A strategy for Estimation in Timed Petri nets

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***Abstract*—The aim of the paper is the estimation of sequences in Timed Petri nets. We propose a a general strategy composed of two phases: The first phase considers the logical aspect only and suggests candidate count vectors where the second one checks the existence of a relevant time sequence for a Timed Petri net and generate a subspace of time sequences for a given candidate vector.**

I. Introduction

In Petri nets, the state observer aims to give an on-line picture of the process and to provide an estimation of the system state based on the observation of a set of events. Exploiting the observation of events available from the Petri net, the state observer estimates the events that cannot be directly detected by an external agent called observer or estimator. In this paper, the goal is to consider a Timed Petri nets and to compute the complete set of possible sequences consistent with the set of the observed events. This topic is crucial as a natural application of estimation is fault detection. Indeed, the correct behavior of a real-world application is the ultimate requirement, particularly for systems such as communication protocols, manufacturing and real-time systems: A drift from an expected behavior can produce material and financial costs. A failure can be of crucial importance as it can even lead to severe consequences including human losses in extreme cases.

The assumptions of acyclic unobservable induced Petri net, backward- or forward-conflict-freeness, contact-freeness,... are current in estimation and approximations of the spaces are usual. Up to now, the consideration of the time factor as intervals in a general Petri net containing conflicts and synchronizations, remains an open problem. In fact, more the model can express complex time phenomena as in P-time models, Time Stream models with complex semantics (AND, OR,...) and any possible generalizations, more the consideration of complex structure of a Petri net with conflicts seems difficult. This point of view is based on the two key-structures which are the Event Graphs and the State Graphs which are clearly described by the figure 25 in [18] and are analyzed by two close but different communities. Event Graphs are based on the synchronizations which operate at given dates over the real numbers: The consideration of complex time synchronizations is made for Event Graphs in many papers. Symmetrically, the resolution of a conflict needs boolean choices. The logical aspect is an important characteristic for this model. To summarize, the introduction of the time factor over the real numbers (Respectively, boolean choices) is difficult for State Graphs (Respectively, Event Graphs). This opposition is also present in the used algebraic tools: in Petri nets without time, the sequences are naturally defined with counter vectors while a more spontaneous tool is the dater vectors if time is considered for Event Graphs. A more detailed description can be seen in Chapter 1 in [8] where Fig. 1.4 gives a synthesis.

Therefore, we can conclude that no approach can solve directly the general estimation problem for any time model. To solve this problem, we introduce a general strategy composed of the following phases.

- The first phase I is to simplify the P-time Petri net and to take the *untimed* Petri net [17]. Therefore, we can consider complex structures and estimate the firing count vectors (or minimal firing count vectors) (or the complete set of ...) relevant to the unobservable transitions such that it leads to the observed events. In fact, only a subset of these vectors corresponds to real sequences and the obtained firing vector are named untimed candidate vectors. A candidate vector is chosen arbitrarily.

- The second phase II is to consider the Timed Petri net and to determine the sequence relevant to the chosen untimed candidate vector. In other words, we analyze the schedulability of this candidate vector by building the relevant schedule, that is, the counters of the transition firings. We assume that the dates of the firings of the observable transitions are known.

As the general principle of estimation of the current unobservable sequence is to treat separately and successively the firing of a unique observable transition, this procedure does not always start from the initial marking and the first dates of transition firings but restarts from the last estimated marking and firing dates which is used as a new initial condition. Therefore, the technique is not based on a sliding horizon: a shorter time horizon is considered and the calculations for each count vector are reduced as they exploit the past computing.

In this paper, no assumption is made on the cyclicity of the unobservable induced subnet. The occurrences of observable events are assumed to be non-simultaneous contrary to the firing of unobservable transitions which can be simultaneous. The incidence matrices and the initial marking are assumed to be known. We assume the feasibility of the system and the presence of observations during the application of the estimation procedure.

The paper is organized as follows. We first present a brief reminder of the basics of untimed Petri nets. The following section covers the procedure of the sequence estimation in the untimed case. Then, we describe the Timed Petri nets and make a schedulability analysis of the untimed sequences. Finally, we shortly discuss the schedulability analysis of P-time Petri nets. A pedagogical example containing a self-loop and a circuit illustrates the approach.

II. Preliminary remarks

The notation $|Z|$ is the cardinality of set Z and the notation A^T corresponds to the transpose of matrix A. A Place/Transition net (P/TR) is the structure $N = (P, TR, W^+, W^-)$, where P is a set of $|P|$ places and TR is a set of $|TR|$ transitions. The matrices W^+ and W^- are respectively the $|P| \times |TR|$ post and pre-incidence matrices over $\mathbb{N}$, where each row $l \in \{1, ..., |P|\}$ specifies the weight of the incoming and outgoing arcs of the place $p_l \in P$. The incidence matrix is $W = W^+ - W^-$. The preset and postset of the node $z \in P \bigcup TR$ are denoted by ${}^\bullet z$ and $z^\bullet$, respectively. The marking of the set of places P is a vector $M \in \mathbb{N}^{|P|}$ that assigns to each place of a P/TR net a non-negative integer number of tokens, represented by black dots. The notation Ω^* represents the set of firing sequences, noted σ, consisting of transitions of the set $\Omega \subset TR$. Given the sequence $\sigma \in TR^*$, we call $\pi : TR^* \to \mathbb{N}^{|TR|}$ the function that associates with σ a vector $\pi(\sigma) \in \mathbb{N}^{|TR|}$, named the firing vector or count vector of σ, where $\pi(\sigma)_i$ is the firing number of transition i which is fired $\pi(\sigma)_i$ times in σ. The marking M reached from the initial marking M^{init} by firing the sequence σ can be calculated by the fundamental relation: $M = M^{init} + W.\pi(\sigma)$. The transition is enabled at M if $M \geq W^-(., t)$ and may be fired yielding the marking $M' = M + W(., t)$. We write $M[\sigma \succ$ to denote that the sequence of transitions σ is enabled at M, and we write $M[\sigma \succ M'$ to denote that the firing of σ yields M'.

III. Phase I for untimed Petri nets

A. Principle of estimation

In a partial observed Petri net, we assume that the set of transitions TR can be partitioned as $TR = TR_{ob} \bigcup TR_{un}$, where TR_{ob} is the set of observable transitions and TR_{un} is the set of unobservable transitions. Notation t_i expresses an observable transition belonging to TR_{ob} and x_i is an unobservable transition, belonging to TR_{un}.

The problem considered in the paper is as follows. Let us consider a Petri net where the incidence matrix W and the initial marking M^{init} are known. Given a sequence of observed firing events of transitions of TR_{obs} generated by the activity of the Petri net, we want to find the sequences of unobservable firing events of transitions of TR_{un} (denoted $\sigma_{un} \in TR^*_{un}$) that are coherent (or consistent) with the observations.

The general principle of estimation of the current unobservable sequence is based on the treatment of the data produced by the observed transitions successively in an on-line procedure. If there is an observed firing of transition $t_{ob}^{<i>}$ for a current marking $M^{<i>}$, then there are an unobservable sequence $\sigma_{un}^{<i>}$ and a marking M' such that $M^{<i>}[\sigma_{un}^{<i>} \succ M'$ and $M'[t_{ob}^{<i>} \succ$. Thus, M' is the marking reached from the marking $M^{<i>}$ by firing the unobservable sequence $\sigma_{un}^{<i>}$ and this marking M' allows the observation of the firing of the observed transition $t_{ob}^{<i>}$.

The general technique taken in this paper is to estimate the firing count vectors associated to the unobservable transitions that are coherent with the firing of the observed transition. The count vector associated with the set TR_{un} of unobservable transitions is denoted x_{un}. When they correspond to a sequence that can be followed by the Petri net, these count vectors are named explanation vectors.

***Definition** 1:* Let $\sigma_{un}^{<i>}$ be a sequence of unobservable transitions that permits the firing of the observed transition $t_{ob}^{<i>}$, from a given marking M. An *explanation vector* of this sequence is the relevant count vector $\pi(\sigma_{un}^{<i>})$. The sets of all possible sequences and explanation vectors are denoted $SEQ(M, t_{ob}^{<i>})$ and $E(M, t_{ob}^{<i>})$, respectively. Formally:

$$\begin{aligned} SEQ(M, t_{ob}^{<i>}) &= \{\sigma_{un}^{<i>} | \sigma_{un}^{<i>} \in TR^*_{un} \text{ such that} \\ M[\sigma_{un}^{<i>} &\succ M' \text{ and } M'[t_{ob}^{<i>} \succ\} \\ E(M, t_{ob}^{<i>}) &= \{\pi(\sigma_{un}^{<i>}) | \sigma_{un}^{<i>} \in SEQ(M, t_{ob}^{<i>})\}. \end{aligned}$$

Moreover, the next marking $M^{<i+1>}$ used in step $< i+1 >$ is obtained from the firing of $t_{ob}^{<i>}$ at marking M'. Hence, $M'[t_{ob}^{<i>} \succ M^{<i+1>}$.

***Definition** 2:* Denoted $\mathcal{M}^{<i+1>}$, the set of current possible markings at iteration $< i+1 >$ is determined by the consideration of all the firing sequences $\sigma_{un}^{<i>} \in SEQ(M, t_{ob})$ for any marking M obtained at iteration $< i-1 >$. After noting that $\mathcal{M}^{<1>} = \{M^{init}\}$ where M^{init} is the initial marking, the notations $SEQ(M, t_{ob}^{<i>})$ and $E(M, t_{ob}^{<i>})$ are extended to $SEQ(\mathcal{M}^{<i>}, t_{ob}^{<i>})$ and $E(\mathcal{M}^{<i>}, t_{ob}^{<i>})$.

B. Polyhedron of the candidate vectors

In this section, we present a linear algebraic approach that is based on the fundamental equation of marking and the conditions of firing transitions in a Petri net. The $TR_{un}-$induced subnet of the Petri net N is defined as the new net $N_{un} = (P, TR_{un}, W^+_{un}, W^-_{un})$, where W^+_{un} (respectively, W^-_{un}) is the restriction of W^+ (respectively, W^-) to $P \times TR_{un}$. Therefore, $W_{un} = W^+_{un} - W^-_{un}$. For simplicity, the index $< i >$ is removed in the following parts as we focus on one iteration of the estimation procedure. For each unobservable transition, the relevant count is denoted with the same notation x_i with $i \in \{1, \ldots, |TR_{un}|\}$; i.e., the count number of unobservable transition x_i is $(x_{un})_i = x_i$.

The algebraic formulation of a possible explanation vector is made using the above reasoning for a given marking $M \in \mathcal{M}^{<i>}$ and the observation t_{ob}. As $M[\sigma_{un} \succ M'$, the marking M' satisfies the following equation:

$$M' = M + W_{un} \cdot x_{un} \tag{1}$$

where $x_{un} = \pi(\sigma_{un})$ is a count vector associated with σ_{un} and W_{un} is the incidence matrix of unobservable transitions. In addition, the transition t_{ob} is enabled for M'. As $M'[t_{ob} \succ$, we can write the inequality:

$$M' \geq W^{-}_{obs}(., t_{ob}) \tag{2}$$

By replacing M' by its expression (1), we obtain:

$$-W_{un}.x_{un} \leq M - W^{-}_{obs}(., t_{ob}) \tag{3}$$

Futhermore, we must have the constraint of non-negativity $x_{un} \geq 0$. Finally, the vector x_{un} must verify the following matrix inequality:

$$A.x_{un} \leq b \tag{4}$$

where $A = \begin{pmatrix} -W_{un} \\ -I_{n\times n} \end{pmatrix}$ and $b = \begin{pmatrix} M - W^{-}_{obs}(., t_{ob}) \\ 0_{n\times 1} \end{pmatrix}$. A is a matrix of an $m \times n$ dimension with $n = |TR_{un}|$ and $m = |P| + n$. The number of rows m can be reduced if we remove the null rows corresponding to places not connected to an unobservable transition or describing a self-loop. The relevant solution is denoted $S^{ad}_{un}(M, t_{ob}) = \{x_{un} \in \mathbb{R}^{n} | A.x_{un} \leq b\}$ (ad=**ad**missible solutions). We denote $S^{nat}_{un}(M, t_{ob}) = S^{ad}_{un}(M, t_{ob}) \cap \mathbb{N}^{n}$ (nat=**nat**ural numbers)

Accordingly, the obtained algebraic model always includes the set of explanation vectors $E(M, t_{ob})$ for an iteration $< i >$. Conversely, this inclusion can possibly be strict as the firing conditions of the unobservable transitions are neglected in this part. As a result, we have:

$$S^{ad}_{un}(M, t_{ob}) \supset S^{nat}_{un}(M, t_{ob}) \supset E(M, t_{ob}) \tag{5}$$

which defines the context of this paper. As the solutions of $S^{ad}_{un}(M, t_{ob})$ and $S^{nat}_{un}(M, t_{ob})$ are possible explanation vectors, they are named *candidate solutions* or *candidate vectors over* $\mathbb{R}$ *or* $\mathbb{N}$.

If each transition in the labeled net is associated with a nonnegative cost which captures its likelihood (e.g., in terms of the amount of workload or power required to execute the transition), we can minimize the cost of the count vector $c.x_{un}$ where c is the cost associate with transition i.

$$\min c.x_{un} \text{ such that } A.x_{un} \leq b$$

The obtained result is relevant to a least-cost transition sequence if $x_{un} \in E(M, t_{ob})$. Otherwise, the same optimisation must be made with a restriction of the considered space. The approach is out the scope of the paper.

In the next section, we focus on the consistency of an arbitrary candidate vector of $S^{nat}_{un}(M, t_{ob})$ with respect to the Timed Petri net which introduces time constraints. Precisely, we desire to know if the Timed Petri net can follow a time trajectory corresponding to the candidate vector.

IV. Phase II: Schedulability analysis in Timed Petri nets

A. Counter

With language misuse, each transition and its corresponding variable is denoted with the same letter. Each transition is associated with the *number of events* which happen *before or at time* t. The numbering is *absolute* as it starts at the origin of time zero contrary to relative numbering in the previous part where the counts are about the new transition firings such that $M[\sigma_{un} \succ M'$ where M is the last "initial" marking. Called a 'counter', the number of events which are the firings of the transition is denoted by $x(t)$. In this paper, time is discrete ($t \in \mathbb{Z}$) and the occurrence of each event is synchronized with an external clock. Assuming that the events can only occur at $t \geq 1$, we have $x(t) = 0$ for $t \leq 0$. For any $t \in \mathbb{N}^{*}$, it may be that no event takes place at t, a single event happens at t, or several events occur simultaneously at t. Remember that it leads to non-decreasing sequences. For a given transition, the arrival of two events at times 3 and 5 implies that the sequence of numbers of events starting at $t = 0$ and finishing at $t = 7$ is 0, 0, 0, 1, 1, 2, 2, 2, that is, $x(t = 3) = 1$ and $x(t = 5) = 2$ but also $x(t = 4) = 1$ and $x(t = 7) = 2$.

B. Objective 2

Let $t_{<i>}$ be the date of last observed transition firing at iteration $< i >$ and the candidate vector x_{un} produced by the phase I. Let $x_{obs}(\theta)$ (respectively, $x_{un}(\theta)$) be the subvector of the state vector $x(\theta)$ such that the relevant transitions belong to the set of observable transitions TR_{obs} (respectively, unobservable transitions TR_{un}). The objective for each time $t_{<i>}$ is to check the candidate vector x_{un} estimated at iteration $< i >$ by analyzing the existence of an estimate sequence $x(\theta)$ for $\theta \leq t_{<i>}$. If it exists, we can conclude the candidate vector x_{un} is an explanation vector.

Let us precise the data. If the observable firing events are known at times $t_{<i-2>}, t_{<i-1>}$ $t_{<i>}$, we can deduce the count vector for the observed transition at time $t_{<i-1>}$ knowing its complete history or past evolution. The same operations for all the observable transitions lead to the complete vector $x_{obs}(t_{<i-1>})$ which is denoted $\underline{x_{obs}}$. We also obtain $x_{obs}(\theta) = \underline{x_{obs}}$ for $\theta = t_{<i-1>}+1, t_{<i-1>}+2, \ldots, \leq t_{<i>}-1$ as there is no new firing. Moreover, $x_{obs}(t_{<i>}) = \underline{x_{obs}} + \Delta$ (denoted $\overline{x_{obs}}$) where a unique non-null component of Δ equal to 1 corresponds to the firing of the observed transition at time $t_{<i>}$.

In addition, if we have the absolute count vector deduced from the past estimations until iteration $< i-1 >$ and denoted $\underline{x_{un}}$, then we have $x_{un}(t_{<i>}) = \underline{x_{un}} + x_{un}$ (denoted $\overline{x_{un}}$) where x_{un} is the relative estimate which is chosen for iteration $< i >$. The unknown vectors are $x_{un}(\theta)$ for $\theta \in \{t_{<i-1>} + 1, t_{<i-1>} + 2, ..., t_{<i>} - 1, t_{<i>}\} = \{t_{<i>} - h + 1, t_{<i>} - h + 2, ..., t_{<i>} - 1, t_{<i>}\}$ if the time horizon $h \in \mathbb{N}^{*}$ is equal to $t_{<i>} - t_{<i-1>}$.

C. Model

Timed Petri nets allow the modeling of discrete event systems with sojourn time constraints of the tokens inside the places. Each place $p_l \in P$ is associated with a temporization $T_l \in \mathbb{N}$. Its initial marking is the entry l of the vector M_0 which is denoted by $(M_0)_l$. A token remains in place p_l at least for time T_l. Assuming that the tokens of the initial marking are immediately available at $t = 1$, the evolution can be described by the following inequalities expressing relations between the firing event numbers of transitions. For each place p_l, we can write that the output flow of tokens at time $t \in \mathbb{N}^*$ is lower than or equal to the addition of the input flow and the initial marking of p_l.

$$\sum_{i \in p_l^\bullet} x_i(t) \leq \sum_{i \in {}^\bullet p_l} x_i(t - T_l) + (M_0)_l \tag{6}$$

In this inequality, each weight 1 of $x_i(t - T_l)$ (respectively, 1 of $x_i(t)$) corresponds to the weight of an incoming arc going from input transition x_i to place p_l (respectively, the outgoing arc going from place p_l to output transition x_i) which is equal to W_{li}^+ (respectively, W_{li}^-).

After applying a classical technique described in [8] , the set of the previous inequalities can be expressed in the following way such that the temporization of each place is equal to zero or one:

$$G \cdot \begin{pmatrix} x(t-1) \\ x(t) \end{pmatrix} \leq M_0 \tag{7}$$

where the l^{th} row of G contains the weights of the incoming and outgoing arcs of place p_l: Roughly speaking, the general idea in [8] is to split each place p_l associated with a temporization $T_l > 1$ into T_l places, such that the temporization of each place is equal to one. Matrix $G = [G_1 \; G_0]$ has an order $(|P| \times 2.|TR|)$ and the submatrices G_1 and G_0 are defined as follows:

- The row $l \in \{1, 2, ..., |P|\}$ of matrix G_i for $i \in \{0, 1\}$ contains the unitary weights of the incoming arcs of place p_l with temporization i ($T_l = 0$ or 1), with negative sign (usually expressed by the entries of $-W^+$).
- In addition, the row l of matrix G_0 contains the unitary weights of the arc outgoing from place p_l, with positive sign (usually expressed by the entries of W^-).

Note that an inequality using 'dater' in the space of real numbers can also be written for Timed Event Graphs [8] and P-time Event Graphs. This form, which presents symmetry with (7), does not directly allow the deduction of the marking $M(t)$ from the fundamental relation of marking, contrary to the counter form used in this paper.

D. Solution space

The following results are relevant to each couple ($\underline{x_{un}}$, $\underline{x_{obs}}$, $\overline{x_{un}}$, $\overline{x_{obs}}$) where ($\underline{x_{un}}$, $\underline{x_{obs}}$) is the starting point and ($\overline{x_{un}}$, $\overline{x_{obs}}$) the final point. System (7) for time $\theta \in \{t_{<i>} - h + 1, t_{<i>} - h + 2, ..., t_{<i>} - 1, t_{<i>}\}$ can be rewritten as follows:

$$\begin{aligned} &\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix} \cdot \begin{pmatrix} x_{un}(\theta-1) \\ x_{un}(\theta) \end{pmatrix} \leq \\ &M_O - \begin{pmatrix} G_{1,obs} & G_{0,obs} \end{pmatrix} \cdot \begin{pmatrix} x_{obs}(\theta-1) \\ x_{obs}(\theta) \end{pmatrix} \end{aligned} \tag{8}$$

after an adequate permutation of the columns of matrix G with respect to the observable/unobservable transitions: The columns of $\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix}$ (respectively, of $\begin{pmatrix} G_{1,obs} & G_{0,obs} \end{pmatrix}$) correspond to the unobservable transitions (respectively, to the observable transitions).

An equivalent form describing the set of trajectories on horizon h is as follows:

$$A_1 \cdot \mathbf{x}_{un} \leq C_1 - B_1 \cdot \mathbf{x}_{obs} \tag{9}$$

with
$\mathbf{x}_{un} = \begin{pmatrix} \underline{x_{un}} \\ x_{un}(t_{<i>} - h + 1) \\ x_{un}(t_{<i>} - h + 2) \\ \cdots \\ x_{un}(t_{<i>} - 1) \\ \overline{x_{un}} \end{pmatrix}$, $\mathbf{x}_{obs} = \begin{pmatrix} \underline{x_{obs}} \\ \underline{x_{obs}} \\ \underline{x_{obs}} \\ \cdots \\ \underline{x_{obs}} \\ \overline{x_{obs}} \end{pmatrix}$, $A_1 = \begin{pmatrix} G_{1,un} & G_{0,un} & 0 & \cdots & 0 & 0 \\ 0 & G_{1,un} & G_{0,un} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & G_{0,un} & 0 \\ 0 & 0 & 0 & \cdots & G_{1,un} & G_{0,un} \end{pmatrix}$

$B_1 = \begin{pmatrix} G_{1,obs} & G_{0,obs} & 0 & \cdots & 0 & 0 \\ 0 & G_{1,obs} & G_{0,obs} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & G_{0,obs} & 0 \\ 0 & 0 & 0 & \cdots & G_{1,obs} & G_{0,obs} \end{pmatrix}$ and $C_1 = \begin{pmatrix} M_0 \\ M_0 \\ \cdots \\ M_0 \\ M_0 \end{pmatrix}$.

The dimension of vector $\mathbf{x}_{un}$ is denoted by $n = (h + 1).|TR_{un}|$ while the dimension of vector $\mathbf{x}_{obs}$ is $(h + 1).|TR_{obs}|$. The dimensions of matrices A_1, B_1, C_1 and column vector $b_1 = C_1 - B_1 \cdot \mathbf{x}_{obs}$ are respectively ($h.|P|$ x n), ($h.|P|$ x $(h+1).|TR_{obs}|$), ($h.|P|$ x 1) and ($h.|P|$ x 1). In addition, below we express that the trajectories are non-decreasing, that is, $x_{un}(\theta - 1) \leq x_{un}(\theta)$ for $\theta \in \{t-h+1, t-h+2, ..., t\}$ which can easily be rewritten under the form of a polyhedron

$$A_2 \cdot \mathbf{x}_{un} \leq 0_{h.|TR_{un}| \text{x} 1} \tag{10}$$

where the dimension of matrix A_2 is ($h.|TR_{un}|$ x n).

As $\underline{x_{un}}$ and $\overline{x_{un}}$ are known, we can write the polyhedron defining the vector $\mathbf{x} = (x_{un}(t_{<i>} - h + 1)^T, x_{un}(t_{<i>} - h + 2)^T, \ldots, x_{un}(t_{<i>} - 1)^T)^T$ and $\mathbf{x} = (\underline{x_{un}}^T, \mathbf{x}^T, \overline{x_{un}}^T)^T$. transpose ?

$A_1 = \begin{pmatrix} G_{1,un} & G_{0,un} & 0 & \cdots & 0 & 0 \\ 0 & G_{1,un} & G_{0,un} & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & G_{0,un} & 0 \\ 0 & 0 & 0 & \cdots & G_{1,un} & G_{0,un} \end{pmatrix}$

The partition of A_1 is $A_1 = \begin{pmatrix} D_1 & D_2 & D_3 \end{pmatrix}$ with

$D_1 = \begin{pmatrix} G_{1,un} \\ 0 \\ \cdots \\ 0 \\ 0 \end{pmatrix}$, $D_2 = \begin{pmatrix} G_{0,un} & 0 & \cdots & 0 \\ G_{1,un} & G_{0,un} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & G_{0,un} \\ 0 & 0 & \cdots & G_{1,un} \end{pmatrix}$ and $D_3 = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ G_{0,un} \end{pmatrix}$

The partition of A_2 is $A_2 = \begin{pmatrix} E_1 & E_2 & E_3 \end{pmatrix}$ with

$E_1 = \begin{pmatrix} I \\ 0 \\ \cdots \\ 0 \\ 0 \end{pmatrix}$, $E_2 = \begin{pmatrix} -I & 0 & \cdots & 0 \\ I & -I & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -I \\ 0 & 0 & \cdots & I \end{pmatrix}$ and $E_3 = \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ -I \end{pmatrix}$

Finally, we obtain

$$A \cdot \mathbf{x} \leq N + Q \cdot \mathbf{x}_{obs} + R. \begin{pmatrix} \underline{x_{un}} \\ \overline{x_{un}} \end{pmatrix} \tag{11}$$

with $A = \begin{pmatrix} D_2 \\ E_2 \end{pmatrix}$, $N = \begin{pmatrix} C_1 \\ 0 \end{pmatrix}$, $Q = -\begin{pmatrix} B_1 \\ 0 \end{pmatrix}$ and $R = -\begin{pmatrix} D_1 & D_3 \\ E_1 & E_3 \end{pmatrix}$.

As the trajectory starts from $\underline{x_{un}} \geq 0$, the vector x is non-negative and the addition of a relevant condition in the system is not necessary. Finally, we obtain a simple polyhedron

$$A \cdot \mathbf{x} \leq b \tag{12}$$

with $b = N + Q \cdot \mathbf{x}_{obs} + R. \begin{pmatrix} \underline{x_{un}} \\ \overline{x_{un}} \end{pmatrix}$.

It is important to note that the polyhedron (12) completely describes the problem without giving an approximation of the counter vectors contrary to (4) which is an over-space.

E. Checking of the existence

The consistency of system (12) can be checked by integer linear programming. We can make an arbitrary optimization:

$$\min c.\mathbf{x}_{un} \text{ with } c > 0 \text{ such that } A \cdot \mathbf{x}_{un} \leq b \text{ over } \mathbb{Z} \tag{13}$$

If the space is non empty, the minimization of $c.\mathbf{x}_{un}$ with $c > 0$ converges to a finite optimal solution as the space is lower-bounded by zero otherwise an error message is given by the software. Therefore, we can conclude that this sequence satisfies the untime Petri net.

Remark. Applying a fixed point approach is tentameous but this technique can only be used if we consider unobservable induced Petri nets which are Backward Conflict Free (BCF), i.e., any two distinct unobservable transitions have no common output place.

Remark. The approach is based on known observable dates of firings produced by a processus. To make a theoretical analysis, it can also be extended to the simulation of an untime Petri net where an arbitrary timed Petri net is associated with: Each temporization is taken unitary and the horizon h is taken sufficiently large such that the time evolution is not limited between two observed events.

V. Example 1

Let us consider the Petri net of figure 1 which contains a self-loop and a circuit. The temporization of each place is unitary and we have $TR_{un} = \{x_1, x_2\}$ and $TR_{obs} = \{y_1\}$.

The incidence matrices relevant to TR_{un} and TR_{obs} and the starting marking are as follows:

$$W_{un} = \begin{pmatrix} +1 & 0 \\ 0 & +1 \\ +1 & -1 \end{pmatrix},\ W_{ob}^- = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } M = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Phase I.

So, the relations describing the problem are $(-W_{un}.x_{un} \leq M - W_{obs}^-(.,t_{ob}))$

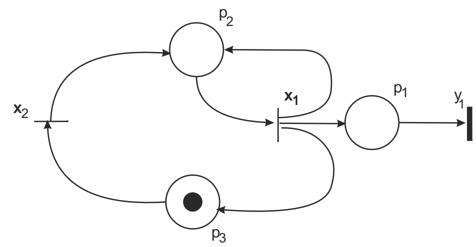

Fig. 1. Petri net of Example 1

$$\begin{cases} -x_1 & \leq -y_1 \\ -x_2 & \leq 0 \\ -x_1 + x_2 & \leq 1 \end{cases} \quad \text{with } y_1 = 1.$$

The following count vectors belong to $S_{un}^{nat}(M^{init},y_1)$: $\begin{pmatrix} 1 & 0 \end{pmatrix}^\top, \begin{pmatrix} 1 & 0 \end{pmatrix}^\top$, $\begin{pmatrix} 1 & 2 \end{pmatrix}^\top$, ... As the TR_{un}-induced subnet is not acyclic, we cannot use the results in [5] [9] and deduce immediately that these vectors are explanation vectors.

Phase II.

Let us check the count vector $\begin{pmatrix} 1 & 2 \end{pmatrix}^\top$. As we consider the first iteration in this example, the starting count vector is $\begin{pmatrix} 0 & 0 \end{pmatrix}^\top$. The problem is now to determine a time sequence connecting these two vectors. The time relations are $\begin{cases} x_1(\theta-1) & \geq y_1(\theta) \\ x_1(\theta-1) + x_2(\theta-1) & \geq x_1(\theta) \\ x_1(\theta-1)+1 & \geq x_2(\theta) \\ x_1(\theta) & \geq x_1(\theta-1) \\ x_2(\theta) & \geq x_2(\theta-1) \end{cases}$ for time $\theta \in \{t_{<i>} - h + 1, t_{<i>} - h + 2, ..., t_{<i>} - 1, t_{<i>}\}$

Consider that the firing of y_1 is at time $t = 3$. So, $y_1(t) = 0$ for $t \leq 2$ and $y_1(t) = 1$ for $t = 3$.

t	0	1	2	3
y_1	0	0	0	1
x_1	0	0	1	1
x_2	0	1	1	2

and the relevant sequence is $x_2\ x_1 x_2$ as the relevant firings appears at time 1, 2 and 3. Note that the second firing of x_2 and the first firing of y_1 are simultaneous. So, the count vector $\begin{pmatrix} 1 & 2 \end{pmatrix}^\top$ is an explanation vector.

Let us check $\begin{pmatrix} 1 & 0 \end{pmatrix}^\top$

t	0	1	2	3
y_1	0	0	0	1
x_1	0	−	1	1
x_2	0	0	0	0

.

We can deduce that $x_2(\theta) = 0$ for $\theta \in \{0, 1, 2, 3\}$ as $x_2(0) = 0$ and $x_2(3) = 0$. Moreover, $x_1(\theta-1) + x_2(\theta-1) \geq x_1(\theta)$ and $x_1(\theta) \geq x_1(\theta-1)$ for $\theta \in \{1, 2, 3\}$. As $x_2(\theta) = 0$ for $\theta \in \{0, 1, 2, 3\}$, we deduce the equality $x_1(\theta-1) = x_1(\theta)$ for $\theta \in \{1, 2, 3\}$ which is not possible since $x_1(0) = 0$ and $x_1(3) = 1$. Therefore, the count vector $\begin{pmatrix} 1 & 0 \end{pmatrix}^\top$ is not an explanation vector. ■

A. Discussion about P-time Petri nets

In a next study, we will show that the consideration of a P-time Petri net which introduces new constraints can also be made. It leads to a phase III which is a final checking based on daters for a given untimed sequence deduced from the phase II. Contrary to the previous part which is based on the numbering of the events, we focus on the firing dates of the sequence. We take here a simplified form of the dater approach used in max-plus community.

As the model of P-time Petri net is rather complex, the Timed Petri net brings an usual simplification by taking the lower bound of each temporization: It allows to check the logical aspect of the candidate vector and to produce the relevant sequence. It is important to note that there is no guarantee that P-time Petri net can follow the minimum trajectory of the Timed Petri net even if the Petri net is an event graph.

We will show that the concept of minimum date vector exists for a non-empty subspace and the vector can be computed. The notion of maximum date vector is pertinent under an algebraic condition. If the subspace is empty, the explanation vector cannot be followed by the P-time Petri net. The technique is as follows:

- Firstly to deduce an *untimed* firing sequence from an arbitrary counter vector obtained for Timed Petri nets;
- Secondly to express the relevant time subspace for a P-time Petri net and check if this model can follow the same sequence. The technique is to determine the evolution of the tokens by exploiting their availability. More details on the determination the firing dates can be found in [16].

VI. Conclusion

In this paper, we have shown that a general strategy can deal with time models. The first phase is based on a simplification leading to an over-space of the admissible sequences. The introduction of time in Timed Petri nets allows the checking of the candidate vector and the verification of the logical aspect of the untimed Petri net: if the relevant time subspace is not empty, the candidate vector is an explanation vector and a sequence which gives the order of the events is generated as shown in the example. The transition firings of the unobservable transitions can be simultaneous. Moreover, we show that the approach can consider circuits and self-loops as illustrated by the example. The general procedure is efficient as the algorithms of linear programming are used.

Natural perspective is a generalization to P-time Petri net (as discussed in the previous part) and also to more general models as Time Stream Petri nets or new types of P-time Petri nets.

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