Trajectory Tracking Control of a Timed Event Graph with Specifications Defined by a P-time Event Graph

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Objective

- Control of Timed Event Graphs with reference model defined by a P-time Event Graph.
- Criteria : use of **strongly**-polynomial algorithms (and not only polynomial (pseudo-polynomial) algorithms).
- Trajectory tracking control

Plan

1. Models
2. Control of Timed Event Graphs with reference model defined by a P-time Event Graph : admissible trajectory
3. Trajectory tracking control on a fixed horizon
4. Trajectory tracking control on a sliding horizon
5. Conclusion

Plan

Models

- $x_i(k)$ is the date of the k^{th} firing of transition x_i.
- $D = \overline{\mathbb{R}}_{max}$ = $(\mathbb{R} \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$
  with : $\oplus$: maximization and $\otimes$: addition
- The Kleene star is defined by : $A^* = \bigoplus_{i=0}^{+\infty} A^i$.

Theorem [bacelli92]

Consider equation $x = A \otimes x \oplus B$ and inequality $x \geq A \otimes x \oplus B$ with A and B in complete dioid D. Then, A^*B is the least solution to these two relations.

[bacelli92] F. Baccelli, G. Cohen, G.J. Olsder and J.P. Quadrat, Synchronization and Linearity. An Algebra for Discrete Event Systems, New York, Wiley, 1992.

Models

The Timed Event Graph

$$\begin{cases} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{cases} \tag{1}$$

The P-time Event Graph

$$\begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \geq \begin{pmatrix} \varepsilon & A^{+} \\ A^{-} & A^{=} \end{pmatrix} \otimes \begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \tag{2}$$

for $k \geq k_s$;

Plan

Control of Timed Event Graphs with reference model defined by a P-time Event Graph.

Objective

The objective is to calculate an admissible trajectory (control u and state x) on horizon $[k_s+1, k_f]$ such that the Timed Event Graph defined by

$$\begin{cases} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{cases} \quad (3)$$

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$$\begin{cases} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{cases} \quad (3)$$

satisfies the following conditions :

1. The state trajectory follows the model of the autonomous P-time Event Graph defined by :

$$\begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \geq \begin{pmatrix} \varepsilon & A^+ \\ A^- & A^= \end{pmatrix} \otimes \begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \quad (4)$$

for $k \geq k_s$;

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   for $k \geq k_s$;
2. The first state vector of the state trajectory $x(k)$ for $k \geq k_s$ is finite and is known vector $\underline{x}(k_s)$.

Characterization of an admissible trajectory

$$X = \begin{pmatrix} x(k_s)^t & x(k_s+1)^t & x(k_s+2)^t & \cdots & x(k_f-1)^t & x(k_f)^t \end{pmatrix}^t$$ and

$$D_h = \begin{pmatrix} \varepsilon & A^+ & \varepsilon & \cdots & \varepsilon & \varepsilon & \varepsilon \\ A \oplus A^- & A^= & A^+ & \cdots & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & A \oplus A^- & A^= & \cdots & \varepsilon & \varepsilon & \varepsilon \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \varepsilon & \varepsilon & \varepsilon & \cdots & A^= & A^+ & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \cdots & A \oplus A^- & A^= & A^+ \\ \varepsilon & \varepsilon & \varepsilon & \cdots & \varepsilon & A \oplus A^- & A^= \end{pmatrix}$$

Matrix D_h presents an original block tridiagonal structure.

Characterization of an admissible trajectory

Theorem

The state trajectories of a Timed Event Graph (3) starting from $\underline{x}(k_s)$ and following the specifications defined by a P-time Event Graph (4) on horizon $[k_s, k_f]$ satisfy the following system

$$\begin{cases} X \geq D_h \otimes X \\ x(k) \geq B \otimes u(k) \ \texttt{for} \ k \in [k_s+1, k_f] \\ x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k) \ \text{for} \ k \in [k_s+1, k_f] \\ x(k_s) = \underline{x}(k_s) \end{cases} \qquad (5)$$

Plan

Trajectory tracking control on a fixed horizon **(problem 1)**

Objective

The objective is to calculate the greatest control u on horizon $[k_s+1, k_f]$ such that its application to the Timed Event Graph defined by :

$$\begin{cases} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{cases} \tag{6}$$

Control on a fixed horizon (**problem 1**)

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1. **Condition (a)** : $y \leq \underline{z}$ knowing the trajectory of the desired output $\underline{z}$ on a fixed horizon $[k_s + 1, k_f]$ with $h = k_f - k_s \in \mathbb{N}$;

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2. **Condition (b)** : The state trajectory follows the model of the autonomous P-time Event Graph defined by

$$\begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \geq \begin{pmatrix} \varepsilon & A^+ \\ A^- & A^= \end{pmatrix} \otimes \begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix}. \tag{7}$$

for $k \geq k_s$;

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   for $k \geq k_s$;
3. **Condition (c)** :The first state vector of the state trajectory $x(k)$ for $k \geq k_s$ is finite and is known vector $\underline{x}(k_s)$.

Greatest trajectory

Theorem

The greatest state and control trajectory of a Timed Event Graph (3) starting from $\underline{x}(k_s)$ *and following specifications defined by a P-time EG (4) on horizon* $[k_s, k_f]$ *is the greatest solution of the following fixed point inequality system*

$$\begin{cases} X \leq D_h \backslash X \\ u(k) \leq B \backslash x(k) \text{ for } k \in [k_s+1, k_f] \\ x(k) \leq [A \otimes x(k-1) \oplus B \otimes u(k)] \wedge C \backslash \underline{z}(k) \text{ for } k \in [k_s+1, k_f] \\ x(k_s) \leq \underline{x}(k_s) \end{cases} \quad (8)$$

with condition $\underline{x}(k_s) \leq x^+(k_s)$.

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Principle of Predictive control

- The date of transition firing of event k : at step $k = k_s$, $\underline{u}_{k_s}$ and $\underline{x}_{k_s}$ are known ;
- A future control sequence $u(k)$ for $k \in [k_s + 1, k_s + h]$ is determined such that this control is the optimal solution of the problem ;
- The first element of the optimal sequence (here $u(k_s + 1)$) is applied to the process ;
- At the next number of event $k_s + 1$, the horizon is shifted : at step $k_s + 1$.The problem is updated with new information $\underline{u}_{k_s+1}$ and $\underline{x}_{k_{s+1}}$ and a new optimization is performed.

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Problem 2

- Condition $\underline{x}_{k_s} = x^+(k_s)$ is satisfied $\Rightarrow$ Control **problem 1** has a solution for data $\underline{z}$ and $\underline{x}_{k_s}$.
- Condition $x_{k_s} \neq x^+(k_s)$ $\Rightarrow$ Control **problem 1** has no solution for data $\underline{z}$ and $\underline{x}_{k_s}$.

Solution : modification of the just in time criteria of **Condition (a)**

The problem is to find the earliest desired output denoted z^{-} such that

- there is control such that its application to the Timed Event Graph generates a state trajectory which starts from the current state $\underline{x}_{k_s}$ (**Condition (c)**)
- this state trajectory follows the additional specifications defined by the P-time Event Graph on horizon $[k_s+1, k_s+h]$ (**Condition (b)**).

The optimal approach of the greatest trajectory can be applied to modified desired output trajectory $z_m(k) = \underline{z}(k) \oplus z^{-}(k)$ for $k \in [k_s+1, k_s+h]$.

Prediction of the earliest desired output z^-

An arbitrary state trajectory obeying the specifications is now described with a fixed point form. From system (5) which describe an admissible trajectory, we deduce the following system

$$\begin{cases} X \geq D_h \otimes X \\ x(k) \geq B \otimes u(k) \text{ for } k \in [k_s+1, k_f] \\ x(k_s) = \underline{x}(k_s) \end{cases} \quad (9)$$

Property

Each trajectory of system (5) which describe an admissible trajectory satisfies (9).

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The resolution makes the prediction of the earliest state trajectory $x^-(k)$ for $k \in [k_s+1, k_s+h]$ and so, of the earliest output trajectory $z^-(k) = C \otimes x^-(k)$.

Earliest firing rule

Assumption : no row of B is null.

Property

A trajectory of (9) x satisfies (5) if this state trajectory x also satisfies condition $B \otimes (B \backslash x(k)) = x(k)$ for $k \in [k_s + 1, k_f]$.

Therefore, condition on state trajectory $B \otimes (B \backslash x(k)) = x(k)$ leads to a control satisfying $x(k) = B \otimes u(k)$ (and not only $x(k) \geq B \otimes u(k)$). The relation expressing the earliest firing rule $x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k)$ can be disregarded in the determination of the trajectory.

Structure 1

Each column of B contains a non-null element at the most.
$\Rightarrow$ There is a control such that $(B \otimes u(k))_i = x_i(k)$ for some i and condition $B \otimes (B \backslash x(k)) = x(k)$ is partially satisfied.

Specific structure 1 : structure 2.

$B = I$ (more generally, a diagonal matrix BD) after a possible reorganization of the rows and the columns.
This assumption also corresponds to the hypothesis of "fully controlled" transitions : the firing of each transition can be delayed in a control way and all the transitions are said to be controllable.
$\Rightarrow$ The control law is obviously $u(k) = x(k)$ (more generally, $BD \otimes u(k) = x(k)$)
.

Generalization of structure

Two classes of internal transitions :

- Transitions whose dates obey the additional constraints. Set T_c is the set of transitions x_i such that there is a non-null coefficient A^{-}_{ij} or $A^{=}_{ij}$ or A^{+}_{ij} . Recall that $x_i(k+1) \geq A^{-}_{ij} \otimes x_j(k)$, $x_i(k+1) \geq A^{=}_{ij} \otimes x_j(k+1)$ and $x_i(k) \geq A^{+}_{ij} \otimes x_j(k+1)$, for $k \geq k_s$.
- The other ones : T_{nc}.

Using the previously calculated state trajectory, the application of control $u(k) = B \backslash x(k)$ must lead to the exact firing dates of the first class but can minimize the firing dates of the second class.

Generalization of structure

After reorganization of the rows and the columns, matrix B is as follows : vector x_c (respectively x_{nc}) expresses the firing dates of transitions $x_i \in T_c$ (respectively $x_i \in T_{nc}$) ;
$\begin{pmatrix} x_c(k) \\ x_{nc}(k) \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \otimes \begin{pmatrix} u_1(k) \\ u_2(k) \end{pmatrix}$ where B_{11} follows structure 2 and $B_{21} = \varepsilon$. There is no condition on B_{12} and B_{22}.
So, the control can satisfy $x_c(k) = B_{11} \otimes u_1(k)$ with $x_c(k) \geq B_{12} \otimes u_2(k)$ and $x_{nc}(k) \geq B_{22} \otimes u_2(k)$.

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Conclusion

- The approach is completely defined in (max, +) algebra.
- Introduction of a special block tridiagonal matrix.
- A **weakly-polynomial** algorithm (pseudo-polynomial algorithm of Mc Millan and Dill) or almost **strongly-polynomial** (E. Walkup and G. Borriello, Y. Cheng and D-Z Zheng...) gives the control and proposes an initial condition which must satisfy a condition of coherence of the state trajectory.
- The trajectory tracking control on a sliding horizon (for specific structures of matrix B), is given by **strongly-polynomial algorithms** : approximately $O(n^2)$ **:**
  1. The calculation time is independant on the magnitude of the coefficients (contrary to the best algorithms of LP (Karmarka :$O(n^{3.5}L)$, Gonzaga :$O(n^3L)$, ... where L is the number of bits)).
  2. Its does not need to start from an admissible solution (its determination is not an obvious problem in LP).
- The problem of causality is discussed.