(u,v) \le \theta \\ 1 & \text{for } X(u,v) > \theta. \end{cases} \tag{6}$$

This hard limiter constitutes a very basic model for imaging systems when they operate, in the low flux domain, close to their threshold. Alternatively, the hard limiter in Eq. (6) also can be viewed as a threshold in a high-level image processing task such as segmentation or detection. In addition, these simple choices for the input image and the image detector are going to allow a complete analytical treatment of our theoretical model.

We are now in a position to study the evolution of the input–output image rms error E_{SY} of Eq. (4) as a function of the level of the speckle noise $N(u,v)$. The input image $S(u,v)$ takes different values over the background (R_0) and over the object (R_1). As a consequence, the rms amplitude of the speckle noise takes different values over these two regions. As a common reference in the sequel, we define the speckle noise level as the rms amplitude $\sqrt{2}\sigma_N$, corresponding to the speckle noise rms amplitude before action of the multiplicative coupling by the object or background in Eq. (1). The quality of the images transmitted by the hard limiter of Eq. (6) is assessed here by the rms error between the output image $Y(u,v)$ and a binary reference $S'(u,v)$ similar to $S(u,v)$, but with $R_0=0$ (the background) and $R_1=1$ (the object). In this context, the input–output image rms error of Eq. (4) becomes

$$E_{S'Y} = \sqrt{p_1 + q_1 - 2p_1 p_{11}}, \tag{7}$$

with conditional probabilities $p_{1k}=\Pr\{Y=1|S=R_k\}$ and $q_1=\Pr\{Y=1\}=p_1 p_{11}+(1-p_1)p_{10}$. The possibility of a useful role of the speckle noise in the image transmission process of Eqs. (1), (2), and (6) is visible in Fig. 3, where, for sufficiently large object-background contrast R_1/R_0 in input image $S(u,v)$, $E_{S'Y}$ follows a nonmonotonic evolution presenting a minimum for an optimal nonzero level $\sqrt{2}\sigma_{N\mathrm{opt}}$ of the speckle noise rms amplitude. This is the signature of a noise-assisted image transmission. Figure 3 also demonstrates a good agreement between experimental and theoretical results. In addition, it is possible to derive the theoretical expression $\sigma_{N\mathrm{opt}}$ minimizing the input–output image rms error $E_{S'Y}$ of Eq. (7) by solving $\partial E_{S'Y}/\partial \sigma_N = 0$, which leads to

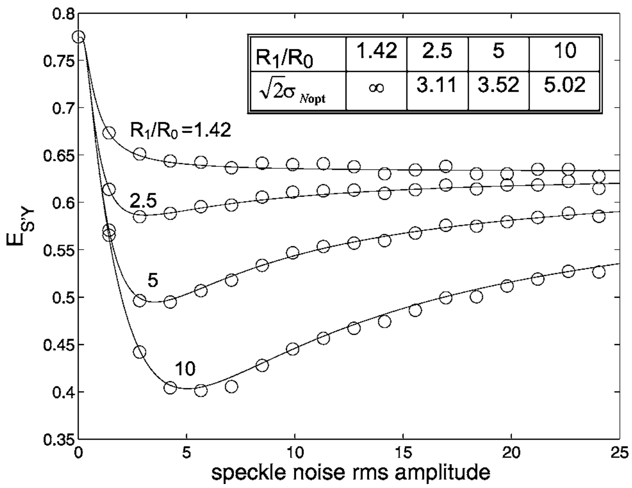

Fig. 3. Input–output image rms error $E_{S'Y}$ of Eq. (7) as a function of the rms amplitude $\sqrt{2}\sigma_N$ of the speckle noise $N(u,v)$ for various values of the input image contrast R_1/R_0. Solid lines stand for the theoretical expression of Eq. (7). The table gives the speckle noise optimal rms amplitude of Eq. (8). The discrete data sets (circles) are obtained by injecting in Eq. (1) real speckle images collected from the experimental setup of Fig. 1. The other parameters are $\theta=0.75$, $p_1=0.6$, $R_1=1$.

$$\sigma_{N\mathrm{opt}} = \frac{R_1 - R_0}{R_0 R_1}\frac{\theta}{\ln(K_a)}, \quad K_a = \frac{R_1}{R_0}\frac{1-p_1}{p_1}. \tag{8}$$

As seen in Eq. (8), there exist domains where the optimal speckle noise level $\sigma_{N\mathrm{opt}}$ is nonzero and positive when $\theta \neq 0$, $p_1 \neq 1$, $R_0 \neq R_1$ if $K_a > 1$. One can check in Fig. 3 that the positions of the optimal speckle noise level $\sqrt{2}\sigma_{N\mathrm{opt}}$ given by Eq. (8) show an exact agreement with the numerical calculations.

Finally, a visual appreciation of the cooperative effect of the speckle noise quantitatively illustrated in Fig. 3 is also presented in Fig. 2, where the multiplicative speckle noise injected in Eq. (1) comes from real speckle images collected from the experimental setup of Fig. 1.

We have demonstrated, theoretically and experimentally, the possibility of a constructive action of the multiplicative speckle noise in the transmission of an image in a coherent imaging system. For what we believe is a first report of this effect, the models for the input image, for the speckle noise, and for the imaging sensor have been purposely taken in their most simple forms. As a result, we obtained a theoretical prediction of the constructive role of the speckle noise, through an explicit theoretical analysis of the behavior of a relevant input–output similarity measure. The theoretical predictions displayed close agreement with experiment. A noticeable feature, in particular, is that our theoretical model authorizes an explicit derivation, without approximation, of an analytical expression (an outcome rarely accessible in studies of stochastic resonance in nonlinear systems) for the optimal level of the noise maximizing the performance in given conditions. The present demonstration of the feasibility of a constructive action of speckle noise in coherent imaging can be extended in various directions. More sophisticated images (with distributed gray levels, for example) could be considered, as well as other types of speckle noise, such as the one appearing in polarimetric imaging [10]. The simple threshold detector chosen here could be replaced by a multilevel quantizer or a linear sensor with saturation, closely matching attributes of digital cameras. It would then be interesting to confront, as done here, experiment and theoretical modeling, and examine how the phenomenon of improvement by noise evolves in these other conditions.

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