Identification of structurally solvable sub.systems for the design of Fault Detection and Isolation schemes , using the Embedding Procedure .

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Abstract. The fault detection in large scale physical system is more and more necessary for technical and economical reasons . Indeed , its aim is the improvement of the reliability and of the availabibity of the system .
The basic principle of fault detection is the comparison of the actual behaviour of the system with the reference one describing the normal operation . Some approaches have been developed which commonly use the linear state and measurement equations representation . However , such a representation is not often available for large scale complex industrial systems . Those systems are characterized by the great number of variables which are necessary for their description , and by the great variety of the types of relationships which link these variables : qualitative or quantitative , statical or dynamical , linear or non linear. The paper presents an approach based on structural analysis in order to exhibit coherence models for fault detection of large scale systems . The initial knowledge upon the normal operation of the system is given by its representation under the form of a network of elementary activities .This network defines the structure of the system under the form of a digraph linking each activity to the physical variables which are constrained by it . We propose an embedding procedure in order to identify the under , just and over determined components of the structural digraph . The overdetermined component represents that part of the overall system in which the fault detection and isolation procedure can be introduced .

Keywords. Structural analysis ; structural solvability ; fault detection and isolation ; embedding procedure ; large scale systems .

INTRODUCTION

The basic principle of fault detection is the comparison of the actual behaviour of the system to a reference behaviour describing its normal operation . The reference behaviour is issued from the knowledge which is available upon the system , this knowledge being expressed under more or less precise terms , and under formalisms which may be very different (knowledge base , analytical models , ...) One of the most frequently used approach is based on the use of Analytical Redundancy Relationships (ARR) : the knowledge available upon the system leads to express its normal operation by a set of invariants : the residuals of the ARR (coherence model) . The fault detection resumes thus to a decision problem : is the variance of the residuals the effect of noise , of normal deviations and errors or the effect of a failure .

Both the parity space and the generalized parity space approach (Potter,1977 ; Chow , 1984) are based on an analytical expression of the knowledge we have about the system : state and measurement equations . Moreover , these equations have to be linear : in fact , the residuals are obtained using a projection operator in the state space .

However , it is the most frequent case that such a representation is not directly available for large scale complex industrial systems . Those systems are characterized by the great number of variables which are necessary for their description , and by the great variety of the types of relationships which link these variables : qualitative or quantitative , statical or dynamical , linear or non linear . Moreover , in practical situations , some models are not known precisely (class of the model , values of its parameters ,...) , although their structure , i.e the different relationships and the variables which intervene , is known . The system may thus be represented by a network of elementary activities , each of them processing a subset of variables . Among the set of all the variables , only some of them are known (computed by elementary activities) or measured (a sensor performs also an elementary activity) . The general framework is to use such a representation in order to identify possible ARR for fault detection , based on the overdetermination , within the system of one or more variables (Staroswiecki ,1989b). The ARR are the result of a systematic approach which can be decomposed into two steps :

* Qualitative step . The structural analysis of the process gives subsets of non independant known or measured variables . It gives also subsets of elementary (or process functions) which link these variables . Each of those subsets will give rise to one or more ARR .

* Quantitative step . This step consists in the computation of the ARR corresponding to each of the previously mentionned subsets .

The present work is concerned whith the qualitative step . It is based on the model presented in (Staroswiecki ,1989a) and uses the embedding procedure in order to exhibit the three canonical components of the structure of the complex system under investigation , namely the structural under , over and just solvable sub-systems .

The first part presents the structural representation of a complex system and defines its sub-systems characterisation . In the second part , the embedding procedure is applied for the structural analysis of the system . The graph theory is used for the identification of the canonical components via a coupling approach . The third part discusses the application of structural analysis to the design of FDI procedures .

STRUCTURAL REPRESENTATION OF COMPLEX MODEL

Structural Model

The large scale system under consideration is represented by a network of elementary activities . These activities represent :
* physical constraints : their model is derived from mass or energy balance considerations
* control constraints : their model is given by the control algorithms which are implemented or by the human operators who act on the system .
* measurement constraints : their model is given by the knowledge of the sensors which are implemented on the system .

To each of the elementary activities corresponds a set of constraints possibly of different kinds which constitute the model of the activity . The overall system is thus represented by a set of m constraints .

$$F = \{ f_1 , f_2 , \dots , f_m \}$$

which are applied to a set of n variables

$$Y = \{ y_1 , y_2 , \dots , y_n \}$$

We write :

$$F (Y) = 0 \qquad (1)$$

We point out that no hypothesis is made about the properties of completeness of the model (Staroswiecki , 1989a) , so that m and n can take any values . Moreover , no use is made of the exact nature of the constraints f_i , and of the values of the parameters which could intervene : we only consider the structure of the system of equations (1) . The following binary relation defines it :

$$S : F \times Y \longrightarrow \{ 0 , 1 \}$$

$$(f_i , y_j) \longrightarrow S (f_i , y_j)$$

such that $S (f_i , y_j) = 1$ iff the constraint f_i applies to the variable y_j .

A digraph $B_0 = G (F , Y ; A_0)$ associated to the function S is defined by:

$$(f_i , x_j) \in A_0 \iff S (f_i , x_j) = 1$$

The following notations are introduced :

$V_0 = F \cup Y$
$(\forall v \in V_0)\ \mu_A (v , B_0) = \{ a \in A_0 \mid (\exists w \in V_0)\ a = (v , w) \}$
$(\forall a \in A_0)\ \mu_V (a , B_0) = \{ v \in V_0 \mid (\exists w \in V_0)\ a = (v , w) \}$
An extension is :
$(\forall V' \subset V_0)\ \mu_A (V' , B_0) = \{ a \mid a = \mu_A (v) \text{ and } v \in V' \}$
$(\forall A' \subset A_0)\ \mu_V (A' , B_0) = \{ v \mid v = \mu_V (a) \text{ and } a \in A' \}$

Let $\{ C , X \}$ be a bi-partition of the set Y . (in application , C will be the subset of known variables and X the subset of the unknown ones) .
A restriction of B_0 is defined by $B_X = G (F_X , X ; A_X)$.
with $A_X = \{ a \mid a \in A_0 \text{ and } P_X (a) \neq \emptyset \}$
$F_X = \{ f \mid f \in F \text{ and } (\exists a \in A_X) (P_F (a) = \{ f \} \}$

Definitions

A subsystem over B_X is said compatible iff at least one solution X exists .A compatible subsystem having a unique solution is said to be determined (underdetermined if more than one solution). Let us consider a determined subsystem . It is said to be overdetermined iff at least two differents means to determine the solution exists . At the opposite , the subsystem is said to be just determined .

CANONICAL REPRESENTATION

The over , under , and just determined subsystems are now structurally characterized . Their properties lead to algorithms for the decomposition of the overall system into three parts .

Canonical Representation of a Digraph

The following definitions are extracted principally from (Dulmage,1958 ; Gondran , 1979).

Definition 1

Let $V_X = F_X \cup X$ and $E \subset V_X$
The projection over E is a function P_E defined by :

$$P_E : \underline{\underline{P}}(A_X) \text{ -------> } V_X$$

$$@ \text{ -------> } P_E (@) = \bigcup_{a \in @} (\mu_V (a) \cap E)$$

Definition 2

$G^D = G (V_X ; @)$ is a disjoint subgraph of B_X iff :
1) $@ \subset A_X$
2) P_{F_X} (P_X) is an injective application of @ in F_X (in X) .

Definition 3

A maximal disjoint subgraph (MDS) of B_X is defined by $G^D = G (V_X ; @)$ such that : $\forall\ @^* \subset @\ ,\ @^* =/= @\ \ G (V_X ; @^*)$ is not disjoint .
The set of the maximal disjoint subgraphs of B_X will be noted E (B_X) .

Definition 4

A disjoint subgraph $G^D = G (V_X ; @)$ of B_X is complete iff $P_F X$ and P_X are surjective .

Definition 5

A pair of sets (α ,β) is an exterior cover of $B = G (F_X' , X' ; A)$ with $F_X' \subset F_X$ and $X' \subset X$ iff :

1) $\alpha \subset F_X'$; $\beta \subset X'$
2) $A \cap (\alpha . \beta) = \emptyset$

Let γ (B) be the set of the exterior covers of B . The number $|\alpha| + |\beta|$ is called the dimension of the covering and B is said to be of finite exterior dimension if there is a covering (α , β) such that $|\alpha| + |\beta|$ is finite .
The exterior dimension (ED) is defined as

$$\dim (B) = \min_{(\alpha,\beta) \in \gamma (B)} (|\alpha| + |\beta|)$$

An exterior cover (α,β) which achieves the minimal dim (B) is called a minimal exterior cover (MEC) .

Definition 6

A subgraph B of Bx is said to be semi-irreducible iff B has a unique MEC (α , $\emptyset$) or ($\emptyset$, β) .

THEOREM 1 (Dulmage , 1958)

For any digraph B_X of finite ED , there exist uniquely determined minimal covers (α_* , β^*) (α^* , β_*) such that if (α , β) is any other minimal cover then :
1) α_* is a proper subset of α or $\alpha_* = \emptyset$
2) α is a proper subset of α^*
3) β_* is a proper subset of β or $\beta_* = \emptyset$
4) β is a proper subset of β^* .
The pair (α_* , β^*) and (α^* , β_*) are named extreme minimal covers (EMEC) .

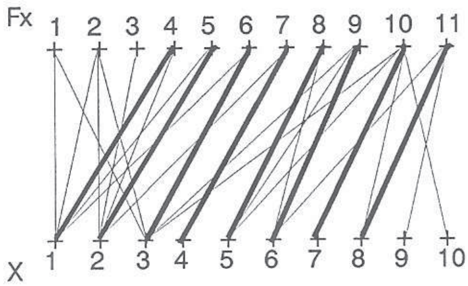

Fig. 1 A bigraph $G^D \in E (B_X)$

THEOREM 2 (Dulmage , 1958)

If (α_* , β^*) and (α^* , β_*) are the EMEC of digraph B_X , the three canonical components are defined as :

$$B^> = G (\overline{\alpha}^* , \beta_* ; A_X^>)$$
$$B^< = G (\alpha_* , \overline{\beta}^* ; A_X^<)$$
$$B^= = G (\alpha^* \backslash \alpha_* , \beta^* \backslash \beta_* ; A_X^=)$$

with

$$A_X^> = A_X \cap (\overline{\alpha}^* . \beta_*)$$
$$A_X^< = A_X \cap (\alpha_* . \overline{\beta}^*)$$
$$A_X^= = A_X \cap (\alpha^* \backslash \alpha_* . \beta^* \backslash \beta_*)$$

They are characterized by :

1) $(\emptyset , \beta_*)$ and $(\alpha_* , \emptyset)$ are respectively the unique MEC of $B^>$ and $B^<$ ($B^>$ and $B^<$ are semi-irreducible).
$(\alpha^* \backslash \alpha_* , \emptyset)$ and $(\emptyset , \beta^* \backslash \beta_*)$ are MEC of $B^=$.

2) If $W \in E (B_X)$ then

$@(W^>) = @(W) \cap (\overline{\alpha}^* . \beta_*) \in E (B^>)$
$@(W^<) = @(W) \cap (\alpha_* . \overline{\beta}^*) \in E (B^<)$
$@(W^=) = @(W) \cap (\alpha^* \backslash \alpha_* . \beta^* \backslash \beta_*) \in E(B^=)$
and $W = W^> \oplus W^< \oplus W^=$

The sum of graphs $\oplus$ is defined by respectively the sum of vertices and arcs of the different graphs .

$| @ (W^>) | = | \beta_* |$
$| @ (W^<) | = | \alpha_* |$
$| @ (W^=) | = | \alpha^* \backslash \alpha_* | = | \beta^* \backslash \beta_* |$

and only the subgraph $W^=$ is complete .

3) $(\overline{\alpha_*} . \overline{\beta}^*) \cup (\overline{\alpha}^* . \overline{\beta_*}) = \emptyset$

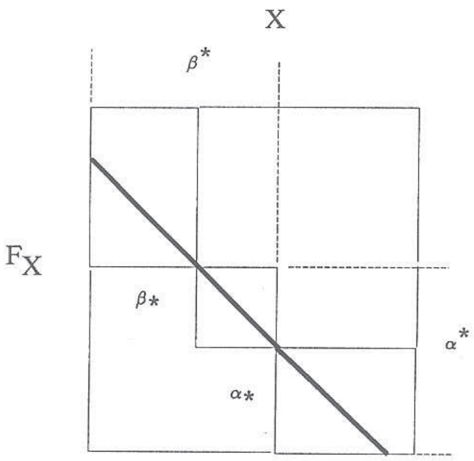

Fig. 2 Canonical decomposition .

The following theorem leads to an algorithm for the decomposition of F_X (resp. X) in $\alpha_* , \alpha^* \backslash \alpha_* , \overline{\alpha}^*$ (resp $\beta_* , \beta^* \backslash \beta_* , \overline{\beta}^*$) . It is based on the construction of alterned chains .

Definition 7

Let $W = G (F_X , X ; @ (W)) \in E (B_X)$. An alterned chain $L = G (F_L , X_L ; A_L)$ on w is defined by :

1) $F_L \cup X_L = \mu_S (A_L)$; $F_L \subset F_X$; $X_L \subset X$; $A_L \subset A_X$.

2) The n arcs of A_L are renamed upon the form a_i such that :

$(\forall i=1.. n) \ (a_i \in A_L)$
$a_i \in @ (W) , a_{i+1} \notin @ (W)$
$\Leftrightarrow P_{FX}(a_i) = P_{FX}(a_{i+1})$
$a_i \notin @ (W) , a_{i+1} \in @ (W)$
$\Leftrightarrow P_X (a_i) = P_X (a_{i+1})$

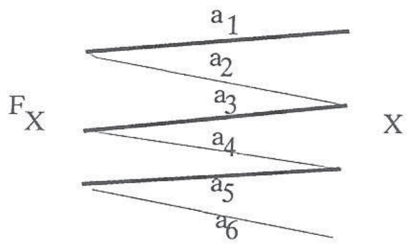

Fig. 3 An alterned chain.

The following sets are associated to W :

$I (W) = F_X \backslash P_{FX} (@ (W))$
$J (W) = X \backslash P_X (@ (W))$

The elements of $I (W)$ ($J (W)$) belong to F_X (X) and are not the extremity of any arc of $@ (W)$. We introduce the following theorem :

THEOREM 3

For any element belonging to $\overline{\alpha}^* \cup \beta_*$, there exists an alterned chain such that:

1) $e \in \mu_V (a_n , B_X)$
2) $P_F (a_1) \in I (W)$
and conversely .

For any element belonging to $\alpha_* \cup \overline{\beta}^*$, there exists an alterned chain such that :

1) $e \in \mu_V (a_1 , B_X)$
2) $P_X (a_n) \in J (W)$
and conversely .

The proof is given in (Declerck,1991).

<u>The Embedding Procedure</u>

Let us define the extended graphs B^+ and W^+ which include initial structures B_X and W :

$W^+ = G (F^+ , X^+ ; @ (W^+))$
$B^+ = G (F^+ , X^+ ; A^+)$
$F^+ = F \cup F_+$
$F_+ = \{ f_{i+} \mid x_i \in J (W) \}$
$X^+ = X \cup X_+$
$X_+ = \{ x_{i+} \mid f_i \in I (W) \}$
$@ (W^+) = @ (W) \cup V_> \cup V_<$
$V_> = \{ (f_i , x_{i+}) \mid f_i \in I (W) \ x_{i+} \in X_+ \}$
$V_< = \{ (f_{i+} , x_i) \mid x_i \in J (W) \ f_{i+} \in F_+ \}$
$A^+ = A_X \cup V_> \cup V_<$

This superstructure is distinguished by the introduction of the variables x_{i+} of X_+ and relations f_{i+} of F_+ whose roles are precised :

- F_+ : for every variable of J (W) , a function f_{i+} which constitutes a new constraint , has been introduced . In order to achieve the equivalence of the models represented by B_X and B^+ , each f_{i+} is given by the following form :

f_{i+} : $x_i = I_i$ with $x_i \in J$ (W)
$I_i \in] -\infty , +\infty [$

The introduced variable is <u>unknown</u> (a).

- X_+ : for every function of I (W) , a variable x_{i+} which constitutes a new degree of freedom , has been introduced . In order to achieve the equivalence of the models represented by B_X and B^+ , each function of I (w) is given by the following form :

f_i : $f_i (C , X) + x_{i+} = 0$ with $x_{i+} = 0$

The introduced variable x_{i+} is <u>known</u> (b)

Property 1

The graph W^+ is a complete disjoint subgraph of B^+ .
The proof , quite simple , is omitted .

Note that the algebraic information (a) and (b) are not contained in the graph B^+ so that the disjoint subgraph W^+ is complete .

The graph G = G (V;A) is associated to B^+.

1) $V = @ (W^+)$
2) A = { (a_h , a_j) such that $a_h \in @ (W^+)$, $a_j \in @ (W^+)$ $\exists a_i \in A^+ \setminus @ (W^+)$ such that $P_{F^+} (a_h) = P_{F^+} (a_i)$ and $P_{X^+} (a_i) = P_{X^+} (a_j)$ }

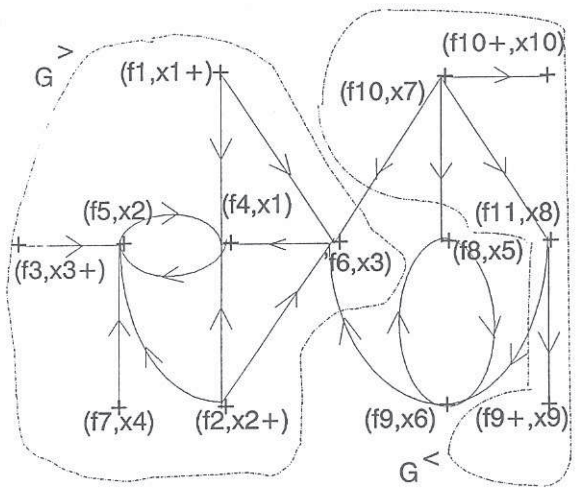

Fig. 4 The graph G = G (V ; A) .

<u>Notation.</u> For two vertices u and v , v is said to be reachable from u on G , which is denoted as " u ---> v " , iff there exists a directed path from u to v on G .

Property 2

$(\forall u \in V_>) (\nexists v \in V)$ such that v ---> u
$(\forall u \in V_<) (\nexists v \in V)$ such that u ---> v

Proof

If $u = a_j = (f_j , x_{j+}) \in V_>$, as $\mu_A (x_{j+}) = \{ a_j \}$ $(\nexists a_i \in A^+)$ with a_i =/= a_j and $P_{X^+} (a_i) = P_{X^+} (a_j)$. Then $\nexists (a_h , a_j) \in A$: the path (u , a_j) (a_j , a_k) ... (a_h , v) does not exist . The proof is symmetrical for the second part .

THEOREM 4

$(\forall u \in V_> \cup V_<) (\nexists v \in V_> \cup V_<)$ such that u ---> v or v ---> u

The proof is given in (Declerck,1991).

<u>Equivalence</u>

Let the graphs $G^> = G (V^> ; A^>)$, $G^< = G (V^< ; A^<)$ and $G^= = G (V^= ; A^=)$ be defined by :

$V^> = \{ v \mid (v \in V) (\exists u \in V_>)$ u ---> v }
$V^< = \{ v \mid (v \in V) (\exists u \in V_<)$ v ---> u }
$V^= = V \setminus (V^> \cup V^<)$
then $V = V^> \cup V^< \cup V^=$
$A^> = \{ a \mid \mu_V (a , G) \subset V^> \}$
$A^< = \{ a \mid \mu_V (a , G) \subset V^< \}$
$A^= = \{ a \mid \mu_V (a , G) \subset V^= \}$

THEOREM 5

The subgraphs $B^>$, $B^<$, $B^=$ of B_X and the subgraphs $G^>$, $G^<$, $G^=$ of G are respectively bound by the following correspondances .

$B^>$ and $G^>$: $\overline{\alpha^*} = P_{F_X} (V^>)$; $\underline{\beta_*} = P_X (V^>)$
$B^<$ and $G^<$: $\alpha_* = P_{F_X} (V^<)$; $\overline{\beta^*} = P_X (V^<)$
$B^=$ and $G^=$: $\alpha^* \setminus \alpha_* = P_{F_X} (V^=)$;
$\beta^* \setminus \beta_* = P_X (V^=)$

The proof is given in (Declerck,1991).

APPLICATION

The canonical decomposition of the digraph B_X which represents the structure of the system of equations $F_X(X) = 0$, gives the following subsystems.
The subproblems corresponding to the semi irreducible composents $B^>$ et $B^<$, if they exist, are not generally solvable. The problem corresponding to $B^<$ is structurally underdetermined , i.e. has more unknowns than equations, and that corresponding to $B^>$ is structurally overdetermined, i.e., has fewer unknowns than equations.
However, the system (1) describes a physical process. For that reason, Y exists and also X : the system is compatible.The subsystem corresponding to $B^<$ is

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underdetermined whereas the subsystem $B^{>}$ and $B^{=}$ are determined. As $B^{>}$ has fewer unknowns than equations, $B^{>}$ gives at least two subsystems which permit the determination of X. In the case of the model (without noise) the two determination must give the same result. In reality, due to measurement noises and modelization errors, a vector of residuals exists. It will be tested by the fault detection procedure. Thus, $B^{>}$ is the structural representation of the part of the system in which the fault detection and isolation procedure can be introduced.

CONCLUSION

The design of model based FDI procedures for complex industrial plants supposes the handling of large scale models. These plants are often constituted by the interconnexion of a great number of elementary activities, each of them being represented by an elementary model, more or less precesely known. Structural analysis gives a means to identify those parts of the overall system whose instrumentation gives enough information for fault detection and isolation. The problem is that of the decomposition of a digraph into its three canonical composents, namely the under, just and overdetermined subsystems. Starting with the initial digraph, we use an embedding procedure in order to construct an overgraph on which some simple manipulations lead to the canonical decomposition. The overdetermined subsystem represents the structure of the part of the overall system which can be monitored via the FDI procedure.

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APPENDICE
EXAMPLE

$F = \{f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}, f_{11}, f_{12}\}$

$C = \{ c_1, c_2, c_3, c_4 \}$

$X = \{ x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \}$

$f_1(c_1, c_4, x_1, x_3) = 0$

$f_2(c_2, x_1, x_2, x_3) = 0$

$f_3 (c_3 , c_4 , x_2) = 0$

$f_4 (x_1, x_2) = 0$

$f_5 (x_1, x_2) = 0$

$f_6 (c_4 , x_1, x_3) = 0$

$f_7 (x_2, x_4) = 0$

$f_8 (c_3, x_5, x_6) = 0$

$f_7 (x_2, x_4) = 0$

$f_8 (c_3, x_5, x_6) = 0$

$f_9 (x_3, x_5, x_6) = 0$

$f_{10} (x_3, x_5, x_7, x_8, x_{10}) = 0$

$f_{11} (c_1, c_3, x_6, x_8, x_9) = 0$

$f_{12} (c_1, c_4) = 0$

$F_X = F \setminus \{ f_{12} \}$ (f_{12} is a RRA)

The bigraph B_X is presented in Fig. 1. The arc of the disjoint subgraph W are :

$@ (W) = \{ (f_4, x_1), (f_5, x_2), (f_6, x_3), (f_7, x_4), (f_8, x_5), (f_9, x_6), f_{10}, x_7), f_{11}, x_8) \}$

$I (W) = \{ f_1, f_2, f_3 \}$; $J(W) = \{ x_9, x_{10} \}$

$\bar{\alpha}^* = \{ f_1, f_2, f_3, f_4, f_5, f_6 \}$; $\beta_* = \{x_1, x_2, x_3\}$

$\alpha_* = \{ f_{10}, f_{11} \}$; $\bar{\beta}^* = \{ x_7, x_8, x_9, x_{10} \}$

$\alpha^* \setminus \alpha_* = \{ f_7, f_8, f_9 \}$; $\beta^* \setminus \beta_* = \{ x_4, x_5, x_6 \}$

$X_+ = \{ x_{1+}, x_{2+}, x_{3+} \}$

$V_> = \{ (f_1, x_{1+}), (f_2, x_{2+}), (f_3, x_{3+}) \}$

$F_+ = \{ f_{9+}, f_{10+} \}$

$V_< = \{ (f_{9+}, x_9) , (f_{10+}, x_{10}) \}$