Trajectory Tracking Control of a Timed Event Graph with Specifications Defined by a P-time Event Graph

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Abstract The aim of this paper is a trajectory tracking control of Timed Event Graphs with specifications defined by a P-time Event Graph. Two problems are solved on a fixed horizon knowing the current state: The optimal control for favorable past evolution; The prediction of the earliest future evolution of the process. These two parts make up an on-line control which is used on a sliding horizon. Completely defined in (max, +) algebra, the proposed approach is a Model Predictive Control using the componentwise order relation.

1 Introduction

In this paper, we focus on the trajectory tracking control of Timed Event Graphs with reference model defined by a P-time Event Graph. The P-time Event Graph describes the desired behavior of the interconnections of all the internal transitions. Some events are stated as controllable, meaning that the corresponding transitions (input) may be delayed from firing until some arbitrary time provided by a supervisor. We wish to determine the greatest input in order to obtain the desired behavior defined by the desired output and the specifications. This problem is denoted problem 1 in the document.

Moreover, the aim of this paper is also the extension of problem 1 to Predictive Control on infinite horizon. This extension is denoted problem 2. Using a receding horizon principle, Model Predictive Control is a form of control in which the current control is obtained by solving on-line, a finite open-loop optimal control problem

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at each sampling instant. The current state of the process is considered as the initial state. The optimization yields an optimal control sequence but only the first control in this sequence is applied to the plant. This procedure can be repeated infinitely.

In this paper, we complete the approaches developed in [4] and [6] by introducing specifications defined by a P-time Event Graph as in the preliminary study [5] which is generalized to a sliding horizon. The framework of this proposed study can be found in [6] where a comparison with [15] is given in the standard algebra. The approach is based on the concept of earliest desired output which was introduced in [4] to the best of our knowledge. A similar concept was also considered in [11]: as this last approach uses the past control and not the current state, we can prove that the relevant updated desired output (called reference input in [11]) can be lower. Let us recall that a simple forward technique gives the earliest desired output while the control is given by the classical backward approach. However, this simple technique does not hold if some specifications are introduced in the problem as shown in parts 2 and 3: the structure of matrix D_h in part 2.2 shows the forward and backward connections of inequality $X \geq D_h \otimes X$ for instance.

In this paper, we consider that each transition is observable: the event date of each transition firing is assumed to be available. Let us note that we have developed software written in Scilab composed of estimation, prediction and control. No hypothesis is taken on the structure of the Event Graphs which does not need to be strongly connected. The initial marking should only satisfy the classical liveness condition and the usual hypothesis that places should be First In First Out (FIFO) is taken. In the context of the trajectory tracking control (problem 2), we consider different structures of matrix B. Defined in part 3.3.2, the case of fully controlled transitions can be found in the modeling of railway system where each departure of train must be controlled [2] [14]. This structure is also considered in urban bus networks where the timetable must be respected at each stop [9].

The paper is structured as follows: The optimal control on a fixed horizon (problem 1) and its extension to a sliding horizon (problem 2) are successively considered. The resolution of problem 2 is based on the prediction of the earliest desired output. By reason on the lack of place, we cannot give a complete presentation of the preliminary remarks but the reader can easily find more information in [1] and [8]. The presentation of the model of the P-time Event Graph is also omitted: the reader can find the preliminaries and the presentation of the models in [5].

Maximization and addition operations are denoted respectively $\oplus$ and $\otimes$. The set of $n.n$ matrices with entries in dioid $D = \overline{\mathbb{R}}_{max}$ = $(\mathbb{R} \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$ including the two operations $\oplus$ and $\otimes$ is a dioid, which is denoted $D^{n.n}$. Mapping f is said to be residuated if for all $y \in D$, the least upper bound of subset $\{x \in D \mid f(x) \leq y\}$ exists and lies in this subset. Mapping $x \in (\overline{\mathbb{R}}_{max})^n \mapsto A \otimes x$, defined over $\overline{\mathbb{R}}_{max}$ is residuated (see [1]) and the left $\otimes-$residuation of B by A is denoted by: $A \backslash B = \max\{x \in (\overline{\mathbb{R}}_{max})^n$ such that $A \otimes x \leqslant B\}$. The following Theorem uses the Kleene star defined by: $A^* = \bigoplus_{i=0}^{+\infty} A^i$.

Theorem 1. *(Theorem 4.75 part 1 in [1]) Consider equation $x = A \otimes x \oplus B$ and inequality $x \geq A \otimes x \oplus B$ with A and B in complete dioid D. Then, A^*B is the least solution to these two relations.* □

Variable $x_i(k)$ is below the date of the k^{th} firing of transition x_i.

2 Control on a fixed horizon (problem 1)

Let us consider the objective of problem 1.

2.1 Objective

The problem of this paper is the determination of the greatest control of a plant described by a Timed Event Graph when the state and control trajectories are constrained by additional specifications defined by a P-time Event graph. Applications of P-time Event Graphs can be found in production systems, microcircuit design, transportation systems, real-time systems and food industry. The objective is to calculate the greatest control u on horizon $[k_s+1, k_f]$ such that its application to the Timed Event Graph defined by

$$\begin{cases} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{cases} \quad (1)$$

satisfies the following conditions:

a) $y \leq \underline{z}$ knowing the trajectory of the desired output $\underline{z}$ on a fixed horizon $[k_s+1, k_f]$ with $h = k_f - k_s \in \mathbb{N}$;

b) The state trajectory follows the model of the autonomous P-time Event Graph defined by

$$\begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \geq \begin{pmatrix} \varepsilon & A^+ \\ A^- & A^= \end{pmatrix} \otimes \begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \quad (2)$$

for $k \geq k_s$;

c) The first state vector of the state trajectory $x(k)$ for $k \geq k_s$ is finite and is known vector $\underline{x}(k_s)$. This “ non-canonical ” initial condition can be the result of a past evolution of a process.

Underlined symbols like $\underline{x}(k_s)$, $\underline{z}(k)$ correspond to known data of the problem and $x(k)$ and $y(k)$ are estimated in the following resolutions based on the information available at number of events k_s .

A simple example of this problem is a production system composed of two tasks which are the cooking of a product and its packaging with an additional constraint: the cooking time must not be too excessive , otherwise, the product would be damaged.

In the following part 2.2, we present the relations which describe a trajectory of a Timed Event Graph satisfying the specifications defined by a P-time Event Graph (constraint b)). The introduction of the ”Just-in-time” objective (constraint a)) in part 2.3 allows the resolution of the control problem on a fixed horizon.

2.2 Trajectory description

From (1) and (2), we deduce a system which describes the trajectories on horizon $[k_s, k_f]$. Let us introduce the following notations. Let $X =$
$\left(x(k_s)^t \; x(k_s+1)^t \; x(k_s+2)^t \; \cdots \; x(k_f-1)^t \; x(k_f)^t \right)^t$ (t: transposed) and $D_h =$
$\begin{pmatrix} \varepsilon & A^+ & \varepsilon & \cdots & \varepsilon & \varepsilon & \varepsilon \\ A \oplus A^- & A^= & A^+ & \cdots & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & A \oplus A^- & A^= & \cdots & \varepsilon & \varepsilon & \varepsilon \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \varepsilon & \varepsilon & \varepsilon & \cdots & A^= & A^+ & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \cdots & A \oplus A^- & A^= & A^+ \\ \varepsilon & \varepsilon & \varepsilon & \cdots & \varepsilon & A \oplus A^- & A^= \end{pmatrix}$. Matrix D_h presents an original block tridiagonal structure: this is a square matrix, composed of a lower diagonal (square submatrices $A \oplus A^-$), a main diagonal (square submatrices $A^=$ except the first element) and an upper diagonal (square submatrices A^+), with all other blocks being zero matrices (ε). As n is the dimension of x, D_h is a $n.(h+1))$ x $n.(h+1))$ matrix.

Theorem 2. *The state trajectories of a Timed Event Graph (1) starting from $\underline{x}(k_s)$ and following the specifications defined by a P-time Event Graph (2) on horizon $[k_s, k_f]$ satisfy the following system*

$$\begin{cases} X \geq D_h \otimes X \\ x(k) \geq B \otimes u(k) \texttt{ for } k \in [k_s+1, k_f] \\ x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k) \text{ for } k \in [k_s+1, k_f] \\ x(k_s) = \underline{x}(k_s) \end{cases} \tag{3}$$

Proof. System (3) is directly deduced from the models of the Timed Event Graph (1) and the P-time Event Graph (2). For instance, equality (1) is equivalent to
$\begin{cases} A \otimes x(k-1) \oplus B \otimes u(k) \leq x(k) \\ x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k) \end{cases} \texttt{for } k \geq k_s.$ □

2.3 Greatest trajectory

We now introduce the "Just-in-time" objective defined by constraint a). Using the previous description of the state and control trajectories (3), the problem is rewritten under a general fixed point formulation $x \leq f(x)$ which allows the resolution of control problem 1. The greatest estimated state trajectory X and its relevant state $x(k)$ are denoted X^+ and $x^+(k)$, respectively.

Theorem 3. *The greatest state and control trajectory of a Timed Event Graph (1) starting from $\underline{x}(k_s)$ and following specifications defined by a P-time EG (2) on horizon $[k_s, k_f]$ is the greatest solution of the following fixed point inequality system*

$$\begin{cases} X \leq D_h \backslash X \\ u(k) \leq B \backslash x(k) \; for \; k \in [k_s+1, k_f] \\ x(k) \leq [A \otimes x(k-1) \oplus B \otimes u(k)] \wedge C \backslash \underline{z}(k) \; for \; k \in [k_s+1, k_f] \\ x(k_s) \leq \underline{x}(k_s) \end{cases} \tag{4}$$

with condition $\underline{x}(k_s) \leq x^+(k_s)$.

Proof. From $D_h \otimes X \leq X$ and $B.u(k) \leq x(k)$, we deduce $X \leq D_h \backslash X$ and $u(k) \leq B \backslash x(k)$ on horizon $[k_s+1, k_f]$.

The constraints of the desired output $y \leq \underline{z}$ and $y(k) = Cx(k)$ can be introduced in the fixed-point formulation with $x(k) \leq C \backslash \underline{z}(k)$. So, $x(k) \leq [Ax(k-1) \oplus B.u(k)] \wedge C \backslash \underline{z}(k)$ on horizon $[k_s+1, k_f]$.

The constraint $x(k_s) = \underline{x}(k_s)$ can be written $x(k_s) \leq \underline{x}(k_s)$ and $\underline{x}(k_s) \leq x(k_s)$. Therefore, a condition is $\underline{x}(k_s) \leq x(k_s)$ □

If condition $\underline{x}(k_s) \leq x^+(k_s)$ is satisfied, then $\underline{x}(k_s) = x^+(k_s)$ and condition c) are satisfied. Therefore, the calculated state trajectory for $k \geq k_s$ is consistent with the past evolution $k \leq k_s$: In other words, the Timed Event Graph can follow calculated trajectory X^+ after k_s which obeys the specifications defined by the P-time Event Graph.

System (4) leads to a fixed-point formulation whose general form is such that $x \leq f(x)$. Containing (min, max, +) term $[A \otimes x(k-1) \oplus B \otimes u(k)] \wedge C \backslash \underline{z}(k)$, f is also a (min, max, +) function. It can be defined by the following grammar: $f = b, x_1, x_2, \ldots, x_n \mid f \otimes a \mid f \wedge f \mid f \oplus f$ where a, b are arbitrary real numbers ($a, b \in \mathbb{R}$). The effective calculation of the greatest control can be made by a classical iterative algorithm of Mc Millan and Dill [12] which particularizes the algorithm of Kleene to (min, max, +) expressions. The general resolution of $x \leq f(x)$ is given by the iterations of $x_i \leftarrow x_{i-1} \wedge f(x_{i-1})$ if the finite starting point is greater than the final solution. Here, number i represents the number of iterations and not the number of components of vector x. The general algorithm of Mc Millan and Dill [12] is known to be pseudo-polynomial in practice.

The aim of the following part is the extension of problem 1 to predictive control.

3 Predictive control (problem 2)

We present below the principle of the sliding horizon in predictive control and the general technique of the proposed approach. Another description can be found in [6] where the control of a Timed Event Graph without specification is described in standard algebra.

We assume that each event date of transition firing is available for current number of event k: at step $k = k_s$, $\underline{u}_{k_s}$ and $\underline{x}_{k_s}$ are known. A future control sequence $u(k)$ for $k \in [k_s+1, k_s+h]$ is determined such that this control is the optimal solution of the problem. The first element of the optimal sequence (here $u(k_s+1)$) is applied to the process. At the next number of event k_s+1, the horizon is shifted: at step k_s+1, the problem is updated with new information $\underline{u}_{k_s+1}$ and $\underline{x}_{k_{s+1}}$ and a new optimization is performed.

3.1 Principle of the proposed approach

After the calculation of state trajectory x^+ and control u at step k_s, condition c) $\underline{x}_{k_s} = x^+(k_s)$ must be checked in order to guarantee the coherence of the state trajectory between each iteration: this verification shows that future trajectory $k \geq k_s + 1$ is the extension of the past trajectory ($k \leq k_s$). The on-line comparison of the two vectors $\underline{x}_{k_s}$ and $x^+(k_s)$, is similar to the comparator of the closed-loop of classical continuous control which compares a desired trajectory and its measure: when the two data are equal, the objective is obtained. In our context, an optimal control is similarly found. Let us consider the different cases.

- If condition $\underline{x}_{k_s} = x^+(k_s)$ is satisfied, we can conclude that control problem 1 has a solution for data $\underline{z}$ and $\underline{x}_{k_s}$: there is an optimal control such that, starting from the current state $\underline{x}_{k_s}$, the Timed Event Graph can follow a trajectory obeying the specifications defined by a P-time Event Graph with a Just-in-time criteria.
- If $x_{k_s} \neq x^+(k_s)$, we can conclude that control problem 1 has no solution for data $\underline{z}$ and $\underline{x}_{k_s}$: the process presents some delays produced by a disruption of the process activity for instance. The Timed Event Graph cannot (provisionally) follow a trajectory obeying the constraints of the problem, i.e. the three conditions a), b) and c).

Consequently, at least a specification and/or the Just-in-time criteria, is not satisfied if we directly apply the calculated control of part 2.3 to the Timed Event Graph starting from the initial condition $\underline{x}_{k_s}$.

Therefore, the problem must be modified such that condition c) $\underline{x}_{k_s} = x^+(k_s)$ is satisfied. In this paper, we consider that the model of the Timed Event Graph cannot be modified. If we assume that the initial condition is the result of a past evolution, $\underline{x}_{k_s}$ is a datum of the problem and only condition a) and/or condition b) can be changed.

3.2 Predictive control objective

Suppose that the fulfillment of the specifications (condition b)) is essential. In consequence, the only possibility is to modify the just in time criteria of condition a) and to put the desired output back such that problem 1 presents a solution.

Therefore, an aim is the determination of a desired output such that control problem 1 presents a solution. Particularly, the state trajectory must start from current state $\underline{x}_{k_s}$. As a minimal desired output allows the limitation of the delays, the problem is to find the earliest desired output denoted z^- such that

- there is control such that its application to the Timed Event Graph generates a state trajectory which starts from the current state $\underline{x}_{k_s}$ (condition c))
- this state trajectory follows the additional specifications defined by the P-time Event Graph on horizon $[k_s+1, k_s+h]$ (condition b)).

This earliest desired output is a limit such that the Timed Event Graph cannot follow a lower trajectory satisfying the different constraints of the problem. Knowing earliest desired output z^-, the optimal approach of part 2.3 can be applied to modified desired output trajectory $z_m(k) = \underline{z}(k) \oplus z^-(k)$ for $k \in [k_s+1, k_s+h]$ such that this procedure yields a control which can be applied to the process. Therefore, condition a) is satisfied for the modified desired output z_m and the relevant calculated control is optimal for z_m.

Below, we characterize an arbitrary state trajectory obeying the specifications (condition b)). System (3) will be rewritten under a fixed point formulation $f(x) \leq x$ allowing the prediction problem of the earliest desired output z^-.

3.3 Prediction of the earliest desired output z^-

An arbitrary state trajectory obeying the specifications is now described with a fixed point form. From system (3), we deduce the following system

$$\begin{cases} X \geq D_h \otimes X \\ x(k) \geq B \otimes u(k) \text{ for } k \in [k_s+1, k_f] \\ x(k_s) = \underline{x}(k_s) \end{cases} \quad (5)$$

which allows the determination of an interesting desired output.

Indeed, this system is a fixed-point form $f(X) \leq X$ where f is a (max,+) function (if we assume that control u is known). Therefore, we can apply the concept of componentwise order to the desired output as follows:

The resolution makes the prediction of the earliest state trajectory $x^-(k)$ for $k \in [k_s+1, k_s+h]$ and so, of the earliest output trajectory $z^-(k) = C \otimes x^-(k)$. The modified desired output z_m is consequently obtained: $z_m(k) = \underline{z}(k) \oplus z^-(k)$ for $k \in [k_s+1, k_s+h]$.

We now characterize the set of trajectories of systems (3) and (5).

Property 1. Each trajectory of system (3) satisfies (5).

Proof. Immediate: As system (3) contains an additional constraint, any trajectory of this system satisfies relaxed system (5).

3.3.1 Earliest firing rule

As $x(k) \geq A \otimes x(k-1) \oplus B \otimes u(k)$ is already satisfied in (5), constraint $x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k)$ guarantees the earliest firing rule. In this part, we determine the conditions such that this last relation can be disregarded in the determination of the trajectory.

If we now consider only inequality $x(k) \geq B \otimes u(k)$ of system (5), the greatest control is obviously $u(k) = B \backslash x(k)$. This control law is considered below. We assume that no row of B is null.

Theorem 4. *A trajectory of (5) x satisfies (3) if this state trajectory x also satisfies condition $B \otimes (B \backslash x(k)) = x(k)$ for $k \in [k_s + 1, k_f]$.*

Proof. Let us prove that inequality $x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k)$ of system (3) is also satisfied in (5). The relaxation of $x(k) = A \otimes x(k-1) \oplus B \otimes u(k)$ for $k \in [k_s + 1, k_f]$ gives $x(k) \geq A \otimes x(k-1) \oplus B \otimes u(k)$ or, $x(k) \geq B \otimes u(k)$ and $x(k) \geq A \otimes x(k-1)$. This last inequality is expressed in (5) with $X \geq D_h \otimes X$. Let us suppose that an arbitrary trajectory denoted X' satisfies system (5). Particularly, X' satisfies $X' \geq D_h \otimes X'$ and so inequality $x'(k) \geq A \otimes x'(k-1)$ is satisfied. We want the Timed Event Graph defined by its state equation to follow given trajectory X' (neither earlier, nor later) by applying a specific control. For given $x'(k)$, a possible control is $u(k) = B \backslash x'(k)$ which is the greatest control satisfying inequality $x'(k) \geq B \otimes u(k)$. As $B \otimes u(k) = B \otimes (B \backslash x'(k)) = x'(k)$ and $x'(k) \geq A \otimes x'(k-1)$, we can deduce that $A \otimes x'(k-1) \oplus B \otimes u(k)$ is equal to $x'(k)$. Particularly, equality $x'(k) = A \otimes x'(k-1) \oplus B \otimes u(k)$ implies inequality $x'(k) \leq A \otimes x'(k-1) \oplus B \otimes u(k)$. This control guarantees the values of trajectory X' and consequently, the consistency of $X' \geq D_h \otimes X'$. □

Therefore, condition on state trajectory $B \otimes (B \backslash x(k)) = x(k)$ leads to a control satisfying $x(k) = B \otimes u(k)$ (and not only $x(k) \geq B \otimes u(k)$). The relation expressing the earliest firing rule $x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k)$ can be disregarded in the determination of the trajectory.

3.3.2 Structures 1 and 2

As above, we assume that no row of B is null. Moreover, we assume that each column of B contains a non-null element at the most (but a row can contain more than one element). With this structure of matrix B (denoted structure 1), there is a control such that $(B \otimes u(k))_i = x_i(k)$ for some i and condition $B \otimes (B \backslash x(k)) = x(k)$ is satisfied. Indeed, as a general result of residuation is $(A \backslash b)_i = \bigwedge_{j=1}^{m} A_{ji} \backslash b_j$ where A is an $m \times n$ matrix, we obtain $u_i = (B \backslash x(k))_i = B_{j'i} \backslash x_{j\prime}(k)$ for a specific row $j\prime$ and equality $B_{j'i} \otimes u_i = x_{j\prime}(k)$ is satisfied.

A more restrictive condition (structure 2) is as follows. We can also assume that each column and each row of B contain a non-null element at the most . This last assumption also corresponds to the hypothesis of "fully controlled" transitions i.e $B = I$. Therefore, the firing of each transition can be delayed in a control way and all the transitions are said to be controllable. Modeling of transportation network with timetable often leads to this assumption [2] [14] [3] [9]. Consequently, $B \otimes (B \backslash x(k)) = x(k)$ is always satisfied for any state trajectory and the control law is obviously $u(k) = x(k)$.

Using the Kleene star, a simple resolution of relaxed fixed-point form (5) in (max, +) algebra can now give the earliest state trajectory denoted X^- and so, the earliest output trajectory $z^-(k) = C \otimes x^-(k)$ where $x^-(k)$ is the earliest state vector for $k \in [k_s+1, k_f]$.

Let us now determine the earliest state trajectory X^- of the prediction problem.

Let $E = \begin{pmatrix} \underline{x}(k_s) \\ \varepsilon \\ \varepsilon \\ \dots \\ \varepsilon \\ \varepsilon \end{pmatrix}$ As constraint $x(k_s) = \underline{x}(k_s)$ can be written $x(k_s) \leq \underline{x}(k_s)$ and $\underline{x}(k_s) \leq x(k_s)$, the earliest state trajectory X^- is given by the resolution of $X \geq D_h \otimes X \oplus E$ with condition $\underline{x}(k_s) \geq x^-(k_s)$. The application of Kleene star by Theorem 1 gives the lowest solution $X^- = (D_h)^* \otimes E$ with condition $\underline{x}(k_s) \geq x^-(k_s)$. The control is given by $u^-(k) = B \backslash x^-(k)$.

3.3.3 Generalization

Condition on state trajectory $B \otimes (B \backslash x(k)) = x(k)$ leads to control $u(k) = B \backslash x(k)$ which produces the exact calculated state trajectory. The same result can be obtained with assumptions on the structure of matrix B (structure 2). In fact, this technique can be generalized as we can only consider only transitions whose dates obey the additional constraints and neglect the other ones. Using the previously calculated state trajectory, the application of control $u(k) = B \backslash x(k)$ must lead to the exact firing dates of the first class but can minimize the firing dates of the second class.

The structure of B is defined as follows: Divide the set of transitions TR into T_c defined below, and its complement T_{nc} with $TR = T_c \cup T_{nc}$; Set T_c is the set of transitions x_i such that there is a non-null coefficient A^-_{ij} or $A^=_{ij}$ or A^+_{ij} . Recall that $x_i(k+1) \geq A^-_{ij} \otimes x_j(k)$, $x_i(k+1) \geq A^=_{ij} \otimes x_j(k+1)$ and $x_i(k) \geq A^+_{ij} \otimes x_j(k+1)$, for $k \geq k_s$.

After reorganization of the rows and columns, matrix B is as follows: vector x_c (respectively x_{nc}) expresses the firing dates of transitions $x_i \in T_c$ (respectively $x_i \in T_{nc}$); With no conditions on B_{12} and B_{22},

$\begin{pmatrix} x_c(k) \\ x_{nc}(k) \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \otimes \begin{pmatrix} u_1(k) \\ u_2(k) \end{pmatrix}$ where B_{11} follows structure 2 and $B_{21} = \varepsilon$

.

So, the control can satisfy $x_c(k) = B_{11} \otimes u_1(k)$ with $x_c(k) \geq B_{12} \otimes u_2(k)$ and $x_{nc}(k) \geq B_{22} \otimes u_2(k)$.

3.4 Causality

Approaches based on a feedback defined by a Petri net are limited by the condition that the temporisation and initial marking of each added place are non-negative. The existence of a linear state feedback is discussed in [10] : this problem is reminiscent of difficulties of the theory of linear dynamical systems over rings [7]. Similarly, Model Predictive Control is limited by the following behavior. Let u_m (respectively, u^-) be the calculated control corresponding to the modified desired output z_m (respectively, earliest desired output z^-). In general, not all components of $x(k)$ are known at the same time and some of the components of $\underline{x}(k_s+j)$ for some $j>0$ might be known when the control u_m is calculated. In this part, we only consider the usual procedure used in model predictive control. Consequently, the application of control $u_m(k_s+1)$ must be made after the dates of $\underline{x}(k_s)$ which are data of the problem. So, each component $(u_m)_i(k_s+1)$ must be greater than the date of the possible application which is the addition (in standard algebra) of the maximum of components of $\underline{x}(k_s)$ and the CPU time T_{CPU}. More formally, the causality condition is $u_m(k_s+1) \geq F_u \otimes \underline{x}(k_s)$ where F_u is the $\otimes-$product of T_{CPU} and a full matrix of zeros with appropriate dimensions. Moreover, each calculated date $x_i(k_s+1)$ is the result of the application of the control and we can similarly write $x(k_s+1) \geq F_x \otimes \underline{x}(k_s)$ where F_x is defined as F_u with appropriate dimensions. As the complete analysis of these conditions needs an extensive study (see part 7.2 "Directions for future research" in [13]), we only give the following results.

Remark. If matrix B has no null row, then the first causality relation implies the second one. Indeed, $x(k_s+1) \geq B \otimes u_m(k_s+1) \geq B \otimes F_u \otimes \underline{x}(k_s) \geq F_x \otimes \underline{x}(k_s)$. Different authors give examples following this assumption on B (see [15], chapter 3 and 4 in [13] for instance).

The following result assumes that the predictive control approach gives $x^-(k_s) = \underline{x}(k_s)$.

Property 2. Suppose that the control procedure gives a control u^- such that $B \otimes u^-(k) = x^-(k)$ and $x^-(k_s) = \underline{x}(k_s)$. The causality conditions $x(k_s+1) \geq F_x \otimes \underline{x}(k_s)$ and $u_m(k_s+1) \geq F_u \otimes \underline{x}(k_s)$ are satisfied for any $\underline{x}(k_s)$ if $I \oplus A \oplus A^- \geq F_x$ and $B \backslash [(I \oplus A \oplus A^-)] \geq F_u$, respectively.

Proof. Let us consider the causality condition on state x. So, $x(k_s+1) \geq x^-(k_s+1) \geq (A \oplus A^-) \otimes x^-(k_s) \oplus B \otimes u^-(k_s+1) = (A \oplus A^-) \otimes x^-(k_s) \oplus x^-(k_s+1)$
$= (I \oplus A \oplus A^-) \otimes x^-(k_s) = (I \oplus A \oplus A^-) \otimes \underline{x}(k_s)$

As relation $x(k_s+1) \geq (I \oplus A \oplus A^-) \otimes \underline{x}(k_s)$ is always satisfied and assumption $I \oplus A \oplus A^- \geq F_x$ is taken, we can deduce that $x(k_s+1) \geq F_x \otimes \underline{x}(k_s)$.

Let us consider the causality condition on control u_m. So, $u_m(k_s+1) \geq u^-(k_s+1) = B \backslash x^-(k_s+1) \geq B \backslash [(I \oplus A \oplus A^-) \otimes \underline{x}(k_s)] \geq B \backslash [(I \oplus A \oplus A^-)] \otimes \underline{x}(k_s)$ (property f12 in [1]). If assumption $B \backslash [(I \oplus A \oplus A^-)] \geq F_u$ is taken, we can deduce that $u_m(k_s+1) \geq F_u \otimes \underline{x}(k_s)$. □

4 Conclusion

In this paper, we present a trajectory tracking control of Timed Event Graphs with specifications defined by a P-time Event Graph. The proposed approach presents the following characteristics.

The approach is completely defined in (max, +) algebra and does not use standard algebra. Except the algorithm of Kleene star, every used mathematical tool is present in the document which gives a complete description of the approach.

The two parts of the trajectory tracking control are: a) the optimal control; b) the updating of a desired output based on a prediction of the earliest possible desired output trajectory. These two parts use a special block tridiagonal matrix. This type of matrix is often encountered in numerical solutions of engineering problems (e.g. computational fluid dynamics, finite element method).

In the general case, a pseudo-polynomial algorithms gives the control and proposes an initial condition which must satisfy a condition of coherence of the state trajectory. This technique is sufficient when the control system can apply the calculated initial condition to the process. For different structures of matrix B, the proposed trajectory tracking control is composed of two polynomial algorithms. Trials show that the approach can be applied on-line for relatively important sizes of Event Graphs and horizon of calculation. It can offset unfavorable initial situations while the specifications are met.

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