

DETECTION OF CHANGES BY OBSERVER IN TIMED EVENT GRAPHS AND TIME STREAM EVENT GRAPHS

Philippe Declerck *

philippe.declerck@istia.univ-angers.fr

* *LISA EA 4014 - Université d'Angers 62, avenue Notre
Dame du Lac - 49000 Angers*

Abstract: A state-based approach for detection of changes in systems modelled as Timed Event Graph and Time Stream Event Graph is presented. We assume that the net in its nominal behavior is known and transitions are partitioned as observable and unobservable transitions. Considered faults are (possibly small) variations of dynamical models by respect to this nominal behavior. Using the algebra of dioids, the approach follows the same principle as the observers used in continuous systems. *Copyright ©2007 IFAC*

Keywords: Time Stream Event Graphs, Observer, Estimation, Detection, Isolation, Diagnostic, (min, max, +) Functions.

1. INTRODUCTION

The detection of changes as limited deteriorations or significant faults in the systems play a crucial role in increasing operational time and productivity. In Discrete-Event Systems, the approaches generally consider drastic failures such as "valve stuck-closed" or "sensor short-circuited" (S. Hashtrudi Zad 2003) (D. Lefebvre 2006). This article focus on a different type of faults, for applications as transportation systems and production systems. We assume that the process has been previously optimized and that a scheduling has been realized. The nominal behavior is expressed by a Petri Net which is an Event Graph in many cases. In this paper, we suppose that the model is a Timed Event Graph or a Time Stream Event Graph. Faults are considered as variations of dynamical models by respect to this nominal behavior expressed by a Petri Net. The conceptual point of view is that each fault is relevant to a specific Petri net. At an upper level, the set of the models (M_0, M_1, M_2, \dots) corresponds to a set of

the system states (normal state, fault case 1, fault case 2, ...) of the process which are linked in a state machine. At this upper level, the appearance of a fault leads to a fault state and its repairing, to the nominal state. These two events are assumed unknown. Firing sequences are observed and the coherence of the model is analyzed using an observer. If the data are incoherent with the model of the nominal behavior, a fault is detected. If the data are coherent with the model of a fault case, the relevant fault is diagnosed. If the fault is repaired, the new data will be again coherent with the nominal model.

We assume that the nominal model is known and is described by an event graph where the delays are associated to the places if the model is a Timed Event Graph. In this paper, we consider a simple fault which is a sudden variation of delay and therefore, the model has the same graph with a different temporisation. The transitions of the set of models are partitioned as $T = T_{ob} \cup T_{un}$ where T_{ob} is the set of observable transitions, and T_{un}

is the set of unobservable transitions. Fault and repairing events are modelled at an upper level in the state machine.

An objective of this paper is to propose an on-line optimal observer (A. Giua 2005) which estimates the greatest state (see (F. Baccelli 1992)). The lowest state is not necessary in the approach. Moreover, only a sub-optimal solution can be calculated) in Timed Event Graphs and Time Stream Event Graphs (M.Diaz 2001) and analyzes the consistency of the data. Moreover, the Petri Net can contain input and output transitions which can be observable or not. For instance, they can express respectively the input or the output of a vehicle in a bus line.

The paper is organized as follows: We first present the motivation and the principle of the proposed approach. Then, the modelling of Timed Event Graphs and Time Stream Event Graphs in the $(\min, \max, +)$ algebra is given. Based on a fixed point approach, we present an observer which allows the detection of changes in the process. Finally, the approach is applied to a simple Timed Event Graph. These parts are preceded by notations and by a brief review of previous results.

2. PRINCIPLE OF THE PROPOSED APPROACH

Let us consider the sequence of two places which describes a non-bounded Event Graph. The first place describes the journey of a vehicle from the town A to B which lasts between 2 and 3 hours. The second represents the following journey from B to C with a temporization between 5 and 6 hours. If the time u_1 of departure of the vehicle is known, the arrival at the intermediate town B can obviously be estimated: $[u_1 + 2, u_1 + 3]$. Symmetrically, If the time y_1 of arrival to the town C is known, the arrival at the intermediate town B can also be estimated: $[y_1 - 6, y_1 - 5]$. Consequently, the estimate of the date associated with B can be calculated by a **forward-backward** approach: $[\max(u_1 + 2, y_1 - 6), \min(u_1 + 3, y_1 - 5)]$. If the date is out this interval, we can conclude that there is a break-down, an unpredictable event or more generally a bad modelling. Here the estimation allows the analysis of the past journey.

But, the model can equivalently be described by the form $x \leq f(x)$ which gives : $x \leq \min(u_1 + 3, y_1 - 5)$, $u_1 \leq x - 2$ and $y \leq x + 6$. The first inequality allows the estimation of the greatest x . This value can be introduced in the two other inequalities: a fault is detected when they are not satisfied. This situation arrives when the model has changed: for instance, a breakdown of the vehicle between the two towns B and C entails

that the temporization associated to the second place equals 9 which does not belong to $[5, 6]$. If the real data are $u = 10$ and $y = 21$, the greatest estimate x is 13 and $u_1 \leq x - 2$ is satisfied ($10 \leq 13 - 2 = 11$) contrary to $y \leq x + 6$ ($21 \leq 13 + 6 = 19$) which shows an incoherence between the used model and the evolution of the current trajectory.

Therefore, this simple example shows an application of fixed point theory in fault detection. Based on the well-known principle of redundancy, the fault detection needs the calculation of only one bound. Therefore, coherence between data (estimate, input and outputs) is checked.

3. PRELIMINARIES

A monoid is a couple (S, \oplus) where the operation \oplus is associative and presents a neutral element. A semi-ring S is a triplet (S, \oplus, \otimes) where (S, \oplus) and (S, \otimes) are monoids, \oplus is commutative, \otimes is distributive relatively to \oplus and the zero element ε of \oplus is the absorbing element of \otimes ($\varepsilon \otimes a = a \otimes \varepsilon = \varepsilon$) . A dioid D is an idempotent semi-ring (the operation \oplus is idempotent, that is $a \oplus a = a$) . Let us notice that contrary to the structures of group and ring, monoid and semi-ring do not have a property of symmetry on S . The unit $\mathbb{R} \cup \{-\infty\}$ provided with the maximum operation denoted \oplus and the addition denoted \otimes is an example of dioid. We have : $\mathbb{R}_{max} = (\mathbb{R} \cup \{-\infty\}, \oplus, \otimes)$. The neutral elements of \oplus and \otimes are represented by $\varepsilon = -\infty$ and $e = 0$ respectively. The absorbing element of \otimes is ε . Isomorphic to the previous one by the bijection: $x \mapsto -x$, another dioid is $\mathbb{R} \cup \{+\infty\}$ provided with the minimum operation denoted \wedge and the addition denoted \odot . The neutral elements of \wedge and \odot are represented by $T = +\infty$ and $e = 0$ respectively. The absorbing element of \odot is ε . The following convention is taken: $T \otimes \varepsilon = \varepsilon$ and $T \odot \varepsilon = T$. The expression $a \otimes b$ and $a \odot b$ are identical if at least either a or b is a finite scalar. The partial order denoted \leq is defined as follows: $x \leq y \iff x \oplus y = y \iff x \wedge y = x \iff x_i \leq y_i$, for i from 1 to n in \mathbb{R}^n . Notation $x < y$ means that $x \leq y$ and $x \neq y$. A dioid D is complete if it is closed for infinite sums and the distributivity of the multiplication with respect to addition extends to infinite sums : $(\forall c \in D) (\forall A \subseteq D) c \otimes (\bigoplus_{x \in A} x) = \bigoplus_{x \in A} c \otimes x$.

For example, $\bar{\mathbb{R}}_{max} = (\mathbb{R} \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$ is complete. The set of $n.n$ matrices with entries in a complete dioid D endowed with the two operations \oplus and \otimes is also a complete dioid which is denoted $D^{n.n}$. The elements of the matrices in the $(\max, +)$ expressions (respectively $(\min, +)$ expressions) are either finite or ε (respectively T). We can deal with non-square matrices if we

complete by rows or columns with entries equals to ε (respectively T). The different operations operate as in the usual algebra: The notation \odot refers to the multiplication of two matrices in which the \wedge -operation is used instead of \oplus . The mapping f is said residuated if for all $y \in D$, the least upper bound of the subset $\{x \in D \mid f(x) \leq y\}$ exists and lies in this subset. The mapping $x \in (\bar{\mathbb{R}}_{max})^n \mapsto A \otimes x$ defined over $\bar{\mathbb{R}}_{max}$ is residuated (F. Baccelli 1992) and the left \otimes -residuation of B by A is denoted by: $A \setminus B = \max\{x \in (\bar{\mathbb{R}}_{max})^n \text{ such that } A \otimes x \leq B\}$.

4. MODELS OF TIME EVENT GRAPHS

4.1 Interval models

The interval models are now described. The variable $x_i(k)$ is the date of the k^{th} firing of transition i and a trajectory of a transition x_i is a firing date sequence $\{x_i(k)\}$ for $k \in \mathbb{Z}$. Any trajectory can be represented by the following formal power series in $\gamma : x(\gamma) = \bigoplus_{k \in \mathbb{Z}} x(k) \otimes \gamma^k$. Variables may also be regarded as the backward shift operator in event domain (formally, $\gamma x(k) = x(k-1)$) and γ -transforms of functions can express this effect. The set of formal power series in one variable γ and coefficients in $\bar{\mathbb{R}} = \mathbb{R} \cup \{\pm\infty\}$, is usually noted $\bar{\mathbb{R}}_{max}[[\gamma]]$. The evolution of the system is described by the following model, called an "interval descriptor system" or "interval model", where f^- and f^+ are (min, max, +) functions on the set of formal power series $\bar{\mathbb{R}}_{max}[[\gamma]]$.

$$f^-(x(\gamma), u(\gamma)) \leq x(\gamma) \leq f^+(x(\gamma), u(\gamma)) \quad (1)$$

The vectors x and u are the state and the input, respectively.

The functions f^- and f^+ will be defined in the following parts for each Event Graph. As the type of the system is defined by the types of the functions f^- and f^+ , we can characterize the model by the following pair (type of f^- , type of f^+). The type ((min, max, +), (min, max, +)) naturally represents the more general mathematical case. Assuming the existence of a solution, they define corresponding classes of interval systems.

In the following part, we will show that Timed Event Graphs, P-time Event Graphs and Time Stream Event Graphs can be modelled under the form of interval model.

4.2 Time Stream Event Graphs

Definition 2.1 (Time Stream Event Graph)

Let I_i be a set of upstream arcs of a transition

i and P_i be the corresponding set of upstream places. A Time Stream Event Graph is an Event-Graph such as:

- 1) an interval $[\alpha_j, \beta_j] \in (\mathbb{R}^+ \cup \{0\}) \times (\mathbb{R}^+ \cup \{+\infty\})$ associated with each $a_j \in I_i$ (usually defined on \mathbb{Q}^+ , the limits of intervals are generalized to \mathbb{R}^+ , which does not introduce new difficulties);
- 2) a special semantic policy of firing associated with each transition is defined below.

Considering one outgoing arc from a given place, when a token is received by that place at time x , the token should remain in the place during an amount of time defined by a value within the range $[x + \alpha, x + \beta]$ associated with the arc. As the firing time of a transition which has more than one input arc, depends on the nature of the processes which will be synchronized, different semantic policies of firing may be associated with a transition. In this paper, we consider two types of semantic policies, And and Weak-And, which we will use later. They are defined by a pair $[x + \alpha_j, x + \beta_j]$ associated to each ingoing arc.

Definition 2.2 A transition i of the type "And" (respectively, "Weak-And") is firing at time x_i if and only if the two following conditions are satisfied:

- 1) transition i is enabled for the current marking: every upstream place j of P_i contains at least one token. Let x_j be the entrance date of the token which is also the date of firing of the upstream transition of this place.

- 2) For the **semantic policy And**, the value of x_i is as follows: $(x_j + \alpha_j) \leq x_i \leq (x_j + \beta_j)$ for every place $p_j \in P_i$ and arc $a_j \in I_i$ (every time condition has to be fulfilled).

Respectively, for the **semantic policy Weak-And**, the value of x_i is as follows: $(x_i + \alpha_i) \leq x_i$ for every place $p_j \in P_i$ and arc $a_j \in I_i$ and $\exists j \in P_i$, $x_i \leq (x_j + \beta_j)$ (the firing may wait until the last time interval).

Therefore, if m_j is the number of the tokens present in each place p_j at the instant $t = 0$ (initial marking), for each transition, we can write

$$\bigoplus_{j \in P_i} (x_j(k - m_j) + \alpha_j) \leq x_i(k) \leq \bigwedge_{j \in P_i} (x_j(k - m_j) + \beta_j) \text{ if the semantic policy is And;}$$

$$\bigoplus_{j \in P_i} (x_j(k - m_j) + \alpha_j) \leq x_i(k) \leq \bigoplus_{j \in P_i} (x_j(k - m_j) + \beta_j) \text{ if the semantic policy is Weak-And.}$$

For Time Stream Event Graphs for semantic policies And and Weak-And, $f^-(x(\gamma), u(\gamma))$ can be a (max, +) function and $f^+(x(\gamma), u(\gamma))$ a (min, max, +) function. Therefore, any Time Stream Event Graph can be modelled under the following general form :

$$\begin{cases} f^-(x(\gamma), u(\gamma)) = A^- \otimes x(\gamma) \oplus B^- u(\gamma) \\ f^+(x(\gamma), u(\gamma)) = \bigwedge_{i=1}^{j(i)} A_i^+ \otimes x(\gamma) \oplus B_i^+ u(\gamma) \end{cases} \quad (2)$$

with $(A^-)_{ij}, (B^-)_{ij}, (A_i^+)_{jk}, (B_i^+)_{jk} \in \overline{\mathbb{R}}_{\max}[[\gamma]]$

Particularly, the general form 2 includes the state equation of Timed Event Graphs because the relevant interval model is found by taking: $i_{\max} = 1$; $A^- = A_1^+ = A$; $B^- = B_1^+ = B$. Consequently, the algebraic model of Timed Event Graphs may be seen as a particular case of the interval model. Consequently, $f^-(x(\gamma), u(\gamma)) = f^+(x(\gamma), u(\gamma)) = A\gamma \otimes x(\gamma) \oplus B \otimes u(\gamma)$

Moreover, it can be proved that P-time Event Graphs can be modelled by inequalities corresponding to semantic policy And. Therefore, the interval model 1 is an algebraic generalization of Timed Event Graphs, P-time Event Graphs and Time Stream Event Graphs for the semantic policies And and Weak-And.

5. ESTIMATION AND DETECTION OF CHANGES

Now, we consider an arbitrary subpart of a given Event Graph, and the notation below are relevant to this graph. The only condition is that the subgraph is an Event Graph. The consideration of the complete graph allows the detection of changes in the process while the consideration of subgraphs allows their isolation.

The transitions of the set of models are partitioned as $T = T_{ob} \cup T_{un}$ where T_{ob} is the set of observable transitions, and T_{un} is the set of unobservable transitions. Some on the input transitions of the Petri Net are observable transitions and $U_{ob} \subset T_{ob}$ is the set of observable input transitions. The set of observable transition t of T_{ob} with $t \notin U_{ob}$ is denoted Y_{ob} and $T_{ob} = U_{ob} \cup Y_{ob}$. Each transition t of Y_{ob} is connected to the graph with places with a null initial marking otherwise some places can be added such that the following equality can be written.

$$y_{ob}(\gamma) = C_{ob} \otimes x(\gamma) \text{ with } (C_{ob})_{ij} \in \overline{\mathbb{R}}_{\max}$$

Therefore, some input and output transitions (which often models the input and output of parts, products, messages,... in the system) can be unobservable and $T_{un} = U_{un} \cup Y_{un}$ with an obvious notation.

The objective is to find the least upper bound of $x(k)$ knowing the values of the input $u_{ob}(k)$ and the output $y_{ob}(k)$ for k going from k_s to k_f with k_s and k_f as the numbers of start and final events. We are supposing the model is known on the same horizon of observation. This problem

of estimation is thus different from the control synthesis which considers that the control, the state and the output are the unknown data.

The initial marking is supposed known contrary to the state $x(k)$ and the initial condition $x(0)$ (the first date of firing of the transitions) which are unknown.

5.1 Fixed point formulation

In this part, the choice of the semantic policies And and Weak-And causes function f^- (2) to be residuated. The problem can consequently, be reformulated as a fixed point problem.

Theorem 4.1 For Time Stream Event Graphs with the semantic policies And and Weak-And, the problem of the greatest estimate of $x(\gamma)$ can be written as follows: from k_s to k_f , search the greatest state of the following inequality $x(\gamma) \leq h(x(\gamma))$ with

$$h(x(\gamma)) = \left(\begin{array}{l} \gamma^{-1} x(\gamma) \wedge [A^- \setminus x(\gamma)] \wedge [C_{ob}(\gamma) \setminus y_{ob}(\gamma)] \wedge \\ \left[\bigwedge_{i=1}^{j_1} ((A_i^+)' \otimes x(\gamma) \oplus (B_{ob,i}^+)' \otimes u_{ob}(\gamma)) \right] \end{array} \right) \quad (3)$$

with the constraints

$$\begin{cases} u_{ob}(\gamma) \leq B_{ob}^- \setminus x(\gamma) \\ y_{ob}(\gamma) \leq C_{ob}(\gamma) \otimes x(\gamma) \end{cases} \quad (4)$$

Proof: omitted.

Clearly, the equation set (3) contains (min, max, +) functions which are defined below. Let us notice, that the first expression presents a usual backward part $A^- \setminus x(\gamma)$ but also, in the case where A_i^+ and B_i^+ have positive exponents, a forward part $\gamma^{-1} x(\gamma) \wedge \left[\bigwedge_{i=1}^{j_1} ((A_i^+)' \otimes x(\gamma) \oplus (B_{ob,i}^+)' \otimes u_{ob}(\gamma)) \right]$. This fact increases the complexity of the problem and forbids the writing of simple equations such as the classical backward equations in control. In other words, we must solve a (min, max, +) fixed-point problem of type $x \leq f(x)$ (if x exists) over the horizon of the known values of the control u and the output y with function f defined by the following grammar: $f = b, x_1, x_2, \dots, x_n \mid f \otimes a \mid f \wedge f \mid f \oplus f$ where a, b are arbitrary reals.

Moreover, the constraints $u_{ob}(\gamma) \leq B_{ob}^- \setminus x(\gamma)$ and $y_{ob}(\gamma) \leq C_{ob}(\gamma) \otimes x(\gamma)$ must always be satisfied in a nominal behavior. They verify coherence between known data and model and show particularly that the Event Graph follows its nominal model. The two constraints make it possible in particular to check that the event graph follows its nominal model.

5.2 Algorithm of calculation of the greatest state

The effective calculation of the greatest control can be made by a classical iterative algorithm. The resolution of (3) whose form is $x \leq f(x)$, is given by the iterations of $x_{i+1} \leftarrow f(x_i) \wedge x_i$ if the starting point is finite and greater than the final solution. Here, the index i represents the number of iterations and not the number of components of the vector x .

Following this framework, we below give an algorithm specific to the estimation of the greatest state for Time Stream Event Graph. It can also be applied to Timed and P-time Event Graphs.

Algorithm

Step 0 (initialization) : $\mu(k_f) \leftarrow T; \lambda(k_f) \leftarrow T$

Repeat

Step 1 : for $k = k_f$ to k_s , $\lambda(k) \leftarrow \mu(k) \wedge \lambda(k+1) \wedge [A^- \setminus \lambda(k+1)] \wedge [C_{ob} \setminus y_{ob}(k)]$

Step 2 : $\mu(k_s) \leftarrow \lambda(k_s)$

for $k = k_s+1$ to k_f , $\mu(k) \leftarrow \lambda(k) \wedge f_i^+(\mu(k), u(k))$

Until $\lambda(k) = \mu(k)$ for $k_s \leq k \leq k_f$

The function $f_i^+(\mu(k), u(k))$ equals $\bigwedge_{l=1}^{j(i)} [A_l^+ \otimes \mu(k-1) \oplus B_{ob,l}^+ \otimes u_{ob}(k)]$ for Time Stream Event Graphs, equals $A^+ \otimes x(\gamma) \wedge B_{ob}^+ \otimes u_{ob}(\gamma)$ for P-time Event Graphs and equals $[A_l^+ \otimes \mu(k-1) \oplus B_{ob,l}^+ \otimes u_{ob}(k)]$ for Timed Event Graphs.

As the general algorithm is known to be pseudo-polynomial, the above algorithm converges to the greatest state for Time Stream Event Graphs (with semantic policies And and Weak-And) in a finished number of iterations.

6. EXAMPLE

In the aim of clearly illustrating the approach, we consider only a Timed Event Graph. Calculations has been realized with Scilab.

Model of the Petri Net

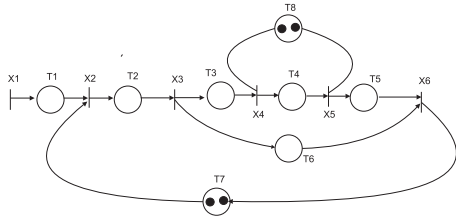


Fig. 1. A Timed Event Graph

We consider a simple Timed Event Graph whose nominal model M_1 is as follows. The temporizations are denoted T .

$T_1 = 1, T_2 = 2, T_3 = 3, T_4 = 10, T_5 = 5, T_6 = 5, T_7 = 1$ and $T_8 = 0$.

$a = T_2 + T_6 + T_7$; $b = d + T_5 + T_7$; $c = T_2 + T_3$; $d = T_4$; $f = T_1 + T_2 + T_3$

$$A = \begin{pmatrix} 0 & 0 & \varepsilon & \varepsilon \\ a & 0 & b & \varepsilon \\ \varepsilon & c & 0 & 0 \\ \varepsilon & \varepsilon & d & 0 \end{pmatrix}, B_1 = \begin{pmatrix} T_1 \\ \varepsilon \\ f \\ \varepsilon \end{pmatrix} \text{ and } C_1 = \begin{pmatrix} T_2 + T_6 & \varepsilon & d + T_5 & \varepsilon \\ T_2 & \varepsilon & \varepsilon & \varepsilon \end{pmatrix}$$

The transition x_1 is an input transition. The observable transitions are: x_1, x_3 and x_6 . An observer on the overall system can be developed using the transitions x_1 and x_6 .

Scenario of the simulation

Now, a control is applied to the system with $x_1 = 0$ and for $i = 1$ to 69, $x_1(i+1) = x_1(i) + 1$. Moreover, the following faults are considered. We successively consider a fault in zone 1 and two faults in zone 2. The normal value of T_2 is 2 and from $k = 10$ to 15, $T_2 = 12$. Then, in zone 2, the normal value of T_4 is 4 for $1 \leq k \leq 28$ and the fault is $T_4 = 13$ from $k = 29$ to 35. The temporisation T_4 is restored to its normal value for $36 \leq k \leq 49$. Finally, $T_4 = 15$ from $k = 50$ to 55 and T_4 is restored to its normal value for $56 \leq k \leq 70$.

Each following curve gives the number of inconsistent relations function of the number of event from 1 to 70. The horizon of calculation of the observers is equal to 5.

Detection

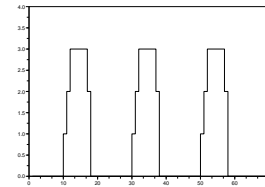


Fig. 2. Global observer

The global observer is sensitive to the three faults.

Isolation

The analysis of the Event Graph show the existence of two input/output sub-models where the input and output are observable. Therefore, each zone can be checked by an observer.

Zone 1

The observer uses observable transitions x_1, x_3 and x_6 .

$$A^- = A^+ = \begin{pmatrix} 0 & 0 & \varepsilon \\ \varepsilon & 0 & T_7 \\ \varepsilon & \varepsilon & 0 \end{pmatrix} B^- = B^+ = \begin{pmatrix} T_1 & \varepsilon \\ \varepsilon & \varepsilon \\ \varepsilon & 0 \end{pmatrix} C^- = C^+ = \begin{pmatrix} T_2 & \varepsilon & \varepsilon \end{pmatrix}$$

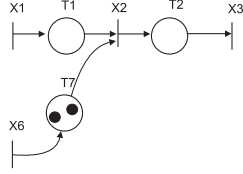


Fig. 3. Observer of the zone 1

Zone 2

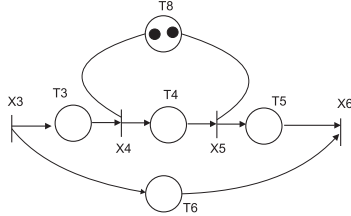


Fig. 4. Observer of the zone 2

The observer uses observable transitions x_3 and x_6 .

$$A^- = A^+ = \begin{pmatrix} 0 & 0 & \varepsilon \\ T_4 & 0 & \varepsilon \\ \varepsilon & \varepsilon & 0 \end{pmatrix} \quad B^- = B^+ = \begin{pmatrix} T_3 \\ \varepsilon \\ 0 \end{pmatrix}$$

$$C^- = C^+ = (T_4 + T_5 \ \varepsilon \ T_6)$$

The observer of the zone 1 reacts to the fault which appeared in zone 1. It is not sensitive to the two faults of zone 2. The observer of the zone 2 has a symmetric behavior. Consequently, the isolation of the fault in zone 1 or zone 2, has been realized.

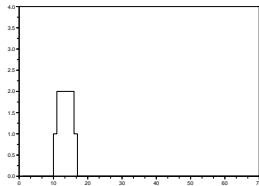


Fig. 5. Observer of zone 1

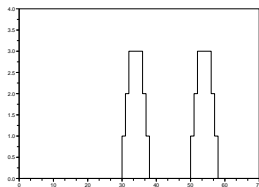


Fig. 6. Observer of zone 2

Diagnostic

The following specific observer checks the coherence of the nominal model of the zone 2 but with T_4 between 4 and 14.

$$A^- = \begin{pmatrix} 0 & 0 & \varepsilon \\ T_4 & 0 & \varepsilon \\ \varepsilon & \varepsilon & 0 \end{pmatrix} \quad A^+ = \begin{pmatrix} 0 & 0 & \varepsilon \\ T_4 + 10 & 0 & \varepsilon \\ \varepsilon & \varepsilon & 0 \end{pmatrix} \quad B^- =$$

$$B^+ = \begin{pmatrix} T_3 \\ \varepsilon \\ 0 \end{pmatrix}$$

$$C^- = (T_4 + T_5 \ \varepsilon \ T_6) \quad C^+ = (T_4 + 10 + T_5 \ \varepsilon \ T_6)$$

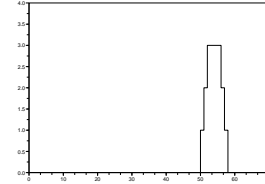


Fig. 7. Observer of T4 in zone 2

The observer does not react to the fault where $T_4 = 13$ but is sensitive to the third fault where $T_4 = 15$.

7. CONCLUSION

The formulation introduces two constraints relevant to input and output data, which can be used to check the correctness of the model and the data and to detect the existence of a change or the deterioration in the process as shown in the simulation. This approach has been improved by considering subparts of the complete model. The simulation illustrates that the different consistencies of faulty subpart and no-faulty subpart allows the isolation of the fault: an observer of a zone is insensible to the fault of another part of the system. Moreover, in the last test, small changes in the process has been detected by a specific observer.

REFERENCES

- A. Giua, Carla Seatzu (2005). Fault detection for discrete event systems using petri nets with unobservable transitions. *44th IEEE Conference on Decision and Control and European Control Conference ECC*.
- D. Lefebvre, C. Delherm (2006). Diagnosis of des with petri net models. *IEEE Trans. On Automation Science and Engineering*.
- F. Baccelli, G. Cohen, G.J. Olsder J.P. Quadrat (1992). *Synchronization and linearity. An Algebra for Discrete Event Systems*. Wiley. New York.
- M.Diaz (2001). *Les réseaux de Petri*. Hermes.
- S. Hashtrudi Zad, R.H. Kwong, W.M. Wonham (2003). Fault diagnosis in discrete-event systems: Framework and model reduction. *IEEE Trans. on Automatic Control*. **48**, 1199–1212.