A strategy for Estimation in Timed Petri nets

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   - Petri Nets
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Outline

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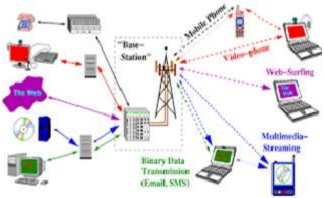
Discrete events systems



Petri Nets

Definition

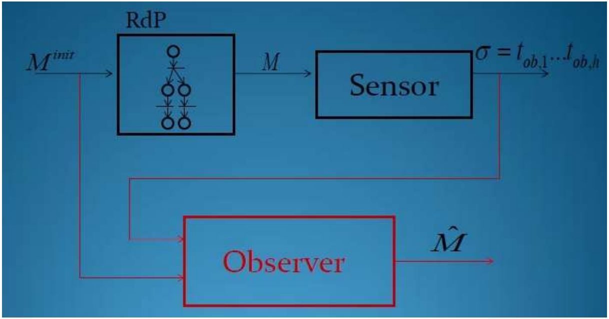
A Petri Net is a structure : $N =< P; Tr; W^+, W^- >$ where :

- P is a set of $|P|$ places and TR is a set of $|TR|$ transitions.
- the matrices W^+ and W^- are resp. the $|P| \times |TR|$ post and pre-incidence matrices over $\mathbb{N}$, where each row $l \in \{1, ..., |P|\}$ specifies the weight of the incoming and the outgoing arcs of the place $p_l \in P$.
- The incidence matrix is $W = W^+ - W^-$.

General principle of the estimation using Petri Nets

Problem

Estimate the states set of a Partially observed Petri Net supposing that the initial marking M^{init} is known.



1. Case of untimed Petri Nets
2. Case of timed Petri Nets

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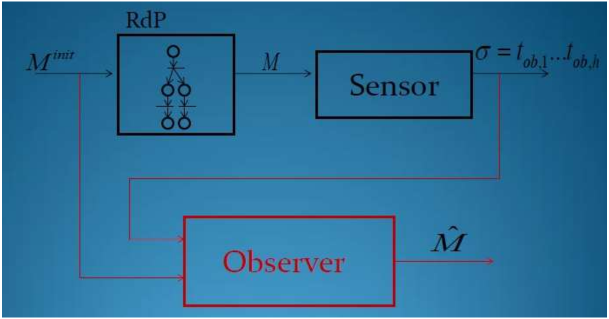

1. Case of untimed Petri Nets
2. Case of timed Petri Nets

Outline

Phase I : for untimed Petri nets

Notations

- $\pi(\sigma) \in \mathbb{N}^{|TR|}$ is the count vector where $\pi(\sigma)_i$ is the firing number of the transition i.
- The evolution of marking M is obtained by : $M = M^{init} + W.\pi(\sigma)$
- A transition is enable for M if $M \geq W^{-}(.,t)$ and gives the marking $M' = M + W(.,t)$.
- $M[\sigma \succ$ express that the transition sequence σ is allowed at M, and we write : $M[\sigma \succ M'$ to denote that the firing of σ gives M'.
- The transition set $TR = TR_{ob} \bigcup TR_{un}$ where :
  - TR_{ob} is the set of observable transitions, it is denoted $t_{ob,i}$
  - TR_{un} is the set of unobservable transitions, it is denoted $x_{un,i}$
- The incidence matrix W and M^{init} are supposed to be known.

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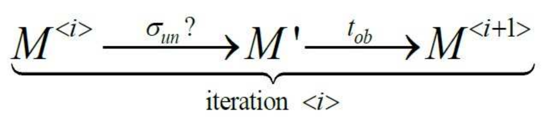
Problem

Given a sequence of observed firing transitions, we want to find the sequences of unobservable firing transitions denoted ($\sigma_{un} \in TR_{un}^{*}$)

Phase I : for untimed Petri nets

Principle

- If there is an observed firing of transition $t_{ob}^{<i>}$ for a current marking $M^{<i>}$, then there are an unobservable sequence $\sigma_{un}^{<i>}$ and a marking M' such that $M^{<i>}[\sigma_{un}^{<i>} \succ M'$ and $M'[t_{ob}^{<i>} \succ$. M' is the marking reached from the marking $M^{<i>}$ by firing the unobservable sequence $\sigma_{un}^{<i>}$ and this marking M' allows the observation of the firing of the observed transition $t_{ob}^{<i>}$.
- The marking $M^{<i+1>}$ used in step $< i+1 >$ is obtained from the firing of $t_{ob}^{<i>}$ at marking M'. Hence, $M'[t_{ob}^{<i>} \succ M^{<i+1>}$.



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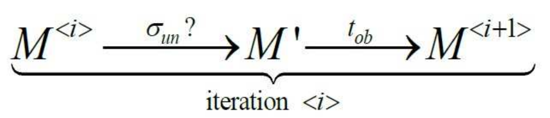

Estimation

- Estimate the count vectors of the firing of the unobserved transitions. The associated vector to the set TR_{un} is denoted x_{un}.
- If this vector is relevant to a possible sequence in the Petri nets, it will be called explanation vector.

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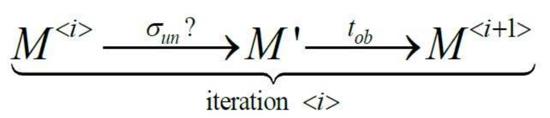

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Phase I : for untimed Petri nets

Definition

- Let $\sigma_{un}^{<i>}$ be a non observed sequence of transitions leading to the firing of the observed sequence $t_{ob}^{<i>}$, from a given marking M.
- An explanation vector of this sequence is the vector relevant to $\pi(\sigma_{un}^{<i>})$.

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Result

Algebraical modelisation of the explanation vectors allowing finding a transition t from a marking M, with :

$$\begin{cases} \text{Marking equation} \;\; M' = M + W_{un}.\hat{x}_{un} \\ \text{firing condition} \;\; M' \geq W^{-}_{obs}(.,t_{ob}^{<i>}) \\ \text{positivity constraint :} \;\; \hat{x}_{un} \geq 0 \end{cases}$$

Phase I : for untimed Petri nets

Polyhedron of candidate vectors

$$A.\hat{x}_{un} \leq b^{<i>} \; ; \; \hat{x}_{un} \in \mathbb{Z}^k$$

with :

$$A = \begin{pmatrix} -W_{un} \\ -I_{k \times k} \end{pmatrix} \text{ and } b^{<i>} = \begin{pmatrix} M^{<i>} - W^{-}_{obs}(., t^{<i>}_{ob}) \\ 0_{k \times 1} \end{pmatrix}$$

1. A depend on the Petri net structure
2. $b^{<i>}$ depend on $M^{<i>}$ and $t^{<i>}_{ob}$

Method of resolution

1. Off-line : Fourier Motzkin
2. On-line : Numerical computing algorithm

Polyhedron of candidate vectors

- The relevant solution is denoted :$S_{un}^{ad}(M,t_{ob}) = \{x_{un} \in \mathbb{R}^{\mathrm{n}} | A.x_{un} \leq b\}$
- We denote : $S_{un}^{nat}(M,t_{ob}) = S_{un}^{ad}(M,t_{ob}) \cap \mathbb{N}^{n}$ (nat= natural numbers)
- $S_{un}^{nat}(M,t_{ob})$ include the set of explanation vectors $E(M,t_{ob})$ for one iteration $< i >$ (firing condition of unobserved transitions is neglected).

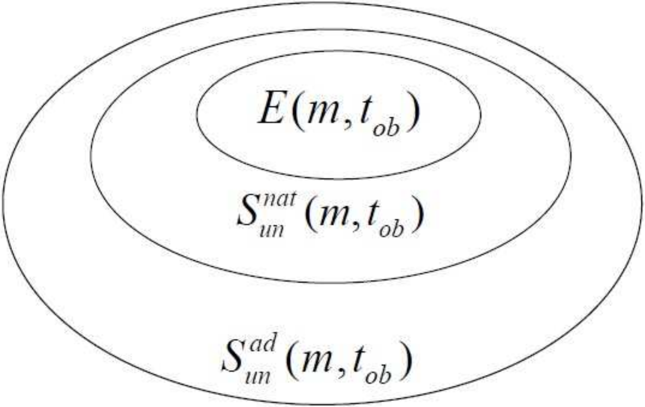
$$S_{un}^{ad}(M,t_{ob}) \supset S_{un}^{nat}(M,t_{ob}) \supset E(M,t_{ob}) \tag{1}$$

- $S_{un}^{ad}(M,t_{ob})$ et $S_{un}^{nat}(M,t_{ob})$ are called *candidate vectors* over $\mathbb{R}$or $\mathbb{N}$.
- If each transition i is associated to a non negative cost c_i, we can minimize the cost of the count vector $c.x_{un}$

$$\min c.x_{un} \text{ such that } A.x_{un} \leq b$$

- The obtained result is relevant to the minimal cost of the transition sequence if $x_{un} \in E(M,t_{ob})$.

Phase I : for untimed Petri nets

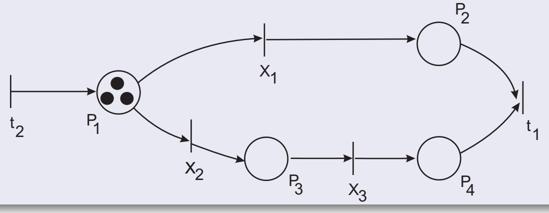


$$E(m,t_{ob}) \subset S_{un}^{nat}(m,t_{ob}) \subset S_{un}^{ad}(m,t_{ob})$$

Example

Description

- Consider the Petri net where : $TR_{un} = \{x_1, x_2, x_3\}$ and $TR_{obs} = \{t_1, t_2\}$. t_2 is a source transition, it firing leads to a possible evolution of the model.



- The incidence matrices relevant to TR_{un} and TR_{obs} are :

$$W_{un} = \begin{pmatrix} -1 & -1 & 0 \\ +1 & 0 & 0 \\ 0 & +1 & -1 \\ 0 & 0 & +1 \end{pmatrix} \text{ et } W^{-}_{ob} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

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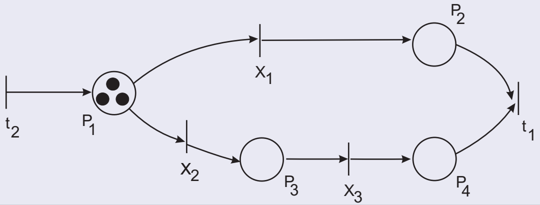

- The relations describing the problem are : $\begin{cases} x_1 + x_2 & \leq 3 \\ -x_1 & \leq -1 \\ -x_2 + x_3 & \leq 0 \\ -x_3 & \leq -1 \end{cases}$

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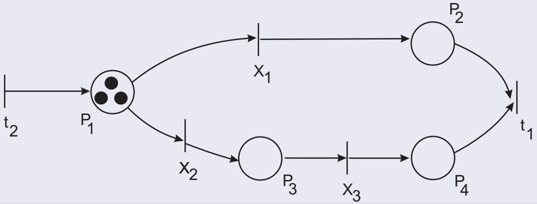

- the relevant polyhedron is : $\begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix} .x \leq \begin{pmatrix} 3 \\ -1 \\ 0 \\ -1 \end{pmatrix}$

Example

Count Vectors

- The set of the count vectors is : $S_{un}^{nat}(M^{init}, t_1) = \{\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^\top, \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}^\top, \begin{pmatrix} 1 & 2 & 1 \end{pmatrix}^\top, \begin{pmatrix} 1 & 2 & 2 \end{pmatrix}^\top\}$.

- Each solution is an explanation vector because TR_{un} is acyclic

- For the vector $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^\top$, more than three possible firing sequences : $x_1x_2x_3$, $x_2x_3x_1$ and $x_2x_1x_3$. etc.
- Pour $\begin{pmatrix} 2 & 1 & 1 \end{pmatrix}^\top$, more than six possibilities $x_1x_1x_2x_3$, $x_1x_2x_3x_1$, $x_1x_2x_1x_3$, $x_2x_1x_1x_3$, $x_2x_1x_3x_1$ et $x_2x_3x_1x_1$. etc.

Outline

Phase II : for timed Petri nets

Definition of counter

The firing number of a transition x before the time τ denoted $x(\tau)$.

Hypothesis

- The time is discrete ($\tau \in \mathbb{Z}$)
- The occurrence of events is synchronized by an external clock
- The events start at $\tau \geq 1, \Rightarrow x(\tau) = 0$ for $\tau \leq 0$.
- The sequences are non decreasing

Consequent

For a given transition, the occurrence of two events at $\tau = 3$ and $\tau = 5$ allows obtaining the sequence 0, 0, 0, 1, 1, 2, 2, 2, between $\tau = 0$ and $\tau = 7$.
$\Rightarrow x(\tau = 3) = 1$ and $x(\tau = 5) = 2$ and also $x(\tau = 4) = 1$ and $x(\tau = 7) = 2$.

Objective

Notations

- $\tau^{<i>}$ is the date of the last observed transition fired at the iteration $< i >$.
- x_{un} the candidate vector produced by the the phase without time.
- $x_{obs}(\theta)$ (resp., $x_{un}(\theta)$) the sub-vector of the state vector $x(\theta)$ such that the relevant transition belongs to the set TR_{obs} (resp., TR_{un}).

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objective

Check the candidate vector x_{un} estimated at the iteration $^{<i>}$ by analyzing the existence of an estimated sequence $x(\theta)$ for a timed Petri net ; if it exists, we conclude that x_{un} is an explanation vector.

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Hypothesis

- We associate to each place $p_l \in P$ a temporization $T_l \in \mathbb{N}$.
- The initial marking is the element l of the vector M^{init} denoted $(M^{init})_l$.
- A token remains in a place p_l at least a time T_l.
- the tokens of the initial marking are available at $t = 1$:

Model

Model

$$\sum_{i \in p_l^\bullet} x_i(\tau) \leq \sum_{i \in {}^\bullet p_l} x_i(\tau - T_l) + (M^{init})_l$$

Each weight 1 of $x_i(\tau - T_l)$ (resp., 1 de $x_i(\tau)$) is relevant to the weight of an incoming arc of a transition x_i to a place p_l (resp., th outgoing arc from a place p_l to an output transition x_i) which is equal to W^+_{li} (resp., W^-_{li}).

- If the temporization of each place is 0 or 1, we obtain

$$G \cdot \begin{pmatrix} x(\tau-1) \\ x(\tau) \end{pmatrix} \leq M^{init}$$

- The order of the matrix $G = [G_1 \; G_0]$ is $(|P| \times 2.|TR|)$.
- The matrices G_1 and G_0 verify :
  - The row l of G_0 or G_1 contains the unitary weight of the incoming arcs from places p_l with ($T_l = 0$ or 1), with a negative sign, generally $(-W^+)$.
  - The row l of G_0 contains the unitary weight of the outgoing arcs from places p_l, with a positive sign, generally (W^-).

Checking of the existence

- We denote $(\underline{x_{un}}, \underline{x_{obs}})$ the starting point and $(\overline{x_{un}}, \overline{x_{obs}})$ the final point.
- We can solve the problem with an arbitrary optimization c :

$$\min c.\mathbf{x}_{un} \text{ with } c > 0 \text{ such that } A \cdot \mathbf{x}_{un} \leq b \text{ over } \mathbb{Z}$$

- If the space is non empty, the minimization of $c.\mathbf{x}_{un}$ with $c > 0$ converges to a finite solution as the space is under-bounded by zero.
- This sequence satisfies the untimed problem.

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Example : continued

Checking

- Let us check the count vector $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^\top$.
- As we consider the first iteration in this example, the initial count vector is : $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^\top$.
- The problem is to determine a time sequence connection the two vectors.
- The time relations with time t are : $\begin{cases} x_1(t)+x_2(t) & \leq 3 \\ y_1(t+1) & \leq x_1(t) \\ x_3(t+1) & \leq x_2(t) \\ y_1(t+1) & \leq x_3(t) \end{cases}$ The resolution gives the following table :

t	0	1	2	3	4
y_1	0	0	0	0	1
x_1	0	0	0	1	1
x_2	0	0	1	1	1
x_3	0	0	0	1	1

- The relevant sequence is x_2 then x_1 and x_3 simultaneously.
- $\Rightarrow$ The count vector is an explanation vector.

Example : continued

Checking

- Let us check the vector $\begin{pmatrix} 1 & 2 & 2 \end{pmatrix}^\top$ The resolution gives the following table with time t :

t	0	1	2	3	4
y_1	0	0	0	0	1
x_1	0	0	0	1	1
x_2	0	0	1	2	2
x_3	0	0	0	1	2

- The relevant sequence is : x_2 then x_1, x_2 x_3 simultaneously, and finally x_3.
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Plan

Conclusion and perspectives

Conclusion

1. The general strategy can deal with timed or untimed models.
2. The first phase based on a simplification gives the candidate count vectors.
3. The introduction of time in Petri nets allows checking the candidate count vector and the verification of its logical aspect in the untimed Petri net : The technique of the second phase is to find an arbitrary sequence.
4. More precisely, in the second phase, if the space is non empty then :
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Perspectives

1. Consider The P-time Petri nets where the temporization T_l of a place p_l is replaced by the interval $[T_{lmin}, \; T_{lmax}]$.
2. Note that the P-time Petri net does not follow, obligatory, the minimum trajectory.

Thank you for your attention