Joint acquisition-processing approach to optimize observation scales in noisy imaging

Agnès Delahaies, David Rousseau,* and François Chapeau-Blondeau

Laboratoire d'Ingénierie des Systèmes Automatisés (LISA), Université d'Angers, 62 avenue Notre Dame du Lac, 49000 Angers, France

**Corresponding author: david.rousseau@univ-angers.fr*

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In imaging, the choice of an observation scale is conventionally settled by the operator in charge of the image acquisition, who is left alone with tuning the framing and zooming parameters of the imaging system. In a somewhat decoupled manner, the operator in charge of processing the data has access to the images after their acquisition, and seeks to extract information from the observed scene. This Letter proposes a manifestation of the interest of an alternative joint acquisition-processing approach. We demonstrate with quantitative informational measures how the choice of an observation scale can be directly related to the performance of the final information processing task. Illustrations are given with various tools from statistical information theory with possible applications of practical interest to any noisy imaging domains.

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Extraction of an object over a background is a common task in imaging. In the absence of noise, the object is perfectly contrasted from the background and the optimal observation scale, the one that allows the most accurate estimate of the surface of the object, for instance, is the observation scale that shows the entire object at the largest magnitude. In the presence of noise, the frontier between object and background is perturbated. Common sense foresees that there exists an optimal intermediate observation scale that should be neither too large nor too small, so that both regions (background and object) are defined with sufficient resolution. In this work, we propose another interesting point of view where the spatial details or shape of the object do not have to be specified. We demonstrate that it is possible to calculate an optimal observation scale when the final information task motivating the image acquisition is used as prior knowledge at the early acquisition stage. We quantitatively address this novel and practical problem of the optimal observation scale in noisy imaging conditions [1] successively with the Shannon information [2] for an information transmission task, and with the Fisher information [2] and the Rissanen stochastic complexity [3,4] for estimation tasks.

Shannon information is a statistical measure useful for image processing tasks such as image quality assessment [5], segmentation [6], or detection [7] in noisy images. We propose to describe the transmission of Shannon information through an imaging system with the statistical information theory where a communication channel consists of a source (the observed scene here) delivering a message (an image here) to a receiver via a channel corrupted with noise (the noisy imaging system here). The informational capacity of a communication channel [2] is defined as $C = \max_{\Pr\{S\}} I(S;Y)$, where S is the input of the channel and Y is the output. $I(S;Y)$ is the input–output mutual information $I(S;Y) = H(Y) - H(Y|S)$ with $H(\cdot)$ the Shannon entropy [2]. The informational capacity C is the upper bound on the amount of Shannon information that can be transmitted through the channel. The noise being imposed, the capacity is obtained by adjusting the only free parameter, the coding of input, S, via its probability distribution, $\Pr\{S\}$. Modeling communication channels and calculating their informational capacity is rather classically useful in telecommunication contexts where unidimensional signals are to be transmitted. Here, $S(u,v)$ and $Y(u,v)$ are images, with (u,v) spatial coordinates where all the physics of a noisy imaging system is incorporated up to any level of required modeling realism in the relation between $Y(u,v)$ and $S(u,v)$ defining the channel. We model the simple situation of a uniform object alone on a uniform background with $S(u,v)$, a binary image. Our scene therefore consists of an object defined with a uniform gray level, I_1, and a background also uniform, I_0. The probability density associated with $S(u,v)$ is $\Pr\{S\}(s) = p_1\delta(s-I_1) + p_0\delta(s-I_0)$, where $p_1 = 1-p_0$ is the fraction of pixels at I_1; that is to say, the relative surface of the object in image $S(u,v)$. Thus, adjusting parameter p_1 is equivalent to modifying the scale at which the object is observed in the image. As a result, the value of p_1 that reaches the capacity of a channel modeling an imaging system defines the optimal observation scale for which the object is, from a Shannon point of view, best observed out of the background. For illustration, we choose, for methodological reasons but with no restriction on the physical noise coupling, to model a simple imaging system where the output images $Y(u,v)$, at the sensor level or after image processing, are binary: $Y(u,v) \in \{0,1\}$. The mutual information $I(S;Y)$ can be calculated from entropies $H(Y) = h[p_{11}p_1 + (1-p_{00})(1-p_1)] + h[(1-p_{11})p_1 + p_{00}(1-p_1)]$ with function $h(u) = -u\log_2(u)$, and $H(Y|S) = (1-p_1)[h(p_{00}) + h(1-p_{00})] + p_1[h(p_{11}) + h(1-p_{11})]$, where $p_{ij} = \Pr\{Y=i|S=I_j\}$. The derivative of $I(S;Y)$ with the observation scale p_1 can then be calculated and it leads to the optimal observation scale p_1^*, which maximizes $I(S;Y)$ and reaches the informational capacity C of the binary channel modeling the imaging system. From this, the question of the optimal observation scale of an object over a background finds a closed-form analytical solution,

$$p_1^* = \frac{a p_{00} - 1}{a(p_{00} + p_{11} - 1)}, \quad (1)$$

with

$$a = 1 + \exp\left[\ln(2) \frac{\sum_{i=0}^{i=1} (-1)^i (h(p_{ii}) + h(1 - p_{ii}))}{p_{00} + p_{11} - 1}\right]. \quad (2)$$

When the object and the background of the imaged scene present the same noise, i.e., when $p_{00} = p_{11}$, one has $p_1^* = 1/2$, according to Eq. (1). The optimal observation scale is obtained when the background and the object are occupying the same surface. In this situation, the common sense mentioned at the beginning of this Letter operates. Such a situation occurs in practice when the signal–noise coupling is additive, for instance, with thermal noise in sensors. Common sense is failing at $p_1^* \neq 1/2$, when $p_{00} \neq p_{11}$. In such cases, the optimal observation scale quantified by Eq. (1) indicates that the less noisy region (object or background) is to be observed so as to occupy a larger relative surface. Such cases are encountered with nonadditive noises such as speckle noise in coherent imaging, which is commonly [1] modeled as a multiplicative noise. Calculation of the optimal observation scale in such practical contexts is directly available from the framework presented here by specifying the physical signal–noise coupling. The result can also easily be extended to multiple gray-level images or to multiple objects, following the same methodology [2].

Information extraction often starts with the estimation of various parameters. With noisy imaging systems, information lies in the statistical properties of the images [1], with parameters of interest that are typically the average flux of photons or the standard deviation of this flux. In such an estimation context, an index to quantify the information contained in a noisy measurement about the value of a parameter attached to a given signal is the Fisher information. Via the Cramér–Rao inequality, the Fisher information fixes the efficiency of any unbiased estimator: the lower bound of the variance of such estimators is the reciprocal of the Fisher information. We consider an imaging system model similar to the one in the discussion of Shannon information, above, with a physical "binary" scene, $S(u,v)$, consisting of a homogeneous object occupying N_1 pixels in $S(u,v)$ placed on a homogeneous background occupying N_0 pixels in $S(u,v)$. The sensor of the imaging system has $N = N_1 + N_0$ pixels and it acquires image $Y(u,v)$, a noisy version of the observed scene $S(u,v)$. With this fixed resolution N of the imaging sensor, an observation scale thus appears for the object that can be defined as $p_1 = \frac{N_1}{N}$. Measurements are then performed on the acquired image $Y(u,v)$ in order to estimate a parameter. With no loss of generality, let us assume we are interested in estimating from a single image, $Y(u,v)$, the average flux of photon I_i with $i \in \{0,1\}$ in each of the two regions (background and object respectively stand for indices 0 and 1), constituting $S(u,v)$. The minimum variance of any unbiased estimator of I_i is $\frac{1}{N_i J_Y(I_i)}$ with the Fisher information $J_Y(I_i)$ contained in $Y(u,v)$ about I_i expressible as [2]

$$J_Y(I_i) = \int_{-\infty}^{+\infty} \frac{1}{p(y|s = I_i)} \left[\frac{\partial}{\partial I_i} p(y|s = I_i)\right]^2 \mathrm{d}y, \quad (3)$$

where $p(y|s = I_i)$ is the probability density function of $Y(u,v)$ in the area corresponding to region i in the observed scene. The minimum of the sum of the variances of estimations of I_1 and I_0 is therefore given by

$$\mathrm{var}_{\min} = \arg\min_{\{N_1\}} \frac{1}{N_1 J_Y(I_1)} + \frac{1}{(N - N_1) J_Y(I_0)}, \quad (4)$$

where we clearly see that the tradeoff on N_1 is defining an optimal observation scale $p_1^* = \frac{N_1^*}{N}$. The derivative of $\mathrm{var}_{\min}$ with N_1 can be calculated, and the question of the optimal observation scale of an object over a background in a general measurement context finds a closed-form analytical solution with

$$p_1^* = \frac{1}{1 + \sqrt{\frac{J_Y(I_1)}{J_Y(I_0)}}}. \quad (5)$$

According to Eq. (5) and similarly to what was found with the Shannon information, one departs from the trivial commonsense optimal scale $p_1^* \neq 1/2$ when the object and the background of the imaged scene do not present the same noise, i.e., here, when Fisher informations $J_Y(I_1) \neq J_Y(I_0)$.

When first-order statistical properties of noisy images are not fully determined by their first moments, it is sometimes useful to make an estimate of the whole probability density. For probability density estimation from observed data, a very common approach proceeds through the construction of an empirical histogram with equal-width bins. The number of bins chosen for the histogram is very important to the quality of the estimation: for a given number of data points, too few bins lead to an estimated histogram with poor resolution, while too many bins lead to a very irregular estimate with strong fluctuations in the counts. An effective method for determining an optimal number of bins based on general information theoretic notions is the principle of Rissanen minimum stochastic complexity. This principle has recently been applied to images in [4], and, as a rule, it amounts to choosing for the data, among a class of possible models, the model allowing the shortest description or coding of these data. We again consider the same imaging system as in the discussions of Shannon and Fisher information, above, where we now want to optimally estimate, in the sense of Rissanen stochastic complexity, the empirical histogram of each of the two regions (object and background) respectively observed with N_1 and $N_0 = N - N_1$ pixels in the acquired image $Y(u,v)$ with N pixels. The description length associated with the empirical histograms of each region $i = 0$ or 1 expresses [4]

$$L(Y_i) = \log[A_{N_i,K_i}] + N_i[H(\hat{f}_{k_i}) - \log(K_i)], \quad (6)$$

where $A_{N_i,K_i} = \frac{(N_i + K_i - 1)!}{N_i!(K_i - 1)!}$ and K_i is the number of bins in the histogram. The standard Shannon entropy function $H(\cdot)$ in Eq. (6) is applied to the empirical probabilities

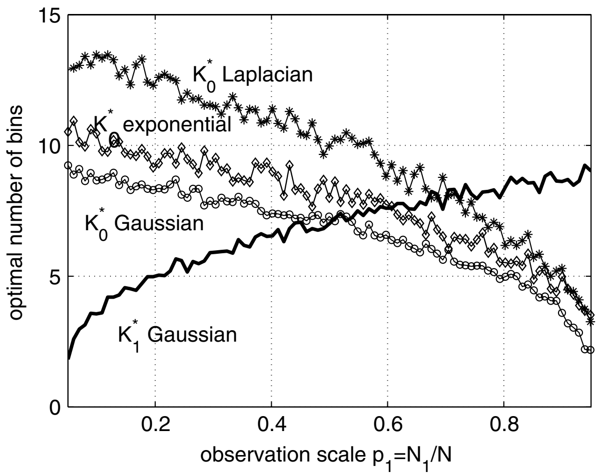

Fig. 1. Optimal number of bins K_0^* and K_1^*, giving the minimum description length associated with the empirical histograms of background $L(Y_0)$ and object $L(Y_1)$ of Eq. (6) as a function of the observation scale $p_1 = N_1/N$ of the object. An optimal scale p_1^* is at $K_1^* = K_0^*$. Object is distributed following a centered Gaussian probability density with standard deviation $\sigma_1 = 1$ and $N = 1024$. Various backgrounds, identical to the object (circles), distributed following a centered Laplacian (stars), or exponential (diamonds) probability densities [1], with $\sigma_0 = 1$ in all cases. Corresponding optimal observation scales are located around $p_1^* = 0.5$ (circles), $p_1^* = 0.7$ (stars), $p_1^* = 0.6$ (diamonds).

$\hat{f}_{k_i} = N_{k_i}/N_i$, with N_{k_i} the number of pixels in bin k_i. For a given acquired image $Y(u,v)$ (see Fig. 1 for illustration), the Rissanen complexity principle applied to each region separately selects $\{\hat{f}^*_{k_i}, k_i = 1, \ldots K_i^* | N_i\} = \min_{\{K_i\}}\{L(Y_i)|N_i\}$, with a criterion for the observation scale that can be taken at $p_1^* = \frac{N_1^*}{N}$ when the two regions, i.e., the whole image, can be quantized with the same algorithmic complexity, i.e., the same number of bins $K_1^* = K_0^*$. Other optimization criteria based on the Rissanen stochastic complexity could be proposed. One could for instance think of a multivariate optimization on $\{K_1, K_0, N_1\}$ with the constraint of fixed N. As visible in Fig. 1, from our criterion proposal $K_1^* = K_0^*$, the observation scale obtained allocates a larger area to the region corrupted with the noise with the heavier distribution.

In this work, we have demonstrated how informational measures could be used to optimize the experimental choice of the observation scale in noisy imaging systems. For a first occurrence of this approach, examples were limited to the determination of optimal observation scales based on informational measures adapted to general information transmission or statistical estimation in a simple scene composed of an object and a background. In this "binary" context, we have established the existence of analytical expressions for the optimal observation scale distinct for each informational criterion. This shows the interest of informational criterion adapted to specific image processing tasks. Other informational contexts, including detection, segmentation, or pattern recognition as in [6] could be analyzed in relation to the experimental determination of an optimal observation scale following our approach. More sophisticated scenes including multiple objects could also receive the same treatment (it just requires increasing the number of classes to be detected or estimated). With such images, it is likely that optimal observation scales would not be accessible under a closed-form analytical expression. It could nevertheless be calculated numerically. The present results are therefore applicable with no restriction to all imaging contexts where physical modeling of the image–noise coupling is accessible [1], and they extend the scope of joint informational strategies (including for instance compressive sensing [8] and source–channel coding [2]) where two stages of the informational chain are jointly performed.

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