State Estimation of Timed Labeled Petri Nets with Unobservable Transitions

Philippe Declerck and Patrice Bonhomme

***Abstract*—The aim of this paper is to reconstruct the least/greatest sequence of unobservable transitions in timed Petri nets based on the on-line observation of firing occurrences of some transitions on a sliding horizon. The Petri net, which can be unbounded and can contain self-loops and circuits, is described under an algebraic form composed of $A.x \leq b$ which expresses the possible time sequence x and the fundamental marking relation. Under the assumption of Backward/Forward Conflict Freeness of the unobservable-induced subnet, we show the existence of a finite least/greatest sequence with respect to the data known on a given horizon. A technique of computation using linear programming is given.**

Note to Practitioners: ***Abstract*—In many processes, it is not always possible to associate a sensor with each state due to the cost and the physical location. In most control applications, not all state variables are measurable. This characteristic can be found in many discrete event systems such as manufacturing systems, microcircuit design, transportation systems, and the food industry. The variables in discrete event systems express events such as the beginning/end of a task, the departure/arrival of a train at a railroad crossing, etc. However, the unknown data can be crucial for the control system which supervises the process. In particular, the knowledge of the timestamps of these past events allows future actions to be determined. The technique proposed in this paper is based on a specific calculation of the unknown numbers of events by using the known data on a sliding horizon.**

***Index Terms*—Petri nets, Time, Observer, Lattice, Linear programming**

I. Introduction

In the framework of discrete event dynamic systems, the observability and the observer design problem have received much attention over the last few years, particularly from a Petri net point of view. In most control applications, not all state variables are measurable: In practice, it is not always possible to associate a sensor with each state. In the classical system theory, this partial knowledge of the system state has led to the introduction of observers in order to estimate these states that cannot be measured directly. In the field of Petri nets, the general objective of the estimation can be defined as follows: It is assumed that a certain number of transitions are labeled with the empty string ε, while a different label taken from a given alphabet is assigned to all other transitions. As the firing of the transitions labeled with the empty string cannot be observed, these transitions are called unobservable or silent. Therefore, the set of transitions is then partitioned into $TR = TR_{obs} \cup TR_{un}$ where TR_{obs} is the set of observable transitions while TR_{un} is the set of unobservable ones. For a sequence (word) ω observed on a given horizon, the aim is to compute firing sequences of unobservable transitions necessary to complete ω into a fireable sequence of the Petri net consistent with its past evolution.

In this paper, we focus on an on-line estimation approach based on a sliding horizon by considering a sequence of observable events at each step of the estimation on a time horizon. This means that after the computation of the state estimate on a given horizon, the horizon shifts to the next sample, and the estimation of the state estimate is restarted using known information of the new horizon. The interest in such estimation methods stems from the possibility of dealing with a limited amount of data, instead of using all the information available from the beginning [5]. In this framework, we focus on the estimation of the least (respectively, greatest) sequence of a timed labeled Petri net system based on the observation of transition labels which are time events for each transition. Coherent with the vocabulary used in lattices, this least (respectively, greatest) sequence (if it exists) is also called minimum (respectively, maximum) sequence and is unique by definition.

Let us illustrate these different sequences by analyzing the schedule of planes in an airport. Two companies 1 and 2 operate a shuttle service between Paris and London with a unique plane 1 and 2 respectively. Number $\{1, 2, \ldots, 7\}$ the days of a week and note $x_i(t)$ the number of past stops of plane i at the end of the day t. If plane 1 can get to the airport twice, and plane 2 three times on a given day (Monday) $t = 1$, then plane 1 has the lowest frequency. Indeed, we have $x_1(t = 1) = 2 \leq x_2(t = 1) = 3$. Moreover, we consider the least sequence with respect to the componentwise order. Now consider an airport which chooses between company 1 and company 2 each day with respect to the frequency of the stops. Suppose that the possible schedules are as follows $x_1(1) = 2$, $x_2(1) = 4$, $x_1(2) = 5$ and $x_2(2) = 3$. The least number of occurrences of the planes is $x_1(1) = 2$, and $x_2(2) = 3$ which is the least sequence with respect to the componentwise order. It is also the least efficient solution for users but limits the deterioration of the airport and the related cost. Symmetrically, the greatest sequence is $x_2(1) = 4$ and $x_1(2) = 5$ which is the most efficient solution for users. It also leads to the optimal earnings of the airport corresponding to the landing fees paid by the aircrafts for landing.

We now consider the following pedagogical problem where

P. Declerck is with Laboratory LISA (EA4094) of University of Angers, 62, avenue Notre Dame du Lac - 49000 Angers, France, e-mail: philippe.declerck@univ-angers.fr.

P. Bonhomme is with Laboratory of Computer Science (EA 2101), Scheduling and Control (ERL CNRS 6305), University François Rabelais of Tours, 64, avenue Jean Portalis, 37200 Tours, France. e-mail: bonhomme@univ-tours.fr

Manuscript received April 19, 2013; revised January 11, 2013.

the time unit is a minute which is more suitable for the description than a day. Let us assume that a person in Paris observes that 10 planes coming from London have landed at time 600 minutes or before. A maintenance department at London can use this observation. Indeed, we can conclude that at least ten planes have taken off from London at time $600-T_{LP}$ minutes where T_{LP} is the travel time from London to Paris. So, the least number of plane take-offs from London is ten at time $600-T_{LP}$ minutes. If each plane has been checked before its take-off, we can estimate the minimum activity of the maintenance department and the relevant invoice giving a (guaranteed) minimum sum. Moreover, at the most 10 planes have taken off at time 650 minutes or before at Paris if we assume that each plane stays at the airport at least 50 minutes. We can conclude that the greatest number of 10 planes have landed in London at time $650+T_{PL}$ minutes where T_{PL} is the travel time from Paris to London. It could be lower: A pilot can decide to return or to land at another airport for technical reasons. The least number of landings is zero in the worst case. Based on the greatest estimated number of landings in London (the highest estimated frequency of the air fleet), the relevant invoice giving the (optimistic) maximum sum can be made. Note that the manager of the maintenance department can also estimate the maximum activity of the maintenance and the working hours of the employees if more observations are used.

Let us briefly put our contribution into a general context and give some related works. A more extensive synthesis about state estimation problems and fault detection using Petri nets and automata models can be found in [9] and numerous references can be found therein. A possible approach is to build an automaton observer for a Petri net model which is a deterministic automaton whose set of events is represented by the set of labels of the observable transitions of the Petri net [2] [18]. The states of the automaton store all the Petri net markings that can be reached from the initial marking which is assumed to be known. An issue that arises with building an observer automaton is that in the worst case it has exponential complexity in the number of states of the original model [11] [15]. Moreover, the state space of the marking is possibly infinite. The starting point of the solution in [18] is a finite state machine which is assumed to be without an unobservable cycle. The technique using an automaton observer also requires that the set of consistent states must be explicitly enumerated. As the set of estimated markings can be huge, and the on-line calculation of this set can be computationally prohibitive, a reduced observer automaton is described in [12].

A second class of approaches directly considers the marking and is often based on an algebraic description of the state space which does not require to be enumerated. An advantage is the possibility to adapt the general results of linear programming to Petri nets. In [8] [17], the algorithm reconstructs the sequence of unobservable transitions, allowing the occurrence of an observable event. Under the acyclicity assumption of the unobservable subnet of the system, the set of markings consistent with the observed word is represented by a linear system with a fixed structure that does not depend on the length of the observed word [8].

Different variations of the state estimation problem for Discrete Event Systems can be distinguished. An issue arises when the labeling function that assigns to each event a label is non-injective (i.e. transitions share the same label). Named ‘non-deterministic transitions’ in some studies, this means that an event occurrence may not be distinguishable from the occurrence of another different event [20] [17]. Another variation is to consider that the Petri net is equipped with sensors that allow observation of the number of tokens in some places [16]. Similar to the state estimation problem in automatic control for continuous systems (Kalman filter, Luenberger observer), we can also consider that the starting point of the system evolution is unknown. In this paper, we consider that the initial counter $x(t=0)$ is unknown. This problem is also considered in [5] developed in the algebra of dioids which presents an estimation approach for Time Event Graphs with complex synchronizations such as P-Time Event Graphs and Time Stream Event Graphs. Considering non-bounded Event Graphs, the approach uses a receding-horizon estimation of the greatest state, and analyzes the consistency of the data. Following the same strategy, we consider simpler synchronizations but more complex incidence matrices in this proposed paper. Note that the state considered in [5] is not a vector of sequences (also called counter) but a vector of dates of the transition firing (also called dater [10]) which presents a duality with the counter form. Another state (a pair composed of a marking and firing temporal interval) is also used in [7] where model checking is applied to acyclic unobservable time Petri nets.

In this paper, the net structure and the initial marking are assumed to be known. The net is assumed to be live. We also take make the assumption that the algebraic model of the timed Petri net described in this paper is ‘time live’ or consistent, that is, it presents at least one time sequence during the application of the on-line approach. We finally assume that the firing of the different observable transitions can be distinguished (a prospective study is to analyze the problem when different transitions share the same label). The firings of the transitions can be simultaneous. No fluidification of the Petri net model is considered in this paper. As in [10] [4][5] and the relevant examples, the assumptions of the non-cyclicity and boundedness of the Petri net, which are common in many papers, are not made. The presence of circuits in the Petri net does not affect the results of this paper (nor our past papers in different fields) because we take time into consideration and choose an algebraic modeling based on a specific state called ‘counter’ (or ‘dater’ in other papers).

The paper is organized as follows: The following section presents basic notions of Petri nets and lattice theory. We then show that the timed Petri net can be described by an algebraic model which is analyzed in the subsequent sections. The next section introduces different theoretical results which show the existence of the least and greatest solutions under some assumptions on the structure of the unobservable subnet. Finally, an estimation approach based on linear programming is proposed. The case of contact-free unobservable transitions is considered. We also discuss the cyclicity and give Example

1 with a self-loop. Finally, Example 2 containing circuits illustrates the approach.

II. Preliminary

In this section, we refer back to the formalism of Petri nets. The notation $|X|$ is the cardinal number of the set X. A Place/Transition net (a P/TR net) is a structure $N = (P, TR, W^+, W^-)$, where P is a set of $|P|$ places and TR is a set of $|TR|$ transitions which are denoted by x (notation t corresponds to the current time, T_l to the temporization of place $p_l \in P$, and T to the transposition of a matrix). Matrices W^+ and W^- are $|P| \times |TR|$ post- and pre-incidence matrices over $\mathbb{N}$ where each row $l \in \{1, \ldots, |P|\}$ specifies the weight of the incoming and outgoing arcs of place $p_l \in P$ respectively. The incidence matrix is $W = W^+ - W^-$. In this paper, we consider that the weight of each arc is unitary which implies that $W_{ij} \in \{-1, 0, 1\}$. The preset and postset of a node $X \in P \cup TR$ are denoted by ${}^\bullet X$ and $X^\bullet$ respectively. The marking of set P is a vector $M \in \mathbb{N}^{|P|}$ that assigns to each place of a P/TR net a non-negative integer number of tokens, represented by black dots. M_l is the marking of place p_l with $l \in \{1, \ldots, |P|\}$. A net system (N, M_0) is a net N with an initial marking $M_0 = M(t = 0)$.

This part briefly recalls lattice vocabulary (see Part 4.3.1 of [1]). This paper deals with partial order $\leq$ defined on set $\mathbb{R}^n$ which is defined componentwise: $x \leq y$ if and only if $x_i \leq y_i$, $\forall i \in \{1, 2, ..., n\}$. The greatest (respectively, least) element of a subset is an element of the subset which is greater (resp., lower) than any other element of the subset. If it exists, it is unique. The greatest (resp., least) element of a subset is also called maximum element (resp., minimum element or smallest element) of a subset. A majorant (resp., minorant) of a subset is an element not necessarily belonging to the subset which is greater (resp., lower) than any other element of the subset. If a majorant (resp., minorant) belongs to the subset, it is the greatest (resp., least) element. The upper bound (resp., the lower bound) is the least majorant (resp., greatest minorant). Sup-semilattice (resp., inf-semilattice) $(\Phi, \leq)$ is an ordered set Φ such that there exists an upper (resp., lower) bound for each pair of elements. It is called a lattice if it is both an inf-semilattice and a sup-semilattice.

III. Model

With language misuse, each transition and its corresponding variable is denoted with the same letter. Each transition is associated with the *number of events* which happen *before or at time* t. Called a 'counter', the number of events which are the firings of the transition is denoted by $x(t)$. In this paper, time is discrete ($t \in \mathbb{Z}$) and the occurrence of each event is synchronized with an external clock. Assuming that the events can only occur at $t \geq 1$, we have $x(t) = 0$ for $t \leq 0$. For any $t \in \mathbb{N}^*$, it may be that no event takes place at t, a single event happens at t, or several events occur simultaneously at t. Remember that it leads to non-decreasing sequences. For a given transition, the arrival of two events at times 3 and 5 implies that the sequence of numbers of events starting at $t = 0$ and finishing at $t = 7$ is 0, 0, 0, 1, 1, 2, 2, 2, that is, $x(t = 3) = 1$ and $x(t = 5) = 2$ but also $x(t = 4) = 1$ and $x(t = 7) = 2$.

Timed Petri nets allow the modeling of discrete event systems with sojourn time constraints of the tokens inside the places. Each place $p_l \in P$ is associated with a temporization $T_l \in \mathbb{N}$. Its initial marking is the entry l of the vector M_0 which is denoted by $(M_0)_l$. A token remains in place p_l at least for time T_l. Assuming that the tokens of the initial marking are immediately available at $t = 1$, the evolution can be described by the following inequalities expressing relations between the firing event numbers of transitions. For each place p_l, we can write that the output flow of tokens at time $t \in \mathbb{N}^*$ is lower than or equal to the addition of the input flow and the initial marking of p_l.

$$\sum_{i \in p_l^\bullet} x_i(t) \leq \sum_{i \in {}^\bullet p_l} x_i(t - T_l) + (M_0)_l \qquad (1)$$

In this inequality, each weight 1 of $x_i(t - T_l)$ (respectively, 1 of $x_i(t)$) corresponds to the weight of an incoming arc going from input transition x_i to place p_l (respectively, the outgoing arc going from place p_l to output transition x_i) which is equal to W^+_{li} (respectively, W^-_{li}).

After applying a technique described in [10] , the set of the previous inequalities can be expressed in the following way such that the temporization of each place is equal to zero or one:

$$G \cdot \begin{pmatrix} x(t-1) \\ x(t) \end{pmatrix} \leq M_0 \qquad (2)$$

where the l^{th} row of G contains the weights of the incoming and outgoing arcs of place p_l: Roughly speaking, the general idea in [10] is to split each place p_l associated with a temporization $T_l > 1$ into T_l places, such that the temporization of each place is equal to one. Matrix $G = [G_1 \ G_0]$ has an order $(|P| \times 2.|TR|)$ and the submatrices G_1 and G_0 are defined as follows:

- The row $l \in \{1, 2, ..., |P|\}$ of matrix G_i for $i \in \{0, 1\}$ contains the unitary weights of the incoming arcs of place p_l with temporization i ($T_l = 0$ or 1), with negative sign (usually expressed by the entries of $-W^+$).
- In addition, the row l of matrix G_0 contains the unitary weights of the arc outgoing from place p_l, with positive sign (usually expressed by the entries of W^-).

Note that an inequality using 'dater' in the space of real numbers can also be written for Timed Event Graphs [10] and P-time Event Graphs. This form, which presents symmetry with (2), does not directly allow the deduction of the marking $M(t)$ from the fundamental relation of marking, contrary to the counter form used in this paper.

IV. Sequence estimation

A. Objective

The aim is the estimation of the sequence of numbers of transition firings and markings by considering system (2) for $\theta \in \{t - h + 1, t - h + 2, ..., t\}$ where $h \in \mathbb{N}^*$ is the horizon of the sequence estimation. Let $x_{obs}(\theta)$ (respectively, $x_{un}(\theta)$)

be the subvector of the state vector $x(\theta)$ such that the relevant transitions belong to the set of observable transitions TR_{obs} (respectively, unobservable transitions TR_{un}). The objective for each time t is the estimation of the least (respectively, greatest) estimate sequence denoted by $x^-_{un}(\theta)$ (respectively, $x^+_{un}(\theta)$) for $\theta \in \{t-h, t-h+1, ..., t\}$ knowing the observable state vector $x_{obs}(\theta)$ in the same window. Knowing this sequence, the relevant markings are directly deduced from the fundamental marking relation.

B. Solution space

System (2) for time $\theta \in \{t-h+1, t-h+2, ..., t\}$ can be rewritten as follows:

$$\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix} \cdot \begin{pmatrix} x_{un}(\theta-1) \\ x_{un}(\theta) \end{pmatrix} \leq M_0 - \begin{pmatrix} G_{1,obs} & G_{0,obs} \end{pmatrix} \cdot \begin{pmatrix} x_{obs}(\theta-1) \\ x_{obs}(\theta) \end{pmatrix} \quad (3)$$

after an adequate permutation of the columns of matrix G with respect to the observable/unobservable transitions: The columns of $\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix}$ (respectively, of $\begin{pmatrix} G_{1,obs} & G_{0,obs} \end{pmatrix}$) correspond to the unobservable transitions (respectively, to the observable transitions).

An equivalent form describing the set of trajectories on horizon h is as follows:

$$A_1 \cdot \mathbf{x}_{un} \leq C_1 - B_1 \cdot \mathbf{x}_{obs} \quad (4)$$

with $\mathbf{x}_{un} = \begin{pmatrix} x_{un}(t-h) \\ x_{un}(t-h+1) \\ x_{un}(t-h+2) \\ \dots \\ x_{un}(t-1) \\ x_{un}(t) \end{pmatrix}$, $\mathbf{x}_{obs} = \begin{pmatrix} x_{obs}(t-h) \\ x_{obs}(t-h+1) \\ x_{obs}(t-h+2) \\ \dots \\ x_{obs}(t-1) \\ x_{obs}(t) \end{pmatrix}$, $A_1 = \begin{pmatrix} G_{1,un} & G_{0,un} & 0 & \dots & 0 & 0 \\ 0 & G_{1,un} & G_{0,un} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & G_{0,un} & 0 \\ 0 & 0 & 0 & \dots & G_{1,un} & G_{0,un} \end{pmatrix}$

$B_1 = \begin{pmatrix} G_{1,obs} & G_{0,obs} & 0 & \dots & 0 & 0 \\ 0 & G_{1,obs} & G_{0,obs} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & G_{0,obs} & 0 \\ 0 & 0 & 0 & \dots & G_{1,obs} & G_{0,obs} \end{pmatrix}$ and $C_1 = \begin{pmatrix} M_0 \\ M_0 \\ \dots \\ M_0 \\ M_0 \end{pmatrix}$.

The dimension of vector $\mathbf{x}_{un}$ is denoted by $n = (h+1).|TR_{un}|$ while the dimension of vector $\mathbf{x}_{obs}$ is $(h+1).|TR_{obs}|$. The dimensions of matrices A_1, B_1, C_1 and column vector $b_1 = C_1 - B_1 \cdot \mathbf{x}_{obs}$ are respectively ($h.|P|$ x n), ($h.|P|$ x $(h+1).|TR_{obs}|$), ($h.|P|$ x 1) and ($h.|P|$ x 1). In addition, below we express that the trajectories are non-decreasing, that is, $x_{un}(\theta-1) \leq x_{un}(\theta)$ for $\theta \in \{t-h+1, t-h+2, ..., t\}$ which can easily be rewritten under the form of a polyhedron

$$A_2 \cdot \mathbf{x}_{un} \leq 0_{h.|TR_{un}|\text{x}1} \quad (5)$$

where the dimension of matrix A_2 is ($h.|TR_{un}|$ x n). Moreover, we have

$$A_3 \cdot \mathbf{x}_{un} \leq 0_{n\text{x}1} \quad (6)$$

where $A_3 = -I_{nxn}$ as the trajectories are non-negative.

Finally, the solution space of the Petri net is characterized by the following polyhedron

$$A \cdot \mathbf{x}_{un} \leq b \quad (7)$$

with $A = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$ and $b = \begin{pmatrix} C_1 - B_1 \cdot \mathbf{x}_{obs} \\ 0_{h.|TR_{un}|\text{x}1} \\ 0_{n\text{x}1} \end{pmatrix}$. The dimensions of matrix A and column vector b are respectively ($n + h.(|P| + |TR_{un}|)$ x n) and ($n + h.(|P| + |TR_{un}|)$ x 1).

After an adequate permutation of the columns of the incidence matrix $W = \begin{pmatrix} W_{obs} & W_{un} \end{pmatrix}$ with respect to the observable/unobservable transitions, each estimated marking of the Petri net $M(\theta)$ for $\theta \in \{t-h, t-h+1, ..., t\}$ based on the observation $x_{obs}(\theta)$ satisfies

$$M(\theta) = M_0 + W. \begin{pmatrix} x_{obs}(\theta) \\ x_{un}(\theta) \end{pmatrix} \quad (8)$$

where $x_{un}(\theta)$ is a subvector of $\mathbf{x}_{un}$ satisfying (7).

***Remark** 1:* System (7) can be completed to handle the case where the initial marking is not completely known but belongs to a finite set of possible initial markings defined by a polyhedron $A_4 \cdot M_0 \leq b_4$. So, vector M_0 becomes an unknown data in (7).

Therefore, the spirit of the paper is to deal with the inequality (7) which completely describes the different trajectories of the Petri net. In the following parts, we show that the concept of componentwise order can give the least and greatest estimates of the trajectories if we add some assumptions.

C. Analysis

1) Structures:

***Definition** 1:* Given a net $N = (P, TR, W^+, W^-)$, and a subset $TR' \subseteq TR$ of its transitions, the TR'-induced subnet of N is defined as the new net $N' = (P, TR', W^{+\prime}, W^{-\prime})$ where $W^{+\prime}$ (respectively, $W^{-\prime}$) is the restriction of W^+ (respectively, W^-) to $P \times TR'$. The net N' is obtained from N by removing all transitions in $TR \setminus TR'$.

We now consider the TR_{un}-induced subnet $(P, TR_{un}, W^{+\prime}, W^{-\prime})$ which is associated with the unobservable transitions TR_{un}. The following definitions, but not the concept of time sequence described above, are also considered in [8] where the Petri net is untimed.

***Definition** 2:* The TR_{un}-induced subnet is Backward Conflict Free (BCF), i.e., any two distinct unobservable transitions have no common output place.

***Definition** 3:* The TR_{un}-induced subnet is Forward Conflict Free (FCF), i.e., any two distinct unobservable transitions have no common input place.

2) Extremum solutions: Considering a TR_{un}-induced subnet, we now focus on the existence of a least/greatest estimated trajectory in our practical problem. We make the connection between the important structures of Petri nets and the mathematical definitions defined in Section II.

***Definition** 4:* The system of linear inequalities $A.x \leq b$ is inf-monotone (respectively, sup-monotone) if each row of matrix A has one strictly negative (respectively positive) element at most.

Let Γ be the solution set of an inf-monotone (respectively, sup-monotone) system $A.x \leq b$. Set Γ is an inf-semilattice (respectively, sup-semilattice) since each pair of elements has an lower (resp., upper) bound. Moreover, the following lemma guarantees that Γ has an extremum element, that is, a least or greatest solution which belongs to Γ. In our practical problem, it implies the existence of a least/greatest estimated trajectory satisfying (7).

***Lemma** 1:* [3]. Let Γ be the solution set of an inf-monotone (respectively, sup-monotone) system $A.x \leq b$. Set Γ has a least (respectively, greatest) element if the set is non-empty and has a minorant (respectively, majorant). ■

***Theorem** 1:* In a BCF TR_{un}-induced subnet, the least estimate $\mathbf{x}_{un}^-$ exists over $\mathbb{R}$. In a FCF TR_{un}-induced subnet, the greatest estimate $\mathbf{x}_{un}^+$ exists over $\mathbb{R}$ if $\mathbf{x}_{un}$ has a finite majorant.

Proof. When the TR_{un}-induced subnet of the considered Petri net is BCF (respectively, FCF), the analysis of system (3) shows that each row of matrix $\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix}$ (and matrix A_1 consequently) has one strictly negative (respectively, positive) coefficient at the most. The same remark can be made for (5) and (6). So, system (7) is inf-monotone (respectively, sup-monotone). It implies that the solution set of (7) is an inf-semilattice (respectively, sup-semilattice). Moreover, the assumption of time liveness shows the existence of a state trajectory and the set is non-empty. Finally, $\mathbf{x}_{un}(\theta) = 0$ for $\theta \in \{t-h, t-h+1, ..., t\}$ is a minorant of the set since the initial condition is $x(\theta = 0) = 0$ and the trajectory is non-decreasing. The application of Lemma 1 shows that the least estimate $\mathbf{x}_{un}^-$ exists over $\mathbb{R}$ in a BCF TR_{un}-induced subnet. In the case of a FCF TR_{un}-induced subnet, the reasoning is similar but the assumption that $\mathbf{x}_{un}$ has a finite majorant must be added as there is no obvious majorant. ■

3) Minorant and majorant: In this part, we consider the determination of the minorant and the majorant used in Theorem 1 showing the existence of extremum solutions. Let us discuss the minorant in the case of a BCF TR_{un}-induced subnet. Remember that it is an element not necessarily belonging to the subset. Note that the least trajectory exists even if all the transitions of the Petri net are unobservable ($TR = TR_{un}$). The minorant $(\mathbf{x}_{un}^-)_i = 0$ which is independent of the Petri net in the above theorem is clearly not a satisfactory answer to the problem. Consideration of the following relations gives a more efficient minorant. Let P_{BCF} be the set of places $p_l \in P$ having a unique unobservable *input transition* $x_i \in ^\bullet p_l$. For each place $p_l \in P_{BCF}$ with $P_{BCF} \subset P$, the relevant relation in system (4) after some elementary transformations becomes

$$\begin{aligned} x_i(\theta - T_l) \geq & \sum_{x_j \in p_l^\bullet | x_j \in TR_{un}} x_j(\theta) - (M_0)_l + \\ & \sum_{x_j \in p_l^\bullet | x_j \in TR_{obs}} x_j(\theta) - \sum_{x_j \in ^\bullet p_l | x_j \in TR_{obs}} x_j(\theta - T_l) \end{aligned} \tag{9}$$

for $\theta \in \{t-h+1, t-h+2, ..., t\}$with $T_l = 0$ or $T_l = 1$. Variable $x_i(\theta - T_l)$ has a finite minorant which is the right-hand term of the above inequality if each variable $x_j(\theta)$ for $x_j \in p_l^\bullet$ with $x_j \in TR_{un}$ also has a finite minorant. Applying the above backward propagation through the Petri net and starting from known data, a finite minorant for each transition of TR_{un} can be determined. The above relations also show a backward propagation through time with a delay T_l for each place, as shown in the above relation, that is, we deduce $x_i(\theta - T_l)$ from $x_j(\theta)$. Note that the variables must also satisfy inequalities $x_i(\theta) \geq x_i(\theta - 1)$ (expressed by (5)) which follow the opposite direction through time, that is, we deduce $x_i(\theta)$ from $x_i(\theta - 1)$. Finally, this resolution generates a minorant and naturally, the constraint propagation on the complete set of inequalities (7) gives the greatest minorant which is the least estimate.

Let us now also sketch the determination of a majorant in the case of a FCF TR_{un}-induced subnet. Let P_{FCF} be the set of places $p_l \in P$ having a unique unobservable *output transition* $x_i \in p_l^\bullet$. For each place $p_l \in P_{FCF}$ with $P_{FCF} \subset P$, the relevant relation in system (4) can be rewritten as

$$\begin{aligned} x_i(\theta) \leq & \sum_{x_j \in ^\bullet p_l | x_j \in TR_{un}} x_j(\theta - T_l) + (M_0)_l + \\ & \sum_{x_j \in ^\bullet p_l | x_j \in TR_{obs}} x_j(\theta - T_l) - \sum_{x_j \in p_l^\bullet | x_j \in TR_{obs}} x_j(\theta) \end{aligned} \tag{10}$$

for $\theta \in \{t-h+1, t-h+2, ..., t\}$ with $T_l = 0$ or $T_l = 1$. Therefore, a determination of the majorant following the direction of the arcs through the Petri net and starting from known data can be made. It is also made with a delay T_l for each place as shown in the above relation. This resolution can also be completed by inequalities $x_i(\theta) \leq x_i(\theta + 1)$ (expressed by (5)) which follow the opposite direction through time. As above, the constraint propagation on the complete set of inequalities (7) gives the least majorant which is the greatest estimate.

Cyclicity

Let us note that we do not use a condition of acyclicity in the previous results: The variables of a TR_{un}-induced subnet can have a minorant even if it contains some circuits or self-loops which lead to null rows in the incidence matrix W. Indeed, the resolution does not strictly follow the paths in the Petri net but a sequence of relations in the corresponding inequality system, where not a unique variable, but a set of variables $\{x_i(\theta), x_i(\theta+1), x_i(\theta+2), ...\}$, is associated to each transition x_i. Roughly speaking, the consideration of counters leads to 'open' the circuits of the Petri net. Moreover, the general conditions $0 \leq x_i(\theta)$ and $x_i(\theta) \leq x_i(\theta+1)$ must be satisfied. The following observer on an elementary Petri net illustrates this point: The approach can consider a self-loop which is expressed by an algebraic relation.

Example 1

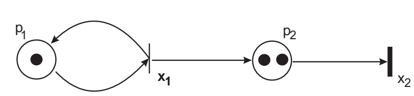

Fig. 1. Example 1: elementary P-timed Petri net with a self-loop (the observable transition is x_2).

In the Petri net given in Fig. 1, each place is associated with a temporization equal to 1 second. The inequalities relevant to places p_1 and p_2 of the Petri net in Fig. 1 are $x_1(t) \leq x_1(t-1) + 1$ and $x_2(t) \leq x_1(t-1) + 2$. Below, underlined symbols like $\underline{x_2}$ correspond to known data of the problem. We take $h = 1$, $t = 5$ and we assume that the transition x_2 is observable. Let us take $\underline{x}_2(t=5)=3$. The observer is given by $max(\underline{x}_2(\theta)-2, x_1(\theta)-1) \leq x_1(\theta-1)$ for $\theta \in \{5\}$ with $0 \leq x_1(4)$, $0 \leq x_1(5)$ and $x_1(4) \leq x_1(5)$. The least solution is $x_1^-(4) = x_1^-(5) = 1$ and $x_1^-(4) = 1$ corresponds to the smallest number of firing of x_1 necessary to the three firings of transition x_2 arriving at $t=5$ or before $t=5$: Transition x_2 can also use the two tokens of the initial marking of its input place p_2. Naturally, other solutions satisfying the inequalities of the observer exist: $x_1(5)-1 = x_1(4) \geq 1$ can be taken, and the solution $x_1(4) = 11$ and $x_1(5) = 12$ leads to the production of ten tokens at $t=4$ or before $t=4$ which are not used by x_2 at $t=5$. ■

D. Technique using objective function

System (7) uses the form $A.x \leq b$ as in linear programming. However, the concept of objective function has not been used up to now. The following result makes the connection between the objective function of linear programming and the componentwise order which has been used in the previous parts. In fact, the optimal solution to the estimation problem is also the solution to a special linear programming problem as indicated by the following result briefly stated in [3] [6]. This assertion is completed by the following proof.

***Lemma** 2:* Let $A.x \leq b$ be a sup-monotone system. The following statements are equivalent:

1. Set $\Gamma = \{x \in \mathbb{R}^{|x|} | A.x \leq b\}$ has a greatest element x^+.
2. x^+ is optimal for the problem $max\{c.x\}$, such that $A.x \leq b$ for any $c > 0$.

***Corollary** 1:* The previous equivalence between 1 and 2 in Lemma 2 holds if: x^+ is replaced by x^-; the words sup-monotone, greatest and max are replaced by inf-monotone, least and min, respectively.

Proof. Let x^+ be the greatest element of set Γ (assertion 1). Then $\forall x \in \Gamma, x \leq x^+ \Rightarrow c.x \leq c.x^+$ as $c > 0$ and so, x^+ is also the optimal solution to assertion 2. The reverse is proved by contradiction. Let x^+ be the optimal solution and assume that assertion 1 is false. As $x^+ \in \Gamma$ is not the greatest element of set Γ, there is $x \in \Gamma$ such that $x \not\leq x^+$ that is, $\exists i$ with $x_i > (x^+)_i$. Including the case where Γ has no greatest element which can replace x^+, we can take $x' = max(x, x^+)$ which belongs to Γ as the system is sup-monotone (Theorem 1 in [6]). So, there is $x' \in \Gamma$ such that $x^+ \leq x'$ with $x^+ \neq x'$. It implies that $c.x^+ < c.x'$ as $c > 0$. Hence x^+ is not the optimal solution to problem 2 which leads to a contradiction. Therefore, the reverse is proved. ■

In fact, vector $x_{un}(t)$ is over the integers as each entry expresses the number of transition firing. Let us now complete the previous study by giving the following definitions and results.

***Definition** 5:* An inf-monotone (respectively, sup-monotone) system of linear inequalities $Ax \leq b$ is also 1-inf-monotone (respectively, 1-sup-monotone) if: A and b are integers; the strictly negative (respectively positive) coefficients of A are equal to -1 (respectively, $+1$).

***Definition** 6:* The TR_{un}-induced subnet is Unitary Backward Conflict Free or UBCF (respectively, Unitary Forward Conflict Free or UFCF) if: The subnet is BCF (respectively, FCF); the weight of each incoming (respectively, outgoing) arc of the subnet is unitary.

Note that the BCF (respectively, FCF) Petri nets considered in this paper are also UBCF (respectively, UFCF) as we have assumed that the weight of each arc is unitary.

***Theorem** 2:* Let the TR_{un}-induced subnet of the considered Petri net be UBCF (respectively, UFCF).

- The least sequences $\mathbf{x}_{un}^-$ (respectively, greatest sequences $\mathbf{x}_{un}^+$) of system (7) in $\mathbb{R}^n$ and $\mathbb{N}^n$ are equal.
- The relevant extremum sequence is given by the following linear programming problem: $\min\{c.\mathbf{x}_{un}\}$ (respectively, $max\{c.\mathbf{x}_{un}\}$) such that $A \cdot \mathbf{x}_{un} \leq b$ for any $c > 0$.

Proof. The following three points shows that system (7) is 1-inf-monotone: 1) Matrix A and vector b in (7) are integers as the initial marking M_0, vector $\mathbf{x}_{obs}$ and the matrices A_1, A_2, A_3, B_1 are integers; 2) System (4) is 1-inf-monotone as the TR_{un}-induced subnet is UBCF; 3) The analysis of matrices A_2 and A_3 shows that systems (5) and (6) are also 1-inf-monotone. As Theorem III.4 in [19] says that the least elements of $\{x \in \mathbb{R}^n : A.x \leq b\}$ and $\{x \in \mathbb{Z}^n : Ax \leq b\}$ are equal when $A.x \leq b$ is 1-inf-monotone, the same equality holds for system (7) which is 1-inf-monotone. In addition, the application of the constraint $x_{un} \geq 0$ expressed by system (6) implies that x_{un}^- is over $\mathbb{N}$. Therefore, the determination of the least sequence $\mathbf{x}_{un}^-$ of system (7) can be made over $\mathbb{R}$ since the result of this resolution is also the least sequence over $\mathbb{N}$. Finally, Corollary 1 of Lemma 2 gives a practical way to obtain the optimal solution which is the resolution of the relevant linear programming problem for any $c > 0$. The reasoning is similar for the UFCF case. ■

Under the above conditions, integer linear programming is not necessary and we can directly apply current algorithms of linear programming such as simplex (although some artificial examples show exponential running times, in practice, and on average, the simplex is efficient) or more recent polynomial algorithms ([14], [13]). If the TR_{un}-induced subnet is BCF and not UBCF (resp. FCF and not UFCF), the uniqueness of the solution is guaranteed, but not its integer type. If the TR_{un}-induced subnet is not BCF (resp. FCF), neither the uniqueness nor the integer type of the solution are guaranteed. In that case, the procedure gives an optimal solution over $\mathbb{R}$ with respect to a given criterion c and not for any $c > 0$.

V. CONTACT-FREE CASE

Let us consider system (7) without the assumption of a BCF and FCF TR_{un}-induced subnet. We now assume that the unobservable transitions are contact-free which is defined below.

***Definition** 7:* The unobservable transitions are contact-free if: For any pair of transitions (x_i, x_j), the set of input and output places of x_i cannot intersect the set of input and output places of x_j; the unobservable transitions do not have self-loops associated with them.

This assumption simplifies matrix A_1 used in system (4) as the TR_{un}-induced subnet is now composed of subgraphs containing a unique unobservable transition. Each row l of $\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix}$ relevant to place p_l is null except for a unique entry which is defined in one of the following cases:

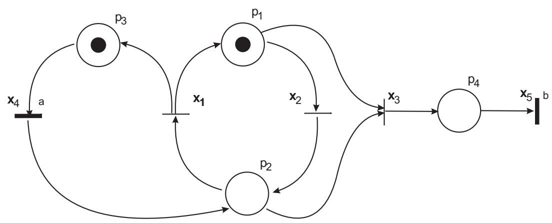

Fig. 2. Example 2: P-timed Petri net with observable transitions x_4 and x_5.

TABLE I
SIMULATION: EVENTS AND MARKINGS

Time t	0	1	2	3	4	5	6	7	8	9
Events		x_4	x_1 x_2	x_3 x_4	x_5 x_1	x_4	x_1	x_4	x_3 x_2	x_5 x_1
$M(t)$	1	1	1	0	1	1	2	2	0	1
	0	1	1	1	0	1	0	1	1	0
	1	0	1	0	1	0	1	0	0	1
	0	0	0	1	0	0	0	0	1	0

- An entry of $G_{i,un}$ for $i \in \{0,1\}$ is equal to -1 if the relevant transition is the (unique) unobservable input transition of place p_l with a temporization i.
- An entry of $G_{0,un}$ is equal to 1 if the relevant transition is the (unique) unobservable output transition of place p_l.

Each row of (4) relevant to the first case (second case, respectively) directly expresses a minorant (majorant, respectively) of the firing count of each transition and the consideration of all the relevant rows gives the least solution (greatest solution, respectively) of the problem. Therefore, the resolution of (4) which is a subsystem of (7) is limited to a maximization of the minorants and a minimization of the majorants. An entry of $\mathbf{x}_{un}$ can have a least solution and a greatest solution if the relevant unobservable transition is an input and output transition of places.

VI. SIMULATION (EXAMPLE 2)

In the Petri net given in Fig. 2, the TR_{un}-induced subnet is BCF and presents a circuit. Each place is associated with a temporization equal to 1 second. The initial marking is $M_0 = \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix}^{\mathrm{T}}$. A possible evolution of the Petri net for $t \in \{0,1,...,9\}$ is given in Table I.

A. Algebraic model

The matrices of the relevant matrix model $G_1 \cdot x(t-1) + G_0 \cdot x(t) \leq M_0$ are:

$$G_1 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix} \text{ and } G_0 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

B. Observer

The labels a and b in the Petri net correspond to the events of the observable transitions x_4 and x_5 (i.e. $TR_{obs} = \{x_4, x_5\}$) while the label ε corresponds to the unobservable transitions x_1, x_2 and x_3 (i.e. $TR_{un} = \{x_1, x_2, x_3\}$). So, we have $x_{obs} = (\underline{x}_4,\ \underline{x}_5)^{\mathrm{T}}$ and $x_{un} = (x_1,\ x_2,\ x_3)^{\mathrm{T}}$. The events associated with label a (respectively, b) are observed at times 1, 3, 5 and 7 (respectively, 4 and 9). The following inequality is deduced from the previous algebraic model: $G_{1,un} \cdot x_{un}(\theta-1) + G_{0,un} \cdot x_{un}(\theta) \leq M_0 - G_{1,obs} \cdot x_{obs}(\theta-1) - G_{0,obs} \cdot x_{obs}(\theta)$ for $\theta \in \{t-h+1, t\}$ where

$$G_{1,un} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},\ G_{0,un} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$G_{1,obs} = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } G_{0,obs} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

TABLE II
EXACT NUMBERS OF FIRING

Time t	6	7	8	9
x_1	3	3	3	4
x_2	1	1	2	2
x_3	1	1	2	2

TABLE III
KNOWN DATA

θ	6	7	8	9
$\underline{x}_4$	3	4	4	4
$\underline{x}_5$	1	1	1	2

C. Estimation

We make an estimation of x_{un} at $t = 9$ and we arbitrarily take $h = 3$. In other words, we estimate the firing numbers of the transitions given by Table II which is directly deduced from Table I. Based on the observations given by Table III on the window $\{t-h, t-h+1, ..., t\} = \{6, 7, 8, 9\}$, Table IV gives the least estimate x_{un}^- calculated by the observer.

The numbers in bold are only deduced from the application of the characteristic that the trajectory is non-decreasing: $x_1^-(8) \leq x_1^-(9)$, $x_2^-(7) \leq x_2^-(8) \leq x_2^-(9)$, and $x_3^-(8) \leq x_3^-(9)$. Each least estimate satisfies $x_i^-(\theta) \leq x_i(\theta)$. The estimates at $\theta = 6$ and $\theta = 7$ are equal to the exact data of the scenario of this simulation. We also have $x_i^-(9) \neq x_i(9)$ and $x_2^-(8) = x_2^-(9) \neq x_2(9)$: The estimation considers only the observations for $\theta \in \{6, 7, 8, 9\}$ and cannot directly use the relations describing the timed Petri net for $\theta \geq 10$: Obviously, $\underline{x}_4(10) - 1 \leq x_1(9)$ and $\underline{x}_5(10) \leq x_3(9)$ cannot be used since $\underline{x}_4(10)$ and $\underline{x}_5(10)$ are unknown at $t = 9$. The analysis of the inequalities shows that the backward propagation of the information through the timed Petri net

TABLE IV
LEAST ESTIMATES FOR $t = 9$ AND $h = 3$

θ	6	7	8	9
x_1^-	3	3	3	**3**
x_2^-	1	1	**1**	**1**
x_3^-	1	1	2	**2**

TABLE V
MEAN ABSOLUTE ERROR FOR $t = 9$ AND $h = 1$ TO 9

h	1	2	3	4	5	6	7	8	9
$e(h)$	0.5	0.33	0.25	0.2	0.16	0.14	0.12	0.11	0.1

TABLE VI
SUBSET OF THE SOLUTION SPACE FOR $t = 9$ AND $h = 3$

θ	6	7	8	9
x_1	$3+k$	$3+k$	$3+k$	$\mathbf{3+k}$
x_2	$1+k$	$1+k$	$\mathbf{1+k}$	$\mathbf{1+k}$
x_3	1	1	2	**2**

produces this transient period (see also inequality (9) and the relevant comment). Symmetrically, the calculation is made without using the observations on the horizon $\{0, ..., 5\}$: So, the observer can only use a part of the past evolution ($\theta \in \{6, 7, 8, 9\}$) and not the complete evolution of the system ($\theta \in \{0, 1, ..., 9\}$). Taking the mean absolute error $e(h) = \frac{1}{n}.\sum_{i\in\{1,2,3\},\theta\in\{t-h,t-h+1,...,t\}} |x_i(\theta) - x_i^-(\theta)|$ which is a way to quantify the difference between the true values $x_i(\theta)$ (Table III) and estimated values $x_i^-(\theta)$ (Table IV), Table V shows that the increase of the horizon in the simulation given by Table I improves the estimation.

Table VI for $h = 3$ and $k \in \mathbb{N}$ describes a subset of the solution space satisfying the inequalities of the observer and the algebraic model. It shows that other sequences that are consistent with the same observations exist.

VII. CONCLUSION

In this paper, we propose to consider the time parameter in the sequence estimation of the Timed Petri net. At first, we show that the solution space is completely described by a polyhedron in the general case. Secondly, we exploit the structure of the Petri net and make the connections between the concept of Backward/Forward Conflict Freeness of the unobservable-induced subnet and the concept of inf-monotone/sup-monotone inequality. The analysis of the relations shows that the resolution is not only limited to a backward/forward propagation of the calculation through the Petri net but must also follow the constraint that the trajectory is non-decreasing. An elementary example illustrates that the assumption of acyclicity is not necessary in the proposed approach. The application of linear programming and its relevant efficient algorithms allows the estimation of the least/greatest sequence with respect to the data known on a given horizon. We also consider the assumption of contact-free unobservable transitions where a simple resolution gives lower and upper bounds on the time sequences.

REFERENCES

[1] F. Baccelli and G. Cohen and G.J. Olsder and J.P. Quadrat, *Synchronization and Linearity*, Wiley, 1992.
[2] CG. Cassandras and S. Lafortune, *Introduction to Discrete Event Systems*, 2nd ed. Springer, New York, USA, 2008.
[3] R.W. Cottle and A.F. Veinott, *Polyhedral sets having a least element*, Mathematical Programming, Volume 3, pages 238-249, 1972.
[4] P. Declerck, *From extremal trajectories to consistency in P-time Event Graphs*, IEEE Transactions on Automatic Control, Volume 56, pages 463-467, 2011.
[5] P. Declerck and G. Abdelhak, *State estimation and detection of changes in time interval models*, Discrete Event Dynamic Systems, Springer US, pages 1-16, 2012.
[6] H. Gabbay, *A note on polyhedral sets having a least element*, Mathematical Programming, Volume 11, pages 94-96, 1976.
[7] M. Ghazel and A. Toguyéni and P. Yim, *State Observer for DES Under Partial Observation with Time Petri Nets*, Discrete Event Dynamic Systems, Volume 19, number 2, pages 137-165, 2009.
[8] A. Giua and C. Seatzu and D. Corona, *Marking estimation of Petri nets with silent transitions*, IEEE Transactions on Automatic Control, Volume 52, number 9, pages 1695-1699, 2007.
[9] A. Giua, *State Estimation and Fault Detection Using Petri Nets*, Applications and Theory of Petri Nets: Proc. 32nd Int. Conf. on Applications and Theory of Petri nets, 2008 Volume 6709, pages 419-428, 2011.
[10] A. Guezzi and P. Declerck and J.L. Boimond, *From monotone inequalities to Model Predictive Control*, IEEE Conference on Emerging Technologies and Factory Automation, pages 1310-1317, 2008.
[11] W. Hill, *Control System Fundamentals*, Discrete Event Systems, S. Levine editor, CRC Press, 2011.
[12] G. Jiroveanu and R.K. Boel and B. Bordbar, *On-Line Monitoring of Large Petri Net Models Under Partial Observation*, Discrete Event Dynamic Systems, Volume 18, number 3, pages 323-354, 2008.
[13] N.K. Karmarkar, *A new polynomial-time algorithm for linear programming*, Combinatorica, Volume 4, pages 373-395, 1984.
[14] L.G. Khachiyan, *Polynomial algorithms in linear programming*, Computational Mathematics and Mathematical Physics, Volume 20, number 1, pages 53-72, 1980.
[15] J. Komenda, *Supervisory Control with Partial Observations in Control of Discrete-Event Systems*, Lecture Notes in Control and Information Sciences series, Springer London, Volume 433, pages 65-84, 2013.
[16] D. Lefebvre and C. Delherm, *Diagnosis of DES With Petri Net Models*, IEEE Transactions on Automation Science and Engineering, Volume 4, number 1, pages 114-118, 2007.
[17] L. Lingxi and C.N. Hadjicostis, *Least-Cost Transition Firing Sequence Estimation in Labeled Petri Nets With Unobservable Transitions*, IEEE Transactions on Automation Science and Engineering, Volume 8, number 2, pages 394-403, 2011.
[18] H. Marchand and O. Boivineau and S. Lafortune, *On optimal control of a class of partially observed discrete event systems*, Automatica, Volume 38, number 11, pages 1935-1943, 2002.
[19] G.A. Narboni, *Un cas remarquable de systèmes linéaires : les systèmes monotones*, Ph.D. Thesis (in french), ENS Cachan, 2001.
[20] Y. Ru and C.N. Hadjicostis, *Bounds on the Number of Markings Consistent With Label Observations in Petri Nets*, IEEE Transactions on Automation Science and Engineering, Volume 6, number 2, pages 334-344, 2009.

Philippe Declerck received the Ph.D. degree in automatic control from the University of Lille I, France, in 1991. His research interests are in modeling, estimation and control of discrete event systems.

Patrice Bonhomme received the PhD in Electronic Engineering in 2001 from the University of Savoie (France). His research interests include discrete event systems, Petri nets, modeling and control of automated systems and real-time systems.