Causality phenomenon and Compromise Technique for Predictive Control of Timed Event Graphs with Specifications Defined by P-time Event Graphs

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13 mai 2014

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- Reference model defined by a P-time Event Graph
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Main advantages of predictive control

- General framework
- Flexibility
- Application to a large class of models

Motivation of the study : the causality phenomenon.

1. The future **calculated** control is applied on-line.
2. The application of the first calculated control must be made after the (standard) addition of :

- the last past date of the known state and
- the computer time.

Objective

- To analyze this causal constraint for predictive control
- To propose some techniques.

Main parts

1. Optimal control problem and fixed point algorithm (IEEE-TAC 10, JDEDS 12)
2. Causality phenomenon
3. Compromise technique and consistency of the constraints
4. Conclusion

Control Problem

Variable $x_i(k)$ is the date of the k^{th} firing of the transition x_i
For one control step, the objective of this paper is the determination of the greatest control u (with respect to the componentwise order) on an arbitrary horizon $[k_s+1, k_f]$ with $h = k_f - k_s \in \mathbb{N}$ such that its application to the Timed Event Graph defined by

$$\left\{ \begin{array}{r} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = C \otimes x(k) \end{array} \right. \quad (1)$$

for $k \geq k_s$, satisfies the following conditions :

Control problem

1. $y \leq \underline{z}$ knowing the trajectory of the desired output $\underline{z}$;
2. The state trajectory follows the model of the P-time Event Graph defined by
$$\begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix} \geq \begin{pmatrix} A^{=} & A^{+} \\ A^{-} & A^{=} \end{pmatrix} \otimes \begin{pmatrix} x(k) \\ x(k+1) \end{pmatrix}; \qquad (2)$$
3. The initial value of the state trajectory $\underline{x}(k_s)$ is finite and is a known vector. This “ non-canonical ” initial condition is the result of a past evolution of a process. $\longrightarrow$ Observers if unknown (JDEDS 12, IEEE-TASE 14)

Relations on horizon

The relations of the Timed Event Graph can be rewritten under the following classical form on horizon $[k_s, k_f]$.

$$X = \Omega_h \otimes x(k_s) \oplus \Psi_h \otimes U \quad (3)$$

where $h = k_f - k_s$,
$X = \begin{pmatrix} x(k_s+1)^t & x(k_s+2)^t & \cdots & x(k_f-1)^t & x(k_f)^t \end{pmatrix}^t$ (t : transposed),
$U = \begin{pmatrix} u(k_s+1)^t & u(k_s+2)^t & \cdots & u(k_f-1)^t & u(k_f)^t \end{pmatrix}^t$, Ω_h is a column of h blocks $(\Omega_h)_i = A^i$ for $i = 1$ to h and Ψ_h is a h x h matrix of blocks $(\Psi_h)_{i,j}$ for $i, j \in \{1, 2, \ldots, h\}$ where $(\Psi_h)_{i,j} = A^{i-j} \otimes B$ for $i > j$ and ε otherwise.

$$\begin{cases} \begin{pmatrix} x(k_s) \\ X \end{pmatrix} \geq D_h \otimes \begin{pmatrix} x(k_s) \\ X \end{pmatrix} \\ x(k_s) = \underline{x}(k_s) \end{cases} \quad (4)$$

In this system, we consider :

- the additional constraints for $k \geq k_s$ and
- an autonomous Timed Event Graph defined by the inequality $x(k) \geq A \otimes x(k-1)$ coming from the state equation (relaxation of the earliest firing rule), starting from $x(k_s) = \underline{x}(k_s)$.

Tridiagonal matrix

Matrix D_h is a tridiagonal matrix of blocks $(D_h)_{i,j}$ for $i, j \in \{1, 2, \ldots, h+1\}$

$$D_h = \begin{pmatrix} A^{=} & A^{+} & \varepsilon & \cdots & & \\ A \oplus A^{-} & A^{=} & A^{+} & \cdots & & \\ \varepsilon & A \oplus A^{-} & A^{=} & \cdots & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & \cdots & A^{=} & A^{+} \\ & & & \cdots & A \oplus A^{-} & A^{=} \end{pmatrix}$$

Fixed point form and algorithm

Extended state vector $\bar{x} = \left(\; (x(k_s))^t \quad (X)^t \; \right)^t$ which expresses the complete state trajectory.
Let $(\bar{x})^+$ be the greatest estimate of state trajectory and
$F =$
$\left(\; \underline{x}(k_s)^t \quad (C \backslash \underline{z}(k_s+1))^t \quad (C \backslash \underline{z}(k_s+2))^t \quad \cdots \quad (C \backslash \underline{z}(k_f))^t \; \right)^t$.

Theorem

The greatest state and control trajectory of the control problem is the greatest solution of the following fixed point inequality system

$$\begin{cases} \bar{x} \le D_h \backslash \bar{x} \wedge F \\ U \le \Psi_h \backslash X \\ X \le \Omega_h \otimes x(k_s) \oplus \Psi_h \otimes U \end{cases} \tag{5}$$

with condition $\underline{x}(k_s) \le x^+(k_s)$. ■

Algorithm 1

$(\overline{x})^1 = \begin{pmatrix} (x^1(k_s))^t & (X^1)^t \end{pmatrix}^t$ and $(\overline{x})^2 = \begin{pmatrix} (x^2(k_s))^t & (X^2)^t \end{pmatrix}^t$ correspond to intermediate values.

Algorithm 1

Step 0 (initialization) : $\langle i \rangle \leftarrow \langle 0 \rangle$; $(\overline{x})^2 \leftarrow F$

Repeat

- $\langle i \rangle \leftarrow \langle i+1 \rangle$ (numbering of the iteration)
- Step 1 : $(\overline{x})^1 \leftarrow D_h^* \backslash (\overline{x})^2$
- Step 2 : $U \leftarrow \Psi_h \backslash X^1$
- Step 3 : $(\overline{x})^2 \leftarrow (\overline{x})^1 \wedge \begin{pmatrix} +\infty \\ \Omega_h \otimes x^1(k_s) \oplus \Psi_h \otimes U \end{pmatrix}$

until $X^1 = X^2$. ■

Analysis of Algorithm 1

The following result shows the minimization of the state trajectory and the property that the state equation is satisfied **at the end of each iteration**.

Property

$X' \leq X^1$ *and* $X^2 = X'$ *where* $X' = \Omega_h \otimes x^1(k_s) \oplus \Psi_h \otimes U$. ■

Causality phenomenon

The application of the control $u(k_s+1)$ must be made after the dates of $\underline{x}(k_s)$ which are the data of the problem.

$$\bigoplus_{i\in[1,n]} \underline{x}_i(k_s)\otimes T_{comp} \leq \bigwedge_{i\in[1,card(u)]} u_i(k_s+1) \tag{6}$$

where $\bigoplus_{i\in[1,n]} \underline{x}_i(k_s)\otimes T_{comp}$ is the availability date of the calculated control. We can also rewrite this causality condition under the form of a (max, +) inequality

$$G_u \otimes \underline{x}(k_s) \leq u(k_s+1) \tag{7}$$

where G_u is the $\otimes-$product of T_{comp} and a full matrix of zeros ($e=0$) with appropriate dimensions.

Compromise technique (technique 3)

$$G_u \otimes \underline{x}(k_s) \leq u(k_s + 1) \quad (8)$$

At each step of Algorithm 1, we have

- The **increase** of the availability date $G_u \otimes \underline{x}(k_s)$ (produced by the increase of the computer time T_{comp} which depends on the number of iterations)
- and, the **decrease** of the dates of the control $u(k_s + 1)$ (minimized by the fixed point algorithm at each iteration) .

$\longrightarrow$ Compromise between :

- the search of optimality of the ideal problem and
- the on-line application which considers the causality phenomenom and the used computer.

Technique 3

In addition, each iteration of the algorithm proposes a control which generates an output satisfying the desired output (expressed by vector F) and the state equation (Property 1). Moreover, a subset of the constraints is satisfied.

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Approach

Since the causal inequality $G_u \otimes \underline{x}(k_s) \leq u(k_s + 1)$ must be satisfied for the current calculated control, the approach is to reduce the CPU time by **stopping Algorithm 1 before the convergence**.

$\Rightarrow$The control is suboptimal as the convergence is not waited and only a subset of the constraints is satisfied. This approach is possible if the crucial constraints are guaranteed :

- The satisfaction of safety regulations for a grade crossing is obligatory
- Non-crucial constraints in the food industry

Constraint consistency

The following theorem highlights an important case where Algorithm 1 gives the final state trajectory at the first iteration $\langle 1 \rangle$.

Theorem

The state trajectory $(\bar{x})^2$ (generated by step 3) satisfies the system

$$\begin{cases} X = \Omega_h \otimes x(k_s) \oplus \Psi_h \otimes U \\ \begin{pmatrix} x(k_s) \\ X \end{pmatrix} \geq D_h \otimes \begin{pmatrix} x(k_s) \\ X \end{pmatrix} \end{cases} \quad (9)$$

when

$$\begin{pmatrix} I & \varepsilon \\ \varepsilon & \Psi_h \end{pmatrix} \otimes \begin{pmatrix} x^0(k_s) \\ U \end{pmatrix} = (\bar{x})^1. \quad (10)$$

Moreover, $(\bar{x})^2 = (\bar{x})^1$. ■

Existence problem

The problem is now to check the solution existence of $\overline{u} \in \mathbb{R}^{\overline{q}}$ in the equality

$$\overline{B} \otimes \overline{u} = \overline{x} \text{ for any } \overline{x} \in \mathbb{R}^{\overline{n}} \text{ satisfying } \overline{x} \geq \overline{A} \otimes \overline{x} \quad (11)$$

with the following notation : $\overline{B} = \begin{pmatrix} I & \varepsilon \\ \varepsilon & \Psi_h \end{pmatrix}$, $\overline{u} = \begin{pmatrix} x(k_s) \\ U \end{pmatrix}$, $\overline{x} = \begin{pmatrix} x(k_s) \\ X \end{pmatrix}$, $\overline{A} = D_h$, $\overline{n}$=card($\overline{x}$) and $\overline{q}$=$card(\overline{u})$.

Complete validity of the constraints

$$\overline{B} \otimes \overline{u} = \overline{x} \text{ for any } \overline{x} \in \mathbb{R}^{\overline{n}} \text{ satisfying } \overline{x} \geq \overline{A} \otimes \overline{x} \tag{12}$$

Theorem

The greatest vector $\overline{u} = \overline{B} \backslash \overline{x}$ *satisfies the above system if and only if* $\overline{B} \otimes (\overline{B} \backslash \overline{A}^*) = \overline{A}^*$. ■

- The relevant algorithm is strongly polynomial (convergence of the algorithm at the first iteration).

Partial consistency

$$A \otimes x = b \qquad (13)$$

where $A \in \mathbb{R}_{\max}^{mxn}$, $b \in \mathbb{R}^m$. The relevant set of solutions over $\mathbb{R}$ is denoted $\mathcal{S}$. The set of indexes for the rows $I = \{1, .., m\}$ and for the columns $J = \{1, .., n\}$ as A is a (mxn) matrix. x^+ is the greatest solution to $A \otimes x \leq b$.
For $j \in J$, $V_j = \{i \in I$ such that $A_{i,j}$ is finite and $x_j^+ = A_{i,j} \backslash b_i\}$.

Lemma

(R.A. Cuninghame-Green, K. Zimmermann, P. Butkovic)
$x \in \mathcal{S}$ *if and only if* $x \leq x^+$ *and* $\bigcup_{j \in J | x_j = x_j^+} V_j = I$. ∎

Result

$$\overline{B} \otimes \overline{v} = \overline{A}^* \ , \tag{14}$$

Corresponding to column $j \in J$ of $\overline{B}$ and column $k \in K$ of $\overline{A}^*$, $V_{j,k}$ is defined by $V_{j,k} = \{i \in I$ such that $\overline{B}_{i,j}$ is finite and $\overline{v}^+_{j,k} = \overline{B}_{i,j} \backslash (\overline{A}^*)_{i,k}\}$.

Property

The system (14) has a solution $\overline{v}$ *if and only if* $\overline{v} \leq \overline{v}^+$ *and* $\bigcap_{k \in K} \bigcup_{j|\ \mathbf{v}_{j,k} = \mathbf{v}^+_{j,k}} V_{j,k} = I$.

The set $\bigcap_{k \in K} \bigcup_{j \in J} V_{j,k}$ *gives the rows of (14) where the equality holds for* $\overline{v} = \overline{v}^+$. ■

Generalization

$$\overline{B} \otimes \overline{u} = \overline{x} \text{ for any } \overline{x} \in \mathbb{R}^{\overline{n}} \text{ satisfying } \overline{x} \geq \overline{A} \otimes \overline{x} \tag{15}$$

Property

For the greatest vector $\overline{u}=\overline{B} \setminus \overline{x}$,

- *each equality* $\overline{B}_{i,.} \otimes \overline{u} = \overline{x}_i$ *with* $i \in I_g = \bigcap_{k\in K} \bigcup_{j\in J} V_{j,k}$ *is always satisfied for any* $\overline{x} \in Im\overline{A}^*$.
- *each equality* $\overline{B}_{i,.} \otimes \overline{u} = \overline{x}_i$ *with* $i \in I_{p,k} = \bigcup_{j\in J} V_{j,k}$ *is always satisfied when* $\overline{x} \in Im(\overline{A}^*)_{.,k}$ *for a given* $k \in K$.
- *An equality* $\overline{B}_{i,.} \otimes \overline{u} = \overline{x}_i$ *with* $i \in I_p = \{i \in \bigcup_{k\in K} \bigcup_{j\in J} V_{j,k}$ *and* $i \notin I_g\}$ *is possibly satisfied when* $\overline{x} \in Im\overline{A}^*$. ■

Example

$$\overline{B} \otimes \overline{u} = \overline{x} \text{ for any } \overline{x} \in \mathbb{R}^{\overline{n}} \text{ satisfying } \overline{x} \geq \overline{A} \otimes \overline{x} \tag{16}$$

Each entry $\Delta_{i,k}$ of the following $\overline{n}$ x $\overline{n}$ symbol matrix gives the row index $i \in \bigcup_{j \in J} V_{j,k}$ for each column $(\overline{A}^*)_{.,k}$ where symbol $=$ expresses that the relevant equality $\overline{B}_{i,.} \otimes \overline{v}_{.,k} = (\overline{A}^*)_{i,k}$ is satisfied while symbol $<$ shows that $\overline{B}_{i,.} \otimes \overline{v}_{.,k} < (\overline{A}^*)_{i,k}$ is obtained.

$\Delta =$

$$\begin{pmatrix}
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= & = & = & = & = & = & = & = & = & = & = & = \\
< & < & < & < & < & < & < & < & < & < & < & <
\end{pmatrix} \quad (17)$$

The analysis of the rows of this matrix Δ gives $I_g = \{1, 2, 3, 5, 8, 11\}$ and $I_p = \{4, 7, 10\}$.
Therefore, the equality $\overline{B} \otimes \overline{u} = (\overline{A}^*)_{.,k}$ does not hold for any $k \in K$ but the equality for the rows $i \in I_g$ is guaranteed.

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- Technique 3 can be applied when only a subset of (crucial) constraints must be satisfied. The suboptimal solution is the result of a compromise between the availability time of application of the control and the calculated dates. Condition $y \leq \underline{z}$ and the model of the Timed Event Graph are satisfied.

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- When the causality phenomenon forbids the application of the calculated control, the proposed techniques enlarge the class of the processes where the predictive control can operate.