Detection of changes by Observer in Timed Event Graphs and Time Stream Event Graphs

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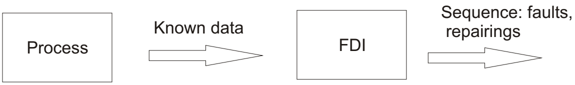
1. General objective : FDI in DEDS using Estimation



Outlines

1. General objective : FDI in DEDS using Estimation

2. Models :
Continuous systems or DEDS *

3. Approaches :
Parity Space, Estimation * or Identification
→Optimal observer

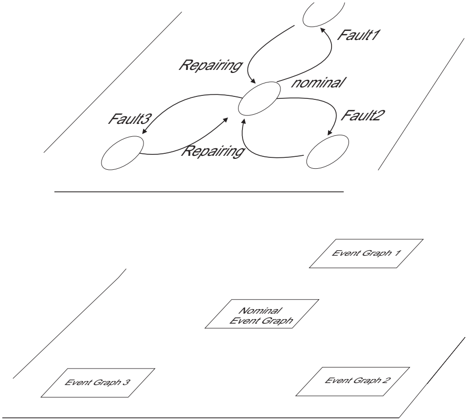
4. Steps of FDI :
Detection *, Isolation * and Diagnostic *

5. Conclusion and perspectives

2. Models : Description of the process and faults

- Process :

a set of Time Event Graphs supervised by a state machine.



- Faults or changes in the process :

Variation of a temporisation (deterioration of a machine, repairing) * ;

Loss or addition of a token (loss of a ressource, addition of a part) ;

Another graph (new schedule)*

Algebraic model

State in Petri Nets : marking, dater *, counter.

$x_i(k)$: date of the kth firing of transition i.

Interval model

$$f^-(x(k-1), x(k), u(k)) \leq x(k) \leq f^+(x(k-1), x(k), u(k)) \quad (1)$$

$$y(k) = C \otimes x(k) \text{ with } C_{ij} \in R_{\max}$$

Lower bound f^- is a (max,+) function.

Upper bound f^+ is a (min, max, +) function.

Timed Event Graphs

$x(k) \geq A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B \otimes u(k)$

$$\left\{ \begin{array}{l} x(k) \geq f^-(x(k-1), x(k), u(k)) = \\ A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B \otimes u(k) \\ x(k) \leq f^+(x(k-1), x(k), u(k)) = +\infty \end{array} \right. \quad (2)$$

Timed Event Graphs + earliest behavior

Usual assumption : there is no extra delay for firing transitions whenever tokens are all available.

$$x(k) = A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B \otimes u(k) \quad (3)$$

$$\begin{cases} x(k) \geq f^-(x(k-1), x(k), u(k)) = \\ A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B \otimes u(k) \\ x(k) \leq f^+(x(k-1), x(k), u(k)) = \\ A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B \otimes u(k) \end{cases} \quad (4)$$

Time Stream Event Graphs

`Semantic And`

If m_j the initial marking of the place p_j, the following expression can be written for each transition,

$$\bigoplus_{j \in P_i} (x_j(k - m_j) + \alpha_j) \leq x_i(k) \leq \bigwedge_{j \in P_i} (x_j(k - m_j) + \beta_j)$$

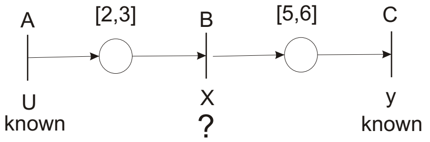
`Semantic Weak-And`

$$\bigoplus_{j \in P_i} (x_j(k - m_j) + \alpha_j) \leq x_i(k) \leq \bigoplus_{j \in P_i} (x_j(k - m_j) + \beta_j)$$

$$\begin{cases} x(k) \geq f^{-}(x(k-1), x(k), u(k)) = \\ A_1^{-} \otimes x(k-1) \oplus A_0^{-} \otimes x(k) \oplus B^{-} u(k) \text{ and} \\ x(k) \leq f^{+}(x(k-1), x(k), u(k)) = \\ \bigwedge_{i=1}^{i_{\max}} g_i(x(k-1), x(k), u(k)) \text{ where} \\ g_i(x(k-1), x(k), u(k)) = \\ A_{1,i}^{+} \otimes x(k-1) \oplus A_{0,i}^{+} \otimes x(k) \oplus B_i^{+} \otimes u(k) \end{cases} \tag{5}$$

3. Approach : Estimation ⟶ Principle

Observable transitions : u and y. Unobservable : x
Time u known, $x \in [u+2, u+3]$.
Time y known, $x \in [y-6, y-5]$.
Therefore, $x \in [\max(u+2, y-6), \min(u+3, y-5)]$
otherwise, model≠reality
Breakdown : the temporization associated to the second place equals 9.
If the real data are $u = 10$ and $y = 21$, $[max(12, 15), \min(13, 16)] = \emptyset$ as $15 \nleq 13 \rightarrow$ incoherence model/reality



Another point of view

$$\begin{cases} u+2 \leq x \text{ and } y-6 \leq x \\ x \leq \min(u+3, y-5) \end{cases}$$

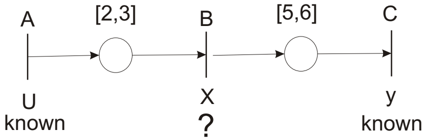
or , $\begin{cases} u \leq x-2 \text{ and } y \leq x+6 \\ x \leq \min(u+3, y-5) \end{cases}$.

Breakdown : the temporization associated to the second place equals 9.

If the real data are $u = 10$ and $y = 21$, the greatest estimate x is 13

$u \leq x - 2$ is satisfied $(10 \leq 13 - 2 = 11)$

but $y \leq x + 6$ $(21 \not\leq 13 + 6 = 19) \rightarrow$ incoherence model/reality



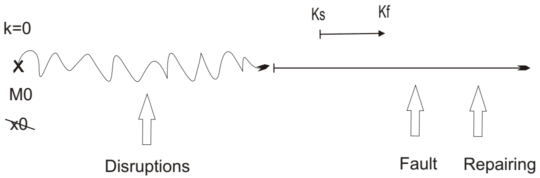
Formulation problem

- Arbitrary Time Event Graph (Timed EG, P-time EG, Time Stream EG with semantic rules "And" and "Weak-And") :

- No conditions of strongly connected graphs, bounded graphs, safe graphs
- Transitions $T = T_{ob} \cup T_{un}$ where T_{ob} is the set of observable transitions (global clock),
and T_{un} is the set of unobservable transitions.
$y_{ob}(k) = C_{ob} \otimes x(k)$ with $(C_{ob})_{ij} \in R_{\max}$

- The **objective** is to find the (least) upper bound of $x(k)$ knowing :

1. The nominal model (structure + initial marking + temporisations)
2. $y_{ob}(k)$ for k going from k_s to k_f with k_s and k_f



Resolution

Time Event Graph

$\Leftrightarrow$

Algebraic model

$\Leftrightarrow$

Fixed point problem :

$$x \leq f(x, x_{obs})$$

- Existence ?

$\rightarrow$ Spectral Theory : $X(f) \geq 0 \rightarrow$ Analogeous to Parity Space : $R(x_{obs})$

- Greatest state x^+?

$\rightarrow$ Algorithm $\rightarrow$ Analogeous to Observers

Transitions $T = T_{ob} \cup T_{un}$ with $T_{ob} = U_{ob} \cup Y_{ob}$ and $T_{un} = U_{un} \cup Y_{un}$

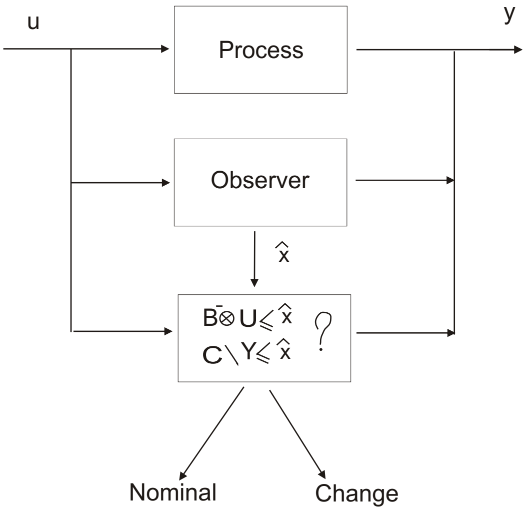
$$\begin{cases} A^- \otimes x(k-1) \oplus B^-_{ob} u_{ob}(k) \oplus B^-_{un} u_{un}(k) \leq x(k) \\ x(k) \leq \bigwedge_{i=1}^{j_1} (A^+_i \otimes x(k-1) \oplus B^+_{ob,i} \otimes u_{ob}(k) \oplus B^+_{un,i} \otimes u_{un}(k)) \\ y_{ob}(k) = C_{ob} \otimes x(k) \end{cases}$$

Theorem. For interval system, search the greatest state of the following inequality $x(\gamma) \leq h(x(\gamma))$ with

$$h(x(k)) = \begin{pmatrix} x(k+1) \wedge [A^- \backslash x(k+1)] \wedge [C_{ob} \backslash y_{ob}(k)] \wedge \\ [\bigwedge_{i=1}^{j_1} ((A^+_i)' \otimes x(k-1) \oplus (B^+_{ob,i})' \otimes u_{ob}(k))] \end{pmatrix} \quad (6)$$

with the constraints

$$\begin{cases} u_{ob}(k) \leq B^-_{ob} \backslash x(k) \\ y_{ob}(k) \leq C_{ob} \otimes x(k) \end{cases} \quad (7)$$


u
y
Process
Observer
x̂
B̄⊗U≤x̂
C\Y≤x̂ ?
Nominal
Change

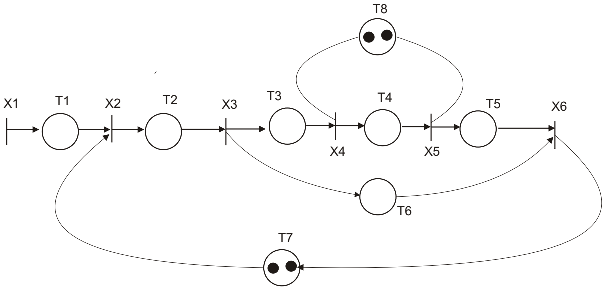
Algorithm 1

$$x_{i+1} \leftarrow f(x_i) \wedge x_i$$

Algorithm 2 (program in Scilab)
Step 0 (initialization) : $\mu(k_f) \leftarrow T; \lambda(k_f) \leftarrow T = +\infty$
Repeat
Step 1 : for $k = k_f$ to k_s ,
$\lambda(k) \leftarrow \mu(k) \wedge \lambda(k+1) \wedge [A^{-} \backslash \lambda(k+1)] \wedge [C_{ob} \backslash y_{ob}(k)]$
Step 2 : $\mu(k_s) \leftarrow \lambda(k_s)$
for $k = k_s + 1$ to k_f , $\mu(k) \leftarrow \lambda(k) \wedge f_i^{+}(\mu(k), u(k))$
Until $\lambda(k) = \mu(k)$ for $k_s \leq k \leq k_f$

4. Example

Model of the Petri Net



Nominal model M_1.

$$A = \begin{pmatrix} 0 & 0 & \varepsilon & \varepsilon \\ a & 0 & b & \varepsilon \\ \varepsilon & c & 0 & 0 \\ \varepsilon & \varepsilon & d & 0 \end{pmatrix}, B_1 = \begin{pmatrix} T_1 \\ \varepsilon \\ f \\ \varepsilon \end{pmatrix} \text{ and}$$

$$C_1 = \begin{pmatrix} T_2 + T_6 & \varepsilon & d + T_5 & \varepsilon \\ T2 & \varepsilon & \varepsilon & \varepsilon \end{pmatrix}$$

with $\varepsilon = -\infty$

Observable transitions : x_1, x_3 and x_6

An observer on the overall system can be developed using transitions x_1 and x_6.

Scenario of the simulation

Control : $x_1 = 0$ and for $i = 1$ to 69, $x_1(i+1) = x_1(i) + 1$.

Faults

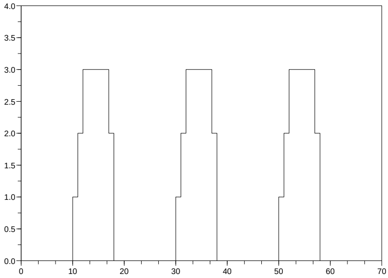
We successively consider a fault in zone 1 (T_2) and two faults in zone 2 (T_4).

1. The normal value of T_2 (zone 1) is 2 from $k = 1$ to 9
2. **Fault 1** : from $k = 10$ to 15, $T_2 = 12$.
3. T_2 is restored to its normal value from $k = 13$ to 70

1. The normal value of T_4 (zone 2) is 4 for $1 \leq k \leq 28$.
2. **Fault 2** : $T_4 = 13$ from $k = 29$ to 35.
3. T_4 is restored to its normal value 4 for $36 \leq k \leq 49$.
4. **Fault 3** : $T_4 = 15$ from $k = 50$ to 55
5. T_4 is restored to its normal value 4 for $56 \leq k \leq 70$.

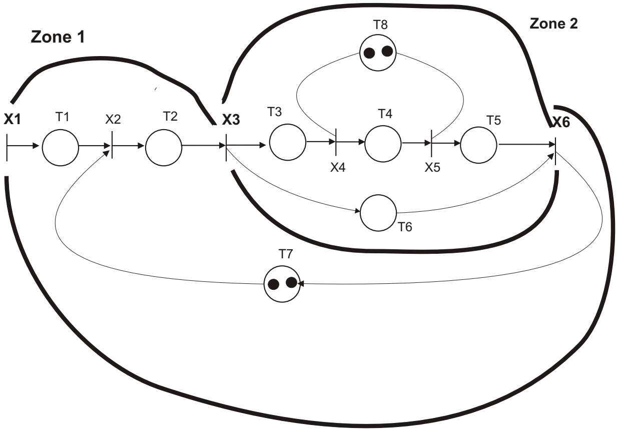
Detection

Each following curve gives the number of inconsistent relations function of the number of event from 1 to 70. The horizon of calculation of the observers is equal to 5.



Isolation

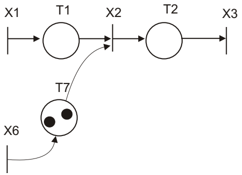
Observable transitions : x_1, x_3 and x_6



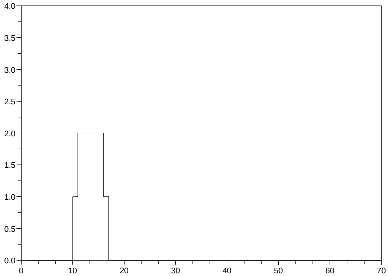
Zone 1 The observer uses observable transitions x_1, x_3 and x_6.

$$A^- = A^+ = \begin{pmatrix} 0 & 0 & \varepsilon \\ \varepsilon & 0 & T_7 \\ \varepsilon & \varepsilon & 0 \end{pmatrix} \quad B^- = B^+ = \begin{pmatrix} T_1 & \varepsilon \\ \varepsilon & \varepsilon \\ \varepsilon & 0 \end{pmatrix}$$

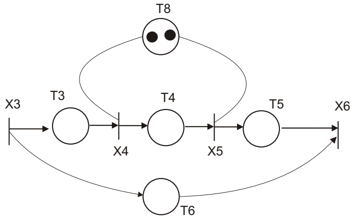
$$C^- = C^+ = \begin{pmatrix} T_2 & \varepsilon & \varepsilon \end{pmatrix}$$



Fault 1 : from $k = 10$ to 15, $T_2 = 12$.



Zone 2 The observer uses observable transitions x_3 and x_6.

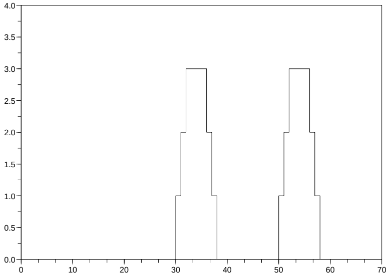


$$A^- = A^+ = \begin{pmatrix} 0 & 0 & \varepsilon \\ T_4 & 0 & \varepsilon \\ \varepsilon & \varepsilon & 0 \end{pmatrix} \quad B^- = B^+ = \begin{pmatrix} T_3 \\ \varepsilon \\ 0 \end{pmatrix}$$

$$C^- = C^+ = \begin{pmatrix} T_4 + T_5 & \varepsilon & T_6 \end{pmatrix}$$

Fault 2 : $T_4 = 13$ from $k = 29$ to 35.

Fault 3 : $T_4 = 15$ from $k = 50$ to 55



Diagnostic

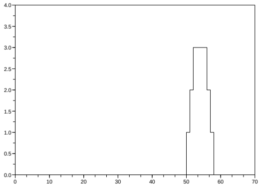
The following specific observer checks the coherence of the nominal model of the zone 2 but with T_4 between 4 and 14.

$$A^- = \begin{pmatrix} 0 & 0 & \varepsilon \\ T_4 & 0 & \varepsilon \\ \varepsilon & \varepsilon & 0 \end{pmatrix} \quad A^+ = \begin{pmatrix} 0 & 0 & \varepsilon \\ T_4 + 10 & 0 & \varepsilon \\ \varepsilon & \varepsilon & 0 \end{pmatrix}$$

$$C^- = \begin{pmatrix} T_4 + T_5 & \varepsilon & T_6 \end{pmatrix} \quad C^+ = \begin{pmatrix} T_4 + 10 + T_5 & \varepsilon & T_6 \end{pmatrix}$$

Fault 2 : $T_4 = 13$ from $k = 29$ to 35.

Fault 3 : $T_4 = 15$ from $k = 50$ to 55



5. Conclusion and perspectives

- Algorithm of optimal state estimation
- Two types of relations check the consistency of the estimate →Fault detection

→Check the correctness of the state estimation

- Subparts of the model →Isolation
- Small variations of model →Diagnostic

Natural perspectives :

- Parity Space in DEDS
- Counter approach →Marking estimation