Trajectory Tracking Control of a Timed Event Graph with Specifications Defined by a P-time Event Graph: On-line control and Off-line preparation.

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Plan

- Overview of the paper
- Technical part of the approach
- Conclusion

Topic

- Control of Timed Event Graphs
- Trajectory tracking control on a sliding horizon
- Reference model defined by a P-time Event Graph.

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- Trajectory tracking control on a sliding horizon
- Reference model defined by a P-time Event Graph.

Objective of the paper

- Model predictive control is an on-line approach which allows flexibility and the consideration of different classes of systems
- But it needs efficient algorithms : Otherwise, the approach is limited to small systems and small horizons.
- The control calculation must obey the calculated dates and not postpone the application of the control.

Previous work based on Linear Programming (Bart De Schutter, Ton van den Boom,...)

The best **generic** algorithms of Linear Programming are **weakly** polynomial algorithms :

- Karmarkar :$O(n^{3.5}L)$, Gonzaga : $O(n^3L)$, ... where L is the number of bits.
- Simplex : The complexity is exponential in the worst case even if this algorithm is relatively good in the average.

– less efficient than the algorithms of graph theory
– limitation of the sizes of the horizons and the systems.

Previous work based on (max, +) algebra (R. D. Katz)

- Modification of the initial Timed Event Graph
- Calculation of the maximal set of the initial states
- Based on the algorithm of P. Butkovic and G. Hegedus

The complexity is doubly exponential (paper IEEE-TAC) $\Rightarrow$ small systems even if the approach needs an horizon [k, k+1] only.

Crucial point

The structure of the matrices present **specific characteristics** :

- the matrices are sparse and contains many rows with two non-null entries (1 and -1) at the most.
- The matrices are close to the ingoing/outgoing incidence matrices of the fundamental marking relation.

– The goal is to make the most of these specific structures of the systems.

First answer : the (max, +) algebra with lattice

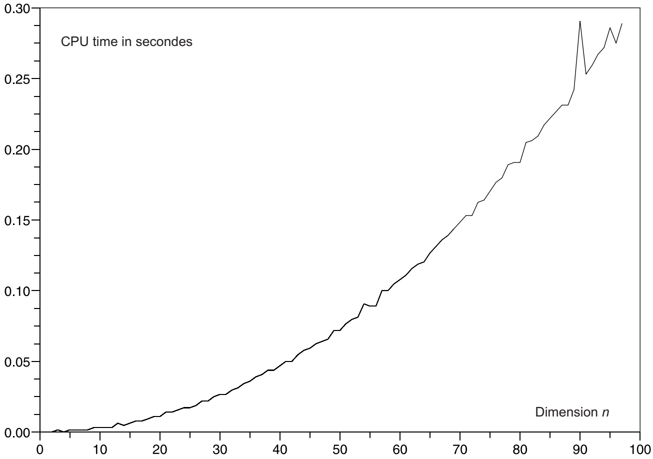
- Use of the efficient algorithms of graph theory : **strongly** polynomial algorithms (Kleene star, matrix residuation,...)
- Calculation of a unique solution and not a complete set of solutions : use of lattice.

Second answer

The predictive control is made on a sliding horizon : the horizon is slightly moved back at each step and the control is calculated.
The idea is to **avoid the repetition of the same calculations** at each step which can be costly in terms of time. Before the application of the on-line control, a preparation can contain these calculations allowing a reduction of the complexity of the on-line procedure.
The initial on-line procedure is divided into two steps :

- Off-line preparation : calculation of a large scale (max, +) matrix
- On-line control

Conclusion : Numerical results



Conclusion : Numerical results

- Consideration of important systems (until 97 transitions) for long horizons (h =50).
- The initial CPU time (approximately 2000 secondes or 33 minutes) is replaced by a new on-line procedure which only needs 0.28 secondes : the ratio is 1 to 7000.

Technical part

1. Models
2. Control of Timed Event Graphs with reference model defined by a P-time Event Graph : admissible trajectory
3. Trajectory tracking control on a fixed horizon
4. Trajectory tracking control on a sliding horizon

Preliminary remarks

- $x_i(k)$ is the date of the k^{th} firing of transition x_i.
- $\overline{\mathbb{R}}_{max}$ = $(\mathbb{R} \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$
  with : $\oplus$: maximization and $\otimes$: addition
- The Kleene star is defined by : $A^* = \bigoplus_{i=0}^{+\infty} A^i$.

Theorem [bacelli92]

Consider equation $x = A \otimes x \oplus B$ and inequality $x \geq A \otimes x \oplus B$ with A and B in complete dioid D. Then, $A^* \otimes B$ is the least solution to these two relations.

[bacelli92] F. Baccelli, G. Cohen, G.J. Olsder and J.P. Quadrat, Synchronization and Linearity. An Algebra for Discrete Event Systems, New York, Wiley, 1992.

Control of Timed Event Graphs with reference model defined by a P-time Event Graph (problem 1).

Objective

The objective is to calculate an admissible trajectory (control u and state x) on horizon $[k_s+1, k_f]$ such that the Timed Event Graph defined by

$$\begin{cases} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{cases} \quad (1)$$

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satisfies the following conditions :

1. The state trajectory follows the model of the autonomous P-time Event Graph defined by :

$$\begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \geq \begin{pmatrix} A^{=} & A^{+} \\ A^{-} & A^{=} \end{pmatrix} \otimes \begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \quad (2)$$

   for $k \geq k_s$;

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   for $k \geq k_s$;
2. The first state vector of the state trajectory $x(k)$ for $k \geq k_s$ is finite and is the known vector $\underline{x}(k_s)$.

Trajectory tracking control on a fixed horizon **(problem 2)**

Objective

The objective is to calculate the greatest control u on horizon $[k_s+1, k_f]$ such that its application to the Timed Event Graph defined by :

$$\left\{ \begin{array}{r} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{array} \right. \quad (3)$$

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3. **Condition (c)** :The first state vector of the state trajectory $x(k)$ for $k \geq k_s$ is finite and is the current known vector $\underline{x}(k_s)$.

Notations : X and D_h.

$$X = \begin{pmatrix} x(k_s)^t & x(k_s+1)^t & x(k_s+2)^t & \cdots & x(k_f-1)^t & x(k_f)^t \end{pmatrix}^t$$ and

$$D_h = \begin{pmatrix} A^{=} & A^{+} & \varepsilon & \cdots & \varepsilon & \varepsilon & \varepsilon \\ A \oplus A^{-} & A^{=} & A^{+} & \cdots & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & A \oplus A^{-} & A^{=} & \cdots & \varepsilon & \varepsilon & \varepsilon \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \varepsilon & \varepsilon & \varepsilon & \cdots & A^{=} & A^{+} & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \cdots & A \oplus A^{-} & A^{=} & A^{+} \\ \varepsilon & \varepsilon & \varepsilon & \cdots & \varepsilon & A \oplus A^{-} & A^{=} \end{pmatrix}$$

Matrix D_h presents an original block tridiagonal structure.

Greatest trajectory

Theorem

The greatest state and control trajectory of a Timed Event Graph (1) starting from $\underline{x}(k_s)$ and following specifications defined by a P-time EG (2) on horizon $[k_s, k_f]$ is the greatest solution of the following fixed point inequality system

$$\begin{cases} X \leq D_h \backslash X \\ u(k) \leq B \backslash x(k) \text{ for } k \in [k_s+1, k_f] \\ x(k) \leq [A \otimes x(k-1) \oplus B \otimes u(k)] \wedge C \backslash \underline{z}(k) \text{ for } k \in [k_s+1, k_f] \\ x(k_s) \leq \underline{x}(k_s) \end{cases} \tag{5}$$

with condition $\underline{x}(k_s) \leq x^+(k_s)$.

- (min, max, +) fixed point problem $\longrightarrow$ Algorithms of Mc Millan and Dill, Walkup and Boriello, Cheng and Zheng, ...

Principle of Predictive control (problem 3)

- The date of transition firing of event k : at step $k = k_s$, $\underline{u}_{k_s}$ and $\underline{x}_{k_s}$ are known ;
- A future control sequence $u(k)$ for $k \in [k_s + 1, k_s + h]$ is determined such that this control is the optimal solution of the problem ;
- The first element of the optimal sequence (here $u(k_s + 1)$) is applied to the process ;
- At the next number of event $k_s + 1$, the horizon is shifted : at step $k_s + 1$.The problem is updated with new information $\underline{u}_{k_s+1}$ and $\underline{x}_{k_{s+1}}$ and a new optimization is performed.

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Problem 2

- Condition $\underline{x}_{k_s} = x^+(k_s)$ is satisfied $\Rightarrow$ Control problem has a solution for data $\underline{z}$ and $\underline{x}_{k_s}$.
- Condition $x_{k_s} \neq x^+(k_s)$ $\Rightarrow$ Control problem has no solution for data $\underline{z}$ and $\underline{x}_{k_s}$.

Solution : modification of the just in time criteria of **Condition (a)**

The problem is to find the earliest desired output denoted z^{-} such that

- there is control such that its application to the Timed Event Graph generates a state trajectory which starts from the current state $\underline{x}_{k_s}$ (**Condition (c)**)
- this state trajectory follows the additional specifications defined by the P-time Event Graph on horizon $[k_s+1, k_s+h]$ (**Condition (b)**).

The optimal approach of the greatest trajectory can be applied to the modified desired output trajectory $z_m(k) = \underline{z}(k) \oplus z^{-}(k)$ for $k \in [k_s+1, k_s+h]$.

Characterization of an admissible trajectory (problem 1)

Theorem

The state trajectories of a Timed Event Graph (1) starting from $\underline{x}(k_s)$ and following the specifications defined by a P-time Event Graph (2) on horizon $[k_s, k_f]$ satisfy the following system

$$\begin{cases} X \geq D_h \otimes X \\ x(k) \geq B \otimes u(k) \texttt{ for } k \in [k_s+1, k_f] \\ x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k) \text{ for } k \in [k_s+1, k_f] \\ x(k_s) = \underline{x}(k_s) \end{cases} \quad (6)$$

Remark. The space solution is not an **inf**-semilattice.

Prediction of the earliest desired output z^-

An arbitrary state trajectory obeying the specifications is now described with a fixed point form. From system (6) which describe an admissible trajectory, we deduce the following system

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Property

Each trajectory of system (6) which describe an admissible trajectory satisfies (7).

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The resolution makes the prediction of the earliest state trajectory $x^-(k)$ for $k \in [k_s+1, k_s+h]$ and so, of the earliest output trajectory $z^-(k) = C \otimes x^-(k)$.

Earliest firing rule

Assumption : no row of B is null.

Property

A trajectory of (7) x satisfies (6) if this state trajectory x also satisfies condition $B \otimes (B \backslash x(k)) = x(k)$ for $k \in [k_s + 1, k_f]$.

Therefore, condition on state trajectory $B \otimes (B \backslash x(k)) = x(k)$ leads to a control satisfying $x(k) = B \otimes u(k)$ (and not only $x(k) \geq B \otimes u(k)$). The relation expressing the earliest firing rule $x(k) \leq A \otimes x(k-1) \oplus B \otimes u(k)$ can be disregarded in the determination of the trajectory.

Conclusion

- The approach is completely defined in (max, +) algebra.
- The trajectory tracking control on a sliding horizon (for specific structures of matrix B), is given by **strongly-polynomial algorithms** : approximately $O(n^2)$ **:**
  1. The calculation time is independant on the magnitude of the coefficients (contrary to the best algorithms of LP (Karmarka :$O(n^{3.5}L)$, Gonzaga :$O(n^3L)$, ... where L is the number of bits)).
  2. Its does not need to start from an admissible solution (its determination is not an obvious problem in LP).
- An important part of the calculations is made off-line in a **preparation**. A consequence is that the approach can be applied to **long horizons** (h =50) and **important systems** (until 97 transitions) with a reduced CPU time.
- The initial CPU time (approximately 2000 secondes or 33 minutes) is replaced by a new on-line procedure which only needs 0.28 secondes : the ratio is 1 to 7000.

Perspectives

- Improvement of the off-line preparation : Kleene star of a tri-diagonal matrix.

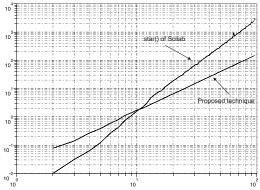

FIGURE: Off-line preparation : Kleene star of a tri-diagonal matrix.

- Analysis of the condition $x(k) = B \otimes u(k)$ and Generation of a corrector such that this condition is always satisfied.
- Generation of a causal control.