Diagnosis Using an Estimation Method for Partially Observed Petri Net

A. Chouchane [*] **P. Declerck** [**] **A. Kheder** [***] **A. Kammoun** [****]

[*] *University of Sfax, Route de l'Aeroport Km 0.5. BP 1169 .3029 Sfax, Tunisia (e-mail: choucheneamira41@yahoo.fr)*
[**] *University of Angers, 62, avenue Notre Dame du Lac - 49000 Angers, France (e-mail: philippe.declerck@univ-angers.fr)*
[***] *University of Gafsa, Route cité Ennour, 2100 Gafsa, Tunisia, e-mail:(khedher_atef@yahoo.fr).*
[****] *University of Sfax, Route de l'Aeroport Km 0.5. BP 1169 .3029 Sfax, Tunisia (e-mail: A.Kamoun@enis.rnu.tn)*

Abstract: In this paper, we design a diagnostic technique for a partially observed labeled Petri net where the faults of the system are modeled by unobservable transitions. The diagnostic method is based on the state estimation principle using the set of observed transitions. The support of the approach is to describe the process under an algebraic form of a polyhedron for each observed firing of an observable transition.

Keywords: Diagnostic, partially observed Petri net, faults, state estimation principle, algebraic form, polyhedron.

1. INTRODUCTION

This paper focuses on the diagnosis of a process that can be modeled in a Discrete Event System (DES) whose state evolves at the occurrence of events. We can cite as examples the transport networks, the computer systems, multimedia systems, and manufacturing systems.

There are two powerful graphical and mathematical tools for DES modeling, which are finite state automata (Bouyer et al., 2005) and Petri nets. In this framework we choose the Petri net formalism, mainly because the presentation of a Petri net can model phenomena of synchronism, assembly and sharing of resources, and thus it is equipped with a much richer structure (Van, 1998). For various economic and/or technical reasons, the presence of a sensor for each system variable is not always possible. As a result, the Petri net may contain transitions that model unobservable events in the system. The relevant transitions are named unobservable transitions and the Petri net is then called a partially observed Petri net. Hence, the model that we use in this paper is a labeled partially observed Petri net.

In the literature, the techniques used for the diagnostic of Petri nets depend on the knowledge available on the system as well as on the detectable faults affecting the system (Wu et al., 2005) (Benveniste et al., 2003). For fault diagnosis, we can find event-based, state-based, and mixed-based faults. The event-based faults model the system faults into a set of transitions, and the occurrence of certain faults is equivalent to firing the associated transitions. The detection and localization of faults are carried out based only on observed events. These event-based models have the advantage of detecting intermittent faults (Genc et al., 2007) (García et al., 2008) (Ramirez et al., 2012). The state-based faults consider that the occurrence of a fault is equivalent to the change in the state of the Petri net deviating from its nominal behavior, which is expressed by losses or duplications of tokens. The disadvantage of the state-based faults' modulation is that it can not detect intermittent faults that are short events leading to unstable states (Wu et al., 2005) (Benveniste et al., 2003). The Mixed-based faults modulation is a combination of the occurrence of fault events and the attainability of fault states (Wu et al., 2005). We find three main techniques for Petri nets diagnosis: state estimation (Benveniste et al., 2003), parity space (Wu et al., 2005), and chronicles (Saddem et al., 2010).

In this paper, the diagnostic method is based on the estimation of a reduced set of unobservable firing sequences that enabled a sequence of observed transitions from a given initial marking without any assumption of acyclicity. The faults of the system are modeled by unobservable transitions. A fault will be detected if there exists at least one unobservable firing sequence of the reduced set that includes a fault transition, i.e., the count vector of fault transitions is different from the null vector. In this paper, the incidence matrices and the initial marking are assumed to be known. The occurrences of observable events are assumed to be non-simultaneous.

The strategy taken in this paper is as follows. Firstly, it is to algebraically describe the count vector denoted x_{un} of a possible unobservable sequence that enabled just one observed transition t of the observation under the form of a polyhedron $A.x_{un} \leq b$ over $\mathbb{Z}$ with an unknown x_{un}. The parameter A only depends on the structure of the Petri net while the parameter b depends on the initial marking and the observed transition. Secondly, using this algebraic model, the observer is designed for the estimation of a reduced subset of unobservable sequences that enabled t while minimizing a cost function. Accordingly, The set of associated count vectors x_{un} is expressed by an Integers' Linear Program (ILP) as $\{\min(c^{\top}.x_{un}) \mid A.x_{un} \leq b \quad \}$. By introducing a new variable, we can rewrite the ILP as a polyhedron that can be solved by the Fourier-Motzkin (FM) algorithm. The reduced observer is designed in two phases. The off-line preparation is based on the variables' elimination of the FM algorithm. The on-line procedure computes the current unobservable sequences and markings by exploiting the current data, which are updated at each occurrence of an observed event. An advantage of this two-phase approach is that only the off-line preparation contains the costly time part of the FM algorithm.

The paper is organized as follows. In the first section, we explain the principle of the state estimation of all sets of unobservable firing sequences and we propose a model of the form $A.x \leq b$ which describes the estimation problem under an algebraic point of view. In section 3, we estimate a reduced set of unobservable firing sequences that is expressed by an integer linear program. The following section introduces the detection and localization procedure based on a set of integer linear programs.

2. ALGEBRAIC DESCRIPTION OF ALL SETS OF UNOBSERVABLE FIRING SEQUENCES

2.1 Principle

In order to estimate the unobservable firing sequences that enabled an observation w, we algebraically characterize this set. We use the problem decomposition to reduce the search space of solutions. The search has is done step by step treating separately and successively each observed transition of the observation. Starting from one observation for each step, we determine the set of count vectors associated to the set of unobservable sequences that enable this transition. These vectors are called the set of explanation vectors. The firing of this observed transition and the unobservable sequences associated to the founded explanation vectors generate new current markings utilized in the following step. We restart the search procedure until the last observed transition.

The set of current markings are reached from the initial marking M_0, by firing the possible sequences associated to the explanation vectors of all steps. We assume that for the initial marking M_0, we have the observation (word) $w = \ell(t_{ob,1}), \ell(t_{ob,2}), ..., \ell(t_{ob,h})$, where $t_{ob,i} \in TR_{ob}$ is the i^{th} observed transition in w, and where $i \in [1..h]$ with h being the number of observed transitions in w. After that, we look for the set of minimal unobservable sequences $\sigma_{un} = \sigma_{un,1}, \sigma_{un,2}, ..., \sigma_{un,h}$ which enable w at M_0. In σ_{un}, the sequence $\sigma_{un,i} \in TR^*_{un}$ is an unobservable sequence that enables $t_{ob,i}$, after being fired, in the order $t_{ob,1}, t_{ob,2}, ..., t_{ob,i-1}$. The sequence σ_{un} can be unique or not.

In step 1, we treat the first observed transition $t_{ob,1}$ from the initial marking M_0 and we search the possible sequences $\sigma_{un,1}$ that enable this transition. The marking M_1, resulting from firing such $\sigma_{un,1} t_{ob,1}$ sequence is a new initial marking treated by step 2 with the new observed transition $t_{ob,2}$. The search procedure is repeated until the last observed transition $t_{ob,h}$. Finally we get:

$$M_0[\sigma_{un,1} t_{ob_1} \succ M_1[\sigma_{un,2} t_{ob,2} \succ M_2 .....M_{h-1} \\ [\sigma_{un,h-1} t_{ob_{h-1}} \succ M_h \tag{1}$$

The marking M_h, resulting from firing the sequence $\sigma_{un,1} t_{ob,1} \sigma_{un,2} t_{ob,2} ......\sigma_{un,h} t_{ob,h}$ and the initial marking M_0, is a possible current marking. The set of current markings are the markings by discovering all the possible unobservable firing sequences $\sigma_{un,1}, \sigma_{un,2}, ..., \sigma_{un,h}$.

In what follows, we describe the estimation problem of the explanation vectors that enabled just one observed transition t of the observation w under the form of a polyhedron $A.x_{un} \leq b$ over $\mathbb{Z}$ with an unknown x_{un}.

2.2 Polyhedron of explanation vectors

Definition *1.* The set of the explanation vectors of the transition t at M is the set of the count vectors associated to the unobservable transitions that enable the transition t, from the marking M.

$$E(M, t) = \{\pi(\sigma_{un}) | \sigma_{un} \in TR^*_{un} \text{ such that} \\ M[\sigma_{un} \succ M' \text{ and } M'[t \succ\}$$

where M' is the marking reached from the marking M by firing the unobservable sequence σ_{un}.

We suppose that from the marking M, we observe the transition $t \in TR_{obs}$, then there exists $\sigma_{un} \in TR^*_{un}$ such that $M[\sigma_{un} > M'$ and $M'[t >$, so we get the system:

$$\begin{cases} M' = M + W_{un}.x_{un} \\ M' \geq W^-_{obs}(.,t) \\ x_{un} \geq 0 \end{cases} \tag{2}$$

Hence, the vector x_{un} must verify the following matrix inequality:

$$A.x_{un} \leq b \tag{3}$$

with $A = \begin{pmatrix} -W_{un} \\ -I_{k \times k} \end{pmatrix}$ and $b = \begin{pmatrix} M - W^-_{obs}(.,t) \\ 0_{k \times 1} \end{pmatrix}$.

For a Petri net with an acyclic TR_{un}-induced subnet, each solution of the matrix inequality (3) in $\mathbb{Z}^k$ corresponds to an explanation vector (Stremersch et al., 2002). In the literature, there exist a lot of works on the determination of the count vectors that are associated with real firing sequences in the case of a cyclic $TR_{un}-$induced subnet, by solving the reachability problem (Stremersch et al., 2002). Then, it consists in demonstrating that there exists a firing sequence from the marking M to reach the target marking $M' = M + W_{un}.x_{un}$.

In the following subsection, we use the off-line the FM elimination algorithm to structurally describe the set of explanation vectors.

2.3 Fourier Motzkin elimination algorithm for off-line characterization of explanation vectors

The FM elimination algorithm transforms the linear inequalities system $A.x \leq b$ with $x = (x_1, ..., x_n)^{\top}$ on a set of n inequalities where each one depends on just one unknown. We are interested in this paper in utilizing the FM eliminations for the off-line generation of a set of inequalities depending only on one unknown whatever the initial marking and the observed transition are, i.e., in function of the data vector b.

The FM elimination algorithm consists of two consecutive phases, repeated until the penultimate variables. The first phase is the computation of bounds for the system variable x_l, which is in function of the variables x_{l+1} until x_n. The second phase is the construction of a new system by eliminating the variable x_l, and consequently the construction of a new system of the variables x_{l+l} until x_n from which we are looking for bounds for x_{l+1}, and so on. For the last variable x_n, we get bounds of known terminals. Thus, by determining x_n we can determine successively x_{n-1} back to x_1. The existence of a solution test, called the feasibility condition is achieved by checking if the lower limit of the last variable x_n is less or equal to its upper limit.

(1) Phase1:Computation of bounds

At the beginning of each step, the system $A^l.X_l \leq b^l$ of unknown $X_l = [x_l \; x_{l+1} ... x_n]^T$ is put in the following form:

$$(S) : \begin{cases} a^l_{pos}.x_l + A^l_{pos}.x^{'} \leq b^l_{pos} & S_+ \\ -a^l_{neg}.x_l + A^l_{neg}.x^{'} \leq b^l_{neg} & S_- \\ a^l_{zer}.x_l + A^l_{zer}.x^{'} \leq b^l_{zer} & S_0 \end{cases} \quad (4)$$

where x_l is the variable to eliminate, $x^{'} = [x_{l+1} \; x_{l+2} ... x_n]^T$ is the vector composed of the other variables after eliminating x_l. The subsystem S_+ corresponds to the subsystem of n^+ inequalities that are associated with the unitary vector a^l_{pos} containing the x_l coefficients of 1. The subsystem S_- corresponds to the subsystem of n^- inequalities that are associated with the unitary vector a^l_{neg} containing the x_l coefficients of -1. The subsystem S_0 corresponds to the subsystem of n^0 inequalities associated with the zeros' vector a^l_{zer} containing the x_l coefficients of 0. The dimensions of matrices can easily be deduced from system (4).

If the two subsystems S_+ and S_- exist, then we can determine the bounds to x_l whose lower limit is a maximum of terms while the upper limit is a minimum:

$$max[(A^l_{neg}.x^{'} - b^l_{neg})_{i,.}] \leq x_l \leq min[(b^l_{pos} - A^l_{pos}.x^{'})_{i,.}]. \quad (5)$$

If S_+ does not exist in (S), then the upper limit of x_l is equal to $+\infty$. If S_- does not exist in (S), then the lower limit of x_l is equal to $-\infty$.

(2) Phase2:Elimination and construction of new system

This phase consists in putting into zeros the column associated with x_l, by adding each row of S_+ with the rows of S_-, and by keeping the rows of S_0. After that, we eliminate the zeros'column associated with x_l.

For the construction of a new system, the system is put in the form (4), after normalizing the values of the first column.

The two phases described above are repeated for the other variables.

3. REDUCED SET OF UNOBSERVABLE FIRING SEQUENCES MINIMIZING A COST FUNCTION

We can determine a subset of the explanation vectors by minimizing a cost function $c^\top.x_{un}$. Accordingly, The set of count vectors x_{un} minimizing $c^\top.x_{un}$ satisfies the ILP:

$$\begin{cases} \min\limits_{x_{un} \in \mathbb{Z}^k} \; c^\top.x_{un} \\ s.t. \quad A.x_{un} \leq b \end{cases} \quad (6)$$

We can write the ILP (6) in the following augmented form:

$$\begin{cases} \min \mu \text{ such that} \\ A_{aug}.y \leq b_{aug} \\ x_{un} \in \mathbb{Z}^k, \mu \in \mathbb{Z}^+ \end{cases} \quad (7)$$

with $A_{aug} = \begin{bmatrix} A & 0_{m\times 1} \\ c^\top & -1 \end{bmatrix}$, the unknown vector $y = \begin{bmatrix} x_{un} \\ \mu \end{bmatrix}$, and the vector $b_{aug} = \begin{bmatrix} b \\ 0 \end{bmatrix}$.

The positive integer variable μ verifying $c^\top.x_{un} \leq \mu$ is chosen as small as possible. We can apply off-line the FM elimination algorithm to get a system of $n+1$ inequalities where each one depends on just one unknown.

In what follows, we show that the optimization problem (6) serves us in detecting and localizing system faults choosing suitable cost functions.

4. SYSTEM DIAGNOSTIC

The aim of this approach is to create a model diagnoser allowing to detect the system faults. The process model describes the normal and faulty behaviors.

Considering that a certain number of faulty behaviors may occur in the system, we assume that the possible faults that can affect the system are modeled by unobservable transitions. Naturally, there may also exist other transitions that represent normal behaviors. We denote TR_f the set of fault modeled by unobservable transitions of an n_f cardinal and TR_n the set of unobservable transitions of an n_n cardinal, which represent normal behaviors. Then, we have:

$$TR_{un} = TR_n \bigcup TR_f \quad (8)$$

Let us consider the count vector x_n of nominal unobservable transitions and the incidence matrix W_n of a nominal unobservable subnet $N_n = (P, TR_n, W^+_n, W^-_n)$. Similarly, x_f is the count vector of faulty unobservable transitions and W_f is the incidence matrix of a faulty unobservable subnet $N_f = (P, TR_f, W^+_f, W^-_f)$.

Then, from the marking M and for the observed transition $t \in TR_{obs}$, the explanation vectors $E(M, t)$ is expressed as follows:

$$A_n.x_n + A_f.x_f \leq b \quad (9)$$

where $x_{un} = (x_n \ x_f)^\top$, the matrix $A_n = \begin{pmatrix} -W_n \\ -I_{n_n \times n_n} \\ 0_{n_f \times n_n} \end{pmatrix}$, the matrix $A_f = \begin{pmatrix} -W_f \\ 0_{n_n \times n_f} \\ -I_{n_f \times n_f} \end{pmatrix}$ and the vector $b = \begin{pmatrix} M - W^-_{obs}(.,t) \\ 0_k \end{pmatrix}$ with $k = n_n + n_f$. In the following, we explain the principle of diagnosis.

4.1 Diagnostic principle

The diagnostic method taken in this paper is based on a set of partial diagnostic decisions by treating separately and successively each observed transition $t_{ob,i}$ of the observed word w. The aim of diagnosis for an observation $t_{ob,i}$ is:

- to determine if the estimated unobservable sequences $\sigma_{un,i}$ enabling the observed transition $t_{ob,i}$ from the marking M correspond to an affected behavior or not,
- and to localize the faults $T^j_f, j \in [1..n_f]$ that leads to a faulty behavior after fault detection.

We determine then the state of the partial diagnosis $\Delta(t_{ob,i})$ and the functions of partial localizations $\Delta(t_{ob,i}, T^j_f)$.

4.2 Fault detection and localization for one observed transition

In this section, we design a fault detection and localization method based on the ILP. We show that the detection of faults is formulated as a problem of minimization of the sums of fault transitions'firing, and the location of faults is based on n_f minimization problems where n_f is the number of fault transitions in the system.

Fault detection A fault is detected if there is at least one fault transition that is fired. This means that the firing sum of the fault transitions is greater or equal to 1. Let us consider the optimization problem defined as follows:

$$\begin{cases} \min(c^\top.x_f) \text{ such that} \\ A_n.x_n + A_f.x_f \le b \\ x_n \in \mathbb{Z}^{n_n}; x_f \in \mathbb{Z}^{n_f} \end{cases} \tag{10}$$

where c is a vector of ones of dimension n_f. Therefore, we can deduce that:

- Case 1: If $\min(c^\top.x_f) = 0$, then we cannot conclude on the existence of a fault.
- Case 2: If $\min(c^\top.x_f) \ge 1$, then a fault is detected.

Moreover, if $\min(c^\top.x_f) \ge 1$, then it gives the minimal number of detected faults that are possibly of the same type. Hence, the detection function for the observed transition $t_{ob,i}$ is defined by:

$$\triangle(t_{ob,i}) = \min(c^\top.x_f) \tag{11}$$

For the off-line application of the FM eliminations on the optimization problem (10), we rewrite the optimization problem (10) under the form $A_{aug}.y \le b_{aug}$ by introducing a new variable μ such that $c.x_f \le \mu$, taking the variable μ as small as possible. The system (10) is then rewritten as below:

$$\begin{cases} \min \mu \\ \text{such that} \begin{bmatrix} A_n & A_f & 0_{m\times 1} \\ 0 & c^\top & -1 \end{bmatrix} . \begin{bmatrix} x_n \\ x_f \\ \mu \end{bmatrix} \le \begin{bmatrix} b \\ 0 \end{bmatrix} \end{cases} \tag{12}$$

For an observed transition $t_{ob,i}$, if we detect faults then we must locate the fault transitions that produce the fault behavior. In the following, we design a method for fault localization.

Fault localization In order to isolate the faults of the system, the occurrence of the faults is tested for each system fault T^j_f separately : For each fault T^j_f with $j \in \{1, \dots, n_f\}$, we test whether this fault has occurred at least once. The optimization problem for each fault T^j_f is as follows:

$$\begin{cases} \min(x^j_f) \text{ such that} \\ A_n.x_n + A_f.x_f \le b \\ x_n \in \mathbb{Z}^{n_n}; x_f \in \mathbb{Z}^{n_f} \end{cases} \tag{13}$$

where x^j_f is the firing count vector of the fault transition T^j_f. Then, we can distinguish two cases:

- Case 1: If $\min(x^j_f) = 0$, then we cannot conclude on the occurrence of the fault T^j_f.
- Case 2: If $\min(x^j_f) \ge 1$, then the fault T^j_f has occurred.

Moreover, if $\min(x^j_f) \ge 1$, then the fault T^j_f can occurred at least $\min(x^j_f)$ times. Therefore, given an observed transition $t_{ob,i}$, the localization function for each fault transition $T^j_f \in TR_f$, $j \in \{1, \dots, n_f\}$ is as follows:

$$\triangle(t_{ob,i}, T^j_f) = \min(x^j_f) \tag{14}$$

The number of isolated faults is always lower or equal to the number of detected faults. Indeed, we have:

$$0 \le \sum_j \min(x^j_f) \le \min(c^\top.x_f) \tag{15}$$

Consequently, it is possible to detect faults without being able to isolate them. Especially, a possible situation is $(\min(c^\top.x_f) \ge 1)$ and $(\min(x^j_f) = 0 \ , \ \forall j)$. In other words, the system faults are detectable but are not always localizable.

For the on-line resolution of the optimization problem (13), **we base again on the** off-line inequalities produced by FM elimination algorithm: The optimization problem (10) must be rewritten under the form $A_{aug}.y \le b_{aug}$ by introducing a new variable μ_j such that $c_j.x_f = x^j_f \le \mu_j$. To minimize $c_j.x_f$, we take the variable μ_j as small as possible. Then, we get:

$$\begin{cases} \min \mu_j \\ \text{such that} \begin{bmatrix} A_n & A_f & 0_{m\times 1} \\ 0 & c_j^\top & -1 \end{bmatrix} . \begin{bmatrix} x_n \\ x_f \\ \mu_j \end{bmatrix} \le \begin{bmatrix} b \\ 0 \end{bmatrix} \end{cases} \tag{16}$$

where c_j is a vector of zeros except the j^{th} elements associated to the firing count vector x_f^j of the fault transition T_f^j, which is equal to 1.

***Example** 1.* Let us consider the Petri net of figure 1 where $TR_{obs} = \{t_1, t_2, t_3\}$, $TR_n = \{\varepsilon_1, \varepsilon_2\}$, and $TR_f = \{T_f^1, T_f^2\}$.

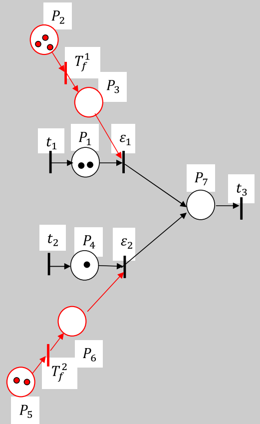

Fig. 1. Petri net example

We note $m'_i = M(p_i) - W^-_{obs}(p_i, t)$

- Off-line inequations for fault detection:
  The FM elimination algorithm applied on the ILP (10) associated with our system gives the following system inequalities:

$$\begin{cases} max(0, -x_2 - m'_7) \leq x_1 \leq m'_1 \\ max(0, -m'_1 - m'_7) \leq x_2 \leq m'_4 \\ max(0, -m'_3) \leq x_f^1 \leq min(m'_2, -x_f^2 + \mu) \\ max(0, -m'_6) \leq x_f^2 \leq min(m'_5, m'_3 + \mu) \\ -\mu \leq m'_3 + m'_6 \end{cases} \tag{17}$$

- Off-line inequations for fault localization:
  The FM eliminations applied on the ILP (13) for the fault transitions T_f^1 and T_f^2 gives the following system inequalities:

$$\text{For } T_f^1 : \begin{cases} max(0, -x_2 - m'_7) \leq x_1 \leq m'_1 \\ max(0, -m'_1 - m'_7) \leq x_2 \leq m'_4 \\ max(0, -m'_3) \leq x_f^1 \leq min(m'_2, \mu) \\ max(0, -m'_6) \leq x_f^2 \leq m'_5 \\ -\mu \leq m'_3 \end{cases} \tag{18}$$

$$\text{For } T_f^2 : \begin{cases} max(0, -x_2 - m'_7) \leq x_1 \leq m'_1 \\ max(0, -m'_1 - m'_7) \leq x_2 \leq m'_4 \\ max(0, -m'_3) \leq x_f^1 \leq m'_2 \\ max(0, -m'_6) \leq x_f^2 \leq min(m'_5, \mu) \\ -\mu \leq m'_6 \end{cases} \tag{19}$$

We suppose that we have the observation t_3 at the initial marking $M_0 = (2\,3\,0\,1\,2\,0\,0)^\top$.

- On-line fault detection:
  For $\mu = 0$, it exist not integer vector verifying (17). For $\mu = 1$,the vector $(1\ 0\ 1\ 0)^\top$ verifying (17) and associated to the firing sequence $\varepsilon_1 T_f^1$. Then, we detect default. Then $\triangle(t_3) = 1$.
- On-line T_f^1 localization:
  For $\mu_1 = 0$, the vector $(0\ 1\ 0\ 1)^\top$ verifying (18) and associated to the firing sequence $\varepsilon_2 T_f^2$. Then, $\triangle(t_3, T_f^1) = 0$. Consequently, T_f^1 is not localisable.
- On-line T_f^2 localization:
  For $\mu_2 = 0$, the vector $(1\ 0\ 1\ 0)^\top$ verifying (19) and associated to the firing sequence $\varepsilon_1 T_f^1$. Then, $\triangle(t_3, T_f^2) = 0$. Consequently, T_f^2 is not localisable.

Then, we detect fault but the localization is not possible.

5. CONCLUSION

In this paper, an on-line approach for fault detection and localization is proposed. The systems that are concerned are the DES modeled by a labeled partially observed Petri net, where a subset of unobservable transitions model the faults. The support of the approach is to describe the process under the algebraic form of a polyhedron for each observed firing of an observable transition. Computing the solution of one ILP gives the diagnostic state of the fault detection. The localization of each fault is made by a specific ILP. A perspective is to develop the parity space for Petri nets when the faults cannot be described by the model.

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