State Estimation and Detection of Changes in Time Interval Models

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Abstract

This paper presents an estimation approach for Time Event Graphs such as P-Time Event Graphs and Time Stream Event Graphs. It is assumed that the nominal behavior is known and that transitions are partitioned as observable and unobservable transitions. The technique is applied to the detection of changes which are (possibly small) finite variations of dynamic models compared to this nominal behavior. The detected changes provide indications that can be used in future maintenance operations. Using the algebra of dioids, the approach uses a receding-horizon estimation of the greatest state and analyzes the consistency of the data.

Index Terms

P-Time Event Graphs; Time Stream Event Graphs; Observer; Estimation; Detection; (min, max, +) Functions.

I. Introduction

In this paper, we consider time interval models which can describe a large class of Time Event Graphs where time intervals can be associated with places, transitions and arcs (Timed Event Graphs, P-time Event Graphs, T-time Event Graphs, Time Stream Event Graphs [4] [5]). The operations of maximization, minimization and addition define the lower and upper bound constraints [10] on the trajectories.

Our objective is the detection of changes such as a (possibly slow) unexpected variation of the holding times of the tasks. These changes can be generated by a deterioration in the process or

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preventive maintenance not modeled by the Petri Net. These changes are clearly non-drastic. Note that the diagnostic approaches in Discrete-Event Systems generally consider *drastic* failures such as ”stuck-closed valve” or ”short-circuited sensor” [13]. These changes can also be produced by switching among different modes of operation in a switching system: In [11], the problem is the determination of the active mode at each time point based on the input/output data.

A possible method of change detection can be based on the identification of the holding times of Timed Event Graphs [24]. The assumption that all transitions are observable is taken (all transition firing times are measurable). The residuals between the estimated values and the given nominal values can be used for fault detection [25]. In our paper, we also consider unobservable transitions and a more general class of Event Graphs described by (min, max, +) functions (see section IV). Another study [18] also considers observers but with a different aim: its objective is the exact determination of some components of the state vector for an autonomous (no input) Timed Event Graph. With the aim of developing a geometric approach, the authors consider a (max, +) system of the form $x(k+1) = A \otimes x(k)$ where some entries of A are unknown but belong to intervals. In this paper, we consider a more general class of models as this model corresponds to the semantic ”Weak-And” of the Time Stream Event Graphs [10]. A second important difference is that we consider that the vector of the initial condition or the first state date on the sliding horizon, is not a datum of the problem but is *unknown*. The working hypothesis of an unknown initial condition is usual for the observers for continuous systems [19] [20].

In this paper, the technique of change detection is based on the on-line analysis of the coherence between a state estimate and known data through specific relations. The general principle of our approach is the transposition in Petri Nets of the classical principle used in fault detection of continuous-variable models (Parity Space [9], Observers [15], Identification [26]). Remember that a large class of fault detection approaches relies on the different types of continuous-variable models while another class considers Discrete-Event Systems such as Petri Nets [17] [1][12] [16] and Automata [22] [14] [23]. In this paper, changes (or faults) are considered as variations of dynamic models compared to a Petri Net which only describes the normal behavior. With the aim of illustrating the efficiency of the approach, the simulation of the examples follows the graph of the non-faulty model, but contains a change which is a (possibly small) variation of a holding time. Firing sequences are observed by an on-line observer. The test of specific relations allows the analysis of the coherence of the data. If the data are incoherent with the model of nominal behavior, a change is detected. If the data are coherent with the model of a fault case, the relevant fault is diagnosed. If the fault is repaired, the new data will be coherent with the nominal model again. The possible existence of faults corresponds to modes of the process that can be in normal (normal mode if the system functions properly) and faulty situations (fault mode 1, fault mode 2,...). The appearance of a fault leads to a fault mode and its repair to the nominal state. Each mode corresponds to a specific model (M_0, M_1, M_2,...) which is a specific Petri net. Therefore, each observer relevant to a Petri net, checks the mode associated with the Petri net considered. The reasoning for the determination of the modes of operation in a switching system is identical and can be easily deduced.

In this paper, the estimation uses a sliding horizon principle. This means that after computation of the state estimate on horizon $\{k_s, \ldots, k_f\}$, the horizon shifts to the next sample, and the estimation of the state estimate on horizon $\{k_s+1, \ldots, k_f+1\}$ is restarted using known information of the new horizon. The interest in such estimation methods stems from the possibility of dealing with a limited amount of data, instead of using all the information available from the beginning. As a consequence, the procedure can detect faults only when the window covers the occurrence of the faults which can be intermittent. It should also be noted that the state estimate allows a prediction of the future trajectory of the process which can be useful in predictive control.

In the sequel, no assumption is made on the Event Graph which can be non-strongly connected. The consistency of the models is beyond the scope of this paper and we assume their consistency (see [8] for more details). No assumption is made on an observer structure: The structure will be deduced from the results obtained in this paper.

The paper is organized as follows: We firstly present the motivations and the principle of the proposed approach. Then, the interval model in the (min, max, +) algebra is briefly presented. Based on a fixed point approach, we describe an observer allowing the detection of changes in the process. With the aim of clearly illustrating the approach, we consider only a simple Timed Event Graph including some uncertainties on the temporizations (more complex Time Event Graphs can be found in [6] and [7]). Calculations have been made with Scilab. These parts are preceded by notations and by a brief review of previous results.

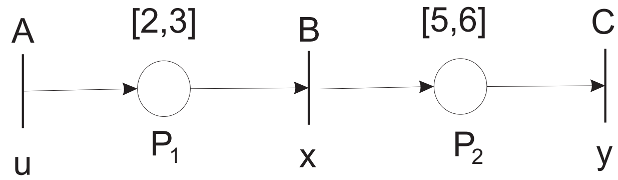

Fig. 1. Example 1: A simple Timed Event Graph

II. Example 1 and Principle of the proposed approach

Many transportation systems can be described as Petri Nets which can be expressed in the (max, +) or (min, max, +) algebra [2]. In railway and urban transportation systems, state variables represent departure and arrival times of vehicles at some stations. In urban transportation systems, buses must follow the given timetables but are subject to environmental variations such as bottlenecks and failures.

Let us consider example 1 which is a simple Event Graph composed of two places (Fig. 1). The first place p_1 describes the journey of a vehicle from town A to town B which lasts between 2 and 3 hours. The second place p_2 represents the following journey from B to C with a temporization between 5 and 6 hours. If the departure time u of the vehicle is known, the arrival at the intermediate town B can obviously be estimated: $[u+2, u+3]$. Symmetrically, if the arrival time y at town C is known, the arrival at the intermediate town B can also be estimated: $[y-6, y-5]$. Consequently, the estimate of the date associated with B can be calculated by a *forward-backward* approach: $[\max(u+2, y-6), \min(u+3, y-5)]$. If this interval is empty, we can conclude that there is a break-down, an unpredicted event, or more generally a poor description of the current situation: In this case, a change detection can be made.

But the model can equivalently be described by: $x \leq \min(u+3, y-5)$ and two additional relations $u \leq x-2$ and $y \leq x+6$. The first inequality has a fixed point form $x \leq f(x)$ which allows the estimation of the greatest value of x. This value can be introduced in the two remaining inequalities: A change is detected when at least one remaining inequality is not satisfied. Assume that a breakdown of the vehicle between towns B and C entails that the temporization associated with the second place is equal to 9: this temporization does not belong to $[5, 6]$. A possible output is $y = 21.5$ for $u = 10$ in this faulty situation. The greatest estimate x is 13 ($x \leq \min(10+3, 21.5-5)$). Inequality $u \leq x-2$ is satisfied $(10 \leq 13-2 = 11)$ but inequality $y \leq x + 6$ is not satisfied ($21 \nleq 13 + 6 = 19$): it shows an incoherence between the used model and the evolution of the current trajectory.

Therefore, this elementary Example 1 shows that the greatest estimate can be calculated with a fixed point form. Moreover, the system contains relations that are not used in the estimation. These redundant relations can check the coherence of the data (estimate, input and outputs).

III. Preliminaries

With the view of keeping the usual (max,+) algebraic notation, maximization, minimization and addition operations are denoted respectively $\oplus$, $\wedge$ and $\otimes$. In this section, we shall review a few basic theoretical notions about dioids. For more extensive presentations, the reader is invited to consult the following reference: [2].

A dioid D is an idempotent semi-ring (operation $\oplus$ is idempotent, that is $a \oplus a = a$). The set $\mathbb{R} \cup \{-\infty\}$ provided with the maximum operation denoted $\oplus$ and the addition denoted $\otimes$ is an example of a dioid which is usually noted $\mathbb{R}_{max}$ = $(\mathbb{R} \cup \{-\infty\}, \oplus, \otimes)$. The neutral elements of $\oplus$ and $\otimes$ are represented by $\varepsilon = -\infty$ and $e = 0$, respectively. Let $\overline{\mathbb{R}}_{max} = (\mathbb{R} \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$ and Γ be a subset of vectors over $\overline{\mathbb{R}}_{max}$. The partial order denoted $\leq$ is defined as follows: $v \leq w \iff v \oplus w = w$ for all $v, w \in \Gamma$. The partial order is also a componentwise order which allows the comparison of any pair of vectors (v, w) i.e. $v \leq w \iff v_i \leq w_i$, for each component i. In the paper, this concept is applied to control and state trajectories. The element $v \in \Gamma$ is called the greatest element or maximum element if and only if $w \leq v$ for all $w \in \Gamma$. In other words, the greatest element is greater than any other element of the subset: (see Part 4.3.1 of [2] for more details). If this greatest element exists, it is unique since the existence of two different maximum elements v and w implies $w \leq v$ and $v \leq w$. Let $v \wedge w$ denote the lower bound of v and w.

A mapping f is monotone or isotone if $x \leq y$ implies $f(x) \leq f(y)$. Let f: $E \rightarrow F$ be an isotone mapping, where (E, $\leq$) and (F,$\leq$) are ordered sets. Mapping f is said to be residuated if for all $y \in F$, the least upper bound of subset $\{x \in E \mid f(x) \leq y\}$ exists and belongs to this subset. The corresponding mapping, denoted $f^\sharp(y)$ is called the residual of f. When f is residuated, $f^\sharp$ is the only isotone mapping, such that $f \circ f^\sharp \leq Id_F$ and $f^\sharp \circ f \geq Id_E$ where Id_F and Id_E are identity mappings. Mapping $x \in (\overline{\mathbb{R}}_{max})^n \mapsto A \otimes x$, defined over $\overline{\mathbb{R}}_{max}$ is residuated [2] and the left $\otimes-$ residual of b by A is denoted by: $A \backslash b = \max\{x \in (\overline{\mathbb{R}}_{max})^n$ such that

$A \otimes x \leq b\}$. Moreover, $(A\backslash b)_i = \bigwedge_{j=1}^{m} A_{ji}\backslash b_j$ where A is an $m \times n$ matrix.

IV. Time interval models

Definition 1. [3] A (min, max, +) function of type $(n, 1)$ is any function $f : \mathbb{R}^n \longrightarrow \mathbb{R}^1$, which can be written as a term in the following grammar: $f = x_1, x_2, \ldots, x_n \mid f \otimes a \mid f \wedge f \mid f \oplus f$ where a is an arbitrary real number ($a \in \mathbb{R}$). The vertical bars separate the different ways in which terms can be recursively constructed. A (min, max, +) function of type (n, m) is any function f: $\mathbb{R}^n \longrightarrow \mathbb{R}^m$, such that each component f_i is a (min, max, +) function of type $(n, 1)$. The set of functions (min, max, +) of the type (n, m) is denoted $F(n, m)$.

The evolution of the system is described by the following model, called an "interval model" or an "interval descriptor system", where f^- and f^+ are (min, max, +) functions. Variable $x_i(k)$ is the date of the k^{th} firing of internal transition denoted x_i. In a similar way, variable $u_i(k)$ is the date of the k^{th} firing of input transition (transitions which have no input place in the Event graph) which is denoted u_i.

$$f^-(x(k-1), x(k), u(k)) \leq x(k) \leq f^+(x(k-1), x(k), u(k)) \quad (1)$$

for $k \geq 1$.

Since the type of the system is defined by the types of functions f^- and f^+, we can characterize the model by the following pair (type of f^-, type of f^+) which defines different types of system. Type ((min, max, +), (min, max, +)) naturally represents the more general mathematical case. In this paper, the following assumption is made: For each interval model, the lower bound denoted f^- is a (max, +) function. Therefore, we obtain the following well-known term used in the model of Timed Event Graphs where multiplication $\otimes$ has priority over addition $\oplus$.

$$f^-(x(k-1), x(k), u(k)) = A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B_0^- \otimes u(k) \quad (2)$$

with $(A_1^-)_{ij}$, $(A_0^-)_{ij}$ and $(B_0^-)_{ij} \in \mathbb{R}_{\max}$. The entry $(A_1^-)_{i,j} = a \geq 0$ (respectively, $(A_0^-)_{i,j} = a \geq 0$) corresponds to the lower bound a of the temporization of the place linking its ingoing transition x_j to its outgoing transition x_i and having a unitary (respectively, null) initial marking. The entry $(B_0^-)_{i,j} = a \geq 0$ corresponds to the lower bound a of the temporization of the place linking its input transition u_j to its outgoing transition x_i and having a null initial marking. More details can be found in [2] [6].

Trajectory $x(k)$ is nondecreasing ($x(k) \leq x(k+1)$) if condition $A_1^- \geq Id$ holds.

Interval model (1) with the assumption (2) will be considered in the sequel. The studies [10] [6] [7] show that this model with the assumption (2) can describe Timed Event Graphs, P-time Event Graphs, T-time Event Graphs and Time Stream Event Graphs [4] [5] for synchronization rules "And" and "Weak-And" (the component f_i^+ associated to a transition x_i is a $(min, +)$ function for synchronization rule "And" and a $(max, +)$ function for synchronization rule "Weak-And"). The initial marking of these Event Graphs is supposed to be known.

In this paper, the interval model can follow a trajectory, which is not extremal (not the earliest or the latest trajectory). State trajectory x and the *initial condition* $x(0)$ (the first date of firing of the transitions) are unknown. We assume that the interval model in the non-faulty mode is consistent, that is, the interval model can describe a trajectory whose components are finite. We also suppose that this model is known on the observation horizon.

V. Change detection using estimation

The transitions of the set of models are partitioned as $TR = TR_{ob} \cup TR_{un}$ where TR_{ob} is the set of observable transitions, and TR_{un} is the set of unobservable transitions [12]. So, the dates of firing of observable transitions are known by definition. With the aim of simplifying the presentation, every input transition is assumed to be observable and we simply keep notation u for the relevant date vector. Output transitions (transitions which have no output place) can be observable or not. We can write for the date vector of observable output transitions, denoted y_{ob} : $y_{ob}(k) = C_{ob} \otimes x(k)$ with $(C_{ob})_{ij} \in \mathbb{R}_{\max}$.

The aim of the paper is as follows: Let k_s and k_f be the numbers of initial and final events of horizon $\{k_s, \ldots, k_f\}$. The first objective of the observer is to find the greatest state estimate $\widehat{x}(k)$ knowing the dates of firing of observable transitions on horizon $\{k_s, \ldots, k_f\}$. The second objective is the change detection.

A. *Fixed point formulation*

The choice of the form of the interval model (assumption (2)) causes function f^- to be residuated. Relations (1) and (2) can consequently, be reformulated as a fixed point problem.

Theorem 1: For interval model (1) with assumption (2), the problem of the greatest estimate of $x(k)$ for $k \geq 1$ can be written as follows: Search for the greatest state estimate $\widehat{x}$ of the following inequality

$$x(k) \leq g\left(x(k-1), x(k), x(k+1), u(k)\right) \tag{3}$$

with $g\left(x(k-1), x(k), x(k+1), u(k)\right) = [C_{ob} \backslash y_{ob}(k)] \wedge [A_0^- \backslash x(k)] \wedge [A_1^- \backslash x(k+1)] \wedge f^+(x(k-1), x(k), u(k))$

with two additional constraints

$$\begin{cases} B_0^- \otimes u(k) \leq \widehat{x}(k) \\ y_{ob}(k) \leq C_{ob} \otimes \widehat{x}(k) \end{cases} \tag{4}$$

Moreover, the interval model (1) with assumption (2) is equivalent to the system composed of (3) and (4).

Proof

Relation $A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B_0^- \otimes u(k) \leq x(k)$ is equivalent to $x(k) \leq A_0^- \backslash x(k) \wedge A_1^- \backslash x(k+1)$ since matrix multiplication $\otimes$ is residuated.

Therefore, inequality $f^-(x(k-1), x(k), u(k)) \leq x(k) \leq f^+(x(k-1), x(k), u(k))$ with $f^-(x(k-1), x(k), u(k)) = A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k) \oplus B_0^- \otimes u(k)$ is equivalent to $x(k) \leq [A_0^- \backslash x(k)] \wedge [A_1^- \backslash x(k+1)] \wedge f^+(x(k-1), x(k), u(k))$ with constraint $B_0^- \otimes u(k) \leq x(k)$ which must be satisfied.

Moreover, relation $y_{ob}(k) = C_{ob} \otimes x(k)$ is equivalent to $y_{ob}(k) \leq C_{ob} \otimes x(k)$ and $x(k) \leq C_{ob} \backslash y_{ob}(k)$.

Finally, inequality (3) and the two constraints (4) are obtained. ■

B. Calculation algorithm of the greatest state

Let us now consider the problem of the state estimation. The relation (3) is a fixed point form $x \leq f(x)$ which can be solved by the general algorithm of McMillan and Dill [21] [27]: The greatest state is given by the iterations of $x_{[i+1]} \leftarrow f(x_{[i]}) \wedge x_{[i]}$ if the starting point is finite and greater than the final solution. Here, index $[i]$ represents the number of iterations and not the number of components of vector x. Following this framework, we give below an algorithm specific to the estimation of the greatest state for interval model (1) with assumption (2). Inequality (3) can be rewritten as follows. Below the first expression presents a backward part of (3) while the other one corresponds to a forward part of (3).

$$\begin{cases} x(k) \leq [C_{ob} \backslash y_{ob}(k)] \wedge [A_0^- \backslash x(k)] \wedge [A_1^- \backslash x(k+1)] \\ x(k) \leq f^+(x(k-1), x(k), u(k)) \end{cases} \quad \text{for } k \geq 1.$$

Following this decomposition, the algorithm considers every possible relation inside the horizon $\{k_s, \dots, k_f\}$. Term $x(k)_{[i]}$ is the state estimate $x(k)$ at iteration i.

Algorithm 1

Step 0 (initialization) : $i = 0$; $x(k)_{[i]} \leftarrow +\infty$ for $k = k_s$ to k_f,

Repeat

Step 1: $i \leftarrow i + 1$; $x(k_f + 1)_{[i]} \leftarrow +\infty$;

$x(k)_{[i]} \leftarrow x(k)_{[i-1]} \wedge [C_{ob} \backslash y_{ob}(k)] \wedge [A_0^- \backslash x(k)_{[i-1]}] \wedge [A_1^- \backslash x(k+1)_{[i]}]$ for $k = k_f$ to k_s

Step 2: $x(k)_{[i]} \leftarrow x(k)_{[i]} \wedge f^+(x(k-1)_{[i]}, x(k)_{[i]}, u(k))$ for $k = k_s + 1$ to k_f

Until no $x(k)_{[i]}$ changes for $k_s \leq k \leq k_f$ ■

The 'Backward' part of inequality (3) corresponds to step 1 while the 'Forward' part of (3) corresponds to step 2. The first iteration of step 1 allows the determination of the starting state trajectory before the following minimizations. Finally, when the minimization of the state stops, the algorithm gives the greatest state which verifies the inequalities of the model. The development of the algorithm requires only the memorization of the matrices of the different models and the estimated trajectory.

C. Horizon

Let us note that the algorithm can converge to $+\infty$ for some situations. This fact depends on the notion of structural observability ([2]) which gives a sufficient condition to observe an effect in the output whose origin comes from at least one internal transition.

***Definition** 1:* An internal transition is structurally observable if, from this transition, there exists at least one path to an output transition in the Event Graph. An Event Graph is structurally observable if each internal transition is structurally observable.

The following properties give sufficient conditions of a possible estimation and change detection. We assume below that the values of the output y are finite.

***Property** 1:* Consider a structurally observable internal transition x_i. Let PA_i be the set of paths connecting the internal transition x_i to an output. Let the marking weight $\delta_{i,j}$ of a path from x_i to y_j be the number of initial tokens in this path and let $\Delta_i = \bigwedge_{PA_i} \delta_{i,j}$ be the minimal marking weight of the paths of PA_i. If $\Delta_i = k_f - k_s$, then the first iteration of step 1 of the algorithm produces a finite upper bound on the component $x_i(k_s)$.

Proof. Algebraically, we have $x(k) \geq A_1^- \otimes x(k-1) \oplus A_0^- \otimes x(k)$ and $y_{ob}(k) \geq C_{ob} \otimes x(k)$. We can easily deduce the different paths by building an associated graph (recall that $(A_1^-)_{i,j}$ corresponds to an arc from vertex $x_j(k-1)$ to vertex $x_i(k)$) . Let us consider a path from a structurally observable internal transition x_i to a known output transition y_j in the Event Graph. From a graphic point of view, successive iterations of step 1 follow the previous path but in the opposite direction: From the known output transition y_j to the internal transition x_i. This path is one of the paths expressed by $[C_{ob} \backslash y_{ob}(k)] \wedge [A_0^- \backslash x(k)] \wedge [A_1^- \backslash x(k+1)]$. Each known value of output transition $y_j(k)$ produces a finite upper bound on the values $x_i(k - \delta_{i,j})$ of the relevant internal transitions x_i with a backward shift of the number of events $\delta_{i,j}$. Finally, we can consider $x_i(k_s)$, Δ_i and the corresponding path from x_i to output y_j, such that $\delta_{i,j} = \Delta_i$. ■

Therefore, a structurally observable Event Graph is a condition whereby the algorithm converges to values different from $+\infty$ for each internal transition. An additional condition is that the considered horizon must be sufficiently large for the backward effect to exist, and produces an upper bound a certain number of events for each internal transition. We use below notation Δ_j defined in Property 1.

Property *2:* Let $h = k_f - k_s$ be the horizon of detection. The detection of changes can be made for $(B_0^-)_{j,.} \neq \varepsilon$ and internal transition x_j for $j \in [1, dim(x)]$ with $(B_0^-)_{j,.} \otimes u(k) \leq (\widehat{x}(k))_j$ if $h \geq \Delta_j$. Similarly, the detection of changes can be made for $(C_{ob})_{i,.} \neq \varepsilon$ and output $(y_{ob})_i$ with $(y_{ob}(k))_i \leq (C_{ob})_{i,.} \otimes \widehat{x}(k)$ if each internal transition x_j such that $(C_{ob})_{i,j} \neq \varepsilon$ satisfies $h \geq \Delta_j$.

Proof. Immediate. ■

If the horizon is the maximal marking weight of the paths of PA_i for every internal transition x_i, a more efficient estimation and change detection is obtained since the algorithm can take into account every path structure and the corresponding output. The choice of a wide horizon which can take into account a great quantity of data is only limited by the CPU time of the algorithm.

D. Complexity in the case of specific interval models

As the general algorithm of McMillan and Dill [21] [27] is known to be pseudo-polynomial (that is, the number of iterations generally depends on the values of the temporizations), we can also conclude that the proposed algorithm V-B is also pseudo-polynomial. The following result however, shows that the proposed algorithm V-B can be fast for some specific interval models such as the Timed Event Graphs where temporizations are unknown but belong to intervals (study [18] also considers this model in the autonomous case). Defined by $x(k) = A \otimes x(k-1) \oplus B \otimes u(k)$ where $A_1^- \leq A \leq A_1^+$ and $B_0^- \leq B \leq B_0^+$, the model of Timed Event Graphs with time uncertainties can be described as follows:

$$\begin{cases} f^-(x(k-1), u(k)) \leq x(k) \leq f^+(x(k-1), u(k)) \text{ with} \\ f^-(x(k-1), u(k)) = A_1^- \otimes x(k-1) \oplus B_0^- \otimes u(k+1) \\ f^+(x(k-1), u(k)) = A_1^+ \otimes x(k-1) \oplus B_0^+ \otimes u(k+1) \end{cases} \tag{5}$$

Time stream Event Graphs for the semantic rule Weak-And presents the same algebraic model. Note that, (5) satisfies $f^+(x(k-1), u(k)) \geq f^-(x(k-1), u(k))$ for any $x(k-1)$ and $u(k)$. This characteristic will be useful in the following property.

***Property** 3:* The algorithm V-B applied to the model defined by (5) converges in one iteration.

Proof.

Considering the model (5), the algorithm converges in one iteration if $x(k)_{[2]} = x(k)_{[1]}$. More precisely, the algorithm can stop at the end of the first iteration if the second iteration of step 1 does not minimize the estimate: Indeed, the second iteration of step 2 does not minimize if step 2 considers the same data. Let $\lambda(k) = x(k)_{[1]}$ (respectively, $\mu(k) = x(k)_{[1]}$) be the calculated estimate $x(k)_{[i]}$ at the first iteration 1 of step 1 (respectively, step 2). Therefore, we must prove that $x(k)_{[2]} = \mu(k)$. We show below that the property holds if we prove that

$$[C_{ob} \backslash y_{ob}(k)] \wedge [A_1^- \backslash \mu(k+1)] \geq \mu(k) \quad . \tag{6}$$

Indeed, the analysis of the second iteration of step 1 shows that step 1 does not produce a new minimization: The following recursion proves that $x(k)_{[2]} = \mu(k)$ for k in $\{k_s, \dots, k_f\}$.

For $k = k_f$, $x(k_f)_{[2]} = x(k_f)_{[1]} \wedge [C_{ob} \backslash y_{ob}(k_f)] = \mu(k_f) \wedge [C_{ob} \backslash y_{ob}(k_f)] = \mu(k_f)$ (we have $\mu(k_f) \leq \lambda(k_f) \leq C_{ob} \backslash y_{ob}(k_f)$)

Assuming $x(k)_{[2]} = \mu(k)$ for a given $k \in \{k_s, \ldots, k_f - 1\}$, we have, $x(k-1)_{[2]} = x(k-1)_{[1]} \wedge [C_{ob} \backslash y_{ob}(k-1)] \wedge [A_1^- \backslash x(k)_{[2]}]$
$= \mu(k-1) \wedge [C_{ob} \backslash y_{ob}(k-1)] \wedge [A_1^- \backslash x(k)_{[2]}]$
$= \mu(k-1) \wedge [C_{ob} \backslash y_{ob}(k-1)] \wedge [A_1^- \backslash \mu(k)]$. The application of (6) gives $x(k-1)_{[2]} = \mu(k-1)$ and the recursion is finished.

Now, let us provide proof of inequality (6). Simple substitutions and the property of distributivity of the left-residual $\backslash$ relative to $\wedge$ are applied below. So, $[C_{ob} \backslash y_{ob}(k)] \wedge [A_1^- \backslash \mu(k+1)] = [C_{ob} \backslash y_{ob}(k)] \wedge [A_1^- \backslash [\lambda(k+1) \wedge f^+(\mu(k-1), u(k))]]$
$= [C_{ob} \backslash y_{ob}(k)] \wedge [A_1^- \backslash \lambda(k+1)] \wedge [A_1^- \backslash f^+(\mu(k-1), u(k))]$ using the expression of μ in the first iteration of step 2.

Therefore,

$$[C_{ob} \backslash y_{ob}(k)] \wedge [A_1^- \backslash \mu(k+1)] = t_1 \wedge t_2 \tag{7}$$

where $t_1 = [C_{ob} \backslash y_{ob}(k)] \wedge [A_1^- \backslash \lambda(k+1)]$ and $t_2 = A_1^- \backslash f^+(\mu(k-1), u(k))$.

- We have $t_1 = [C_{ob} \backslash y_{ob}(k)] \wedge [A_1^- \backslash \lambda(k+1)] = \lambda(k)$ at the first iteration of step 1 because of the initialization $x(k)_{[0]} = +\infty$.

- Now, let us prove that $t_2 \geq \mu(k)$ by using the monotonie of the residuation $\backslash$. Since the model (5) satisfies $f^+(x(k-1), u(k)) \geq f^-(x(k-1), u(k))$, we have $t_2 = A_1^- \backslash f^+(\mu(k-1), u(k)) \geq A_1^- \backslash f^-(\mu(k-1), u(k))$
$= A_1^- \backslash [A_1^- \otimes \mu(k) \oplus B^- \otimes u(k+1)] \geq A_1^- \backslash [A_1^- \otimes \mu(k)] \geq \mu(k)$.

The last inequality is an application of residuation (see Preliminaries on residuation): Indeed, if a function f is residuated, then $f^\sharp$ exists such that $f \circ f^\sharp \leq Id_F$ and $f^\sharp \circ f \geq Id_E$. The direct application gives $A_1^- \otimes (A_1^- \backslash x) \leq x$ and $A_1^- \backslash [A_1^- \otimes x] \geq x$.

Finally, relation (6) is proved as $t_1 \wedge t_2 = \lambda(k) \wedge t_2 \geq \mu(k)$: Indeed, $\lambda(k) \geq \mu(k)$ (step 2 minimizes the result of step 1) and $t_2 \geq \mu(k)$. ■

E. Application to change detection

Described in the previous parts, the estimation of the greatest state is based on a relation (3) whereas the change detection considers the two remaining constraints (4). Let us analyze these relations (4).

- If constraints (4) are satisfied, then no change is detected. *The trajectory follows the model of the non-faulty process on the considered horizon.*
- If there is a constraint in (4) which is not satisfied, there is an incoherence between (3) and (4) and a change is detected. Indeed, the analysis of the two constraints (4) and a direct application of the residuation shows that the state estimate $\widehat{x}(k)$ has the lower bound $B_0^- \otimes u(k)$ and $C_{ob} \backslash Y_{ob}(k)$. In other words, the greatest state estimate $\widehat{x}(k)$ must always be greater than these lower bounds. If these conditions are not satisfied, the value of $\widehat{x}(k)$ is not high enough to satisfy (4) while $\widehat{x}(k)$ is limited by (3). *The trajectory on the considered horizon does not follow the model of the non-faulty process* expressed by (1) and (2).

Using the known values of $y_{ob}(k)$ and $u(k)$ on horizon $\{k_s, \ldots, k_f\}$, the procedure of change detection is as follows:

- 1) Estimation of $\widehat{x}$: we must solve a (min, max, +) fixed-point problem of type $x \leq f(x)$ over horizon $\{k_s, \ldots, k_f\}$ (the algorithm is given in part V-B).
- 2) Determination of the non-satisfied inequalities over horizon $\{k_s, \ldots, k_f\}$ in the relations: $(y_{ob}(k))_i \leq (C_{ob})_{ij} \otimes \widehat{x}(k)_j$ and $(B_0^-)_{jl} \otimes u_l(k) \leq \widehat{x}(k)_j$ for $i \in [1, dim(y_{ob})]$, $j \in [1, dim(x)]$, $k \in \{k_s, \ldots, k_f\}$ and $l \in [1, dim(u)]$.

VI. EXAMPLE 2

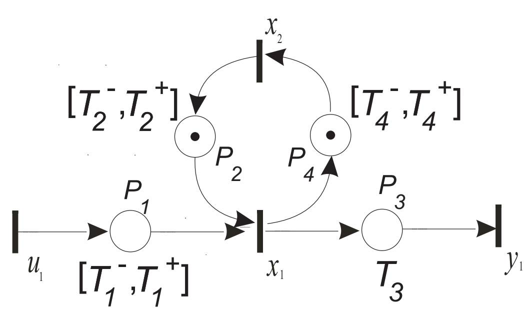

Fig. 2. Example 2: A Timed Event Graph with time intervals

Let us consider a Timed Event Graph including some uncertainties on the temporizations (Fig. 2). The transitions are: Input transition $\{u_1\}$, output transition $\{y_1\}$ and internal transitions x_1 and x_2. Recall that input and output transitions often model the input and output of parts, products, messages,... in the system. Associated with places p_i, the intervals of temporizations are denoted $[T_i^-, T_i^+]$ for index $i \in \{1, 2, 4\}$. We have $[T_1^-, T_1^+] = [1, 3]$, $[T_2^-, T_2^+] = [5, 7]$, $[T_4^-, T_4^+] = [2, 3]$ and $T_3 = 4$. Functions f^- and f^+ are defined as follows: $x(k) = \begin{pmatrix} x_1 & x_2 \end{pmatrix}^t$, $u(k) = u_1(k)$, $A_0^- = A_0^+ = \begin{pmatrix} \varepsilon & \varepsilon \\ \varepsilon & \varepsilon \end{pmatrix}$, $A_1^- = \begin{pmatrix} 0 & T_2^- \\ T_4^- & 0 \end{pmatrix}$, $A_1^+ = \begin{pmatrix} 0 & T_2^+ \\ T_4^+ & 0 \end{pmatrix}$, $B_0^- = \begin{pmatrix} T_1^- \\ \varepsilon \end{pmatrix}$ and $B_0^+ = \begin{pmatrix} T_1^+ \\ \varepsilon \end{pmatrix}$. We have $TR_{ob} = \{u_1, y_1\}$, $TR_{un} = \{x_1, x_2\}$and $C_{ob} = \begin{pmatrix} T_3 & \varepsilon \end{pmatrix}$. Written in conventional algebra, the relations allowing the estimation are as follows:

$$\begin{cases} \widehat{x_1}(k) \leq [y_1(k) - T_3] \wedge \widehat{x_1}(k+1) \wedge [\widehat{x_2}(k+1) - T_4^-] \\ \wedge max[\widehat{x_1}(k-1), \widehat{x_2}(k-1) + T_2^+, u_1(k) + T_1^+] \\ \widehat{x_2}(k) \leq \widehat{x_2}(k+1) \wedge [\widehat{x_1}(k+1) - T_2^-] \wedge max[\widehat{x_1}(k-1) + T_4^+, \widehat{x_2}(k-1)] \end{cases} \tag{8}$$

The sliding horizon of calculation of the observer is $[k_f - 2, k_f]$.

Constraints allowing the detection are as follows:

$$\begin{pmatrix} u_1(k) + T_1^- \\ y_1(k) \end{pmatrix} \leq \begin{pmatrix} \widehat{x_1}(k) \\ \widehat{x_1}(k) + T_3 \end{pmatrix} \tag{9}$$

for $k \in [k_f - 2, k_f]$.

1) Simulation 1: **Scenario of the simulation on** $\{0, \ldots, 10\}$

The normal values of the temporization of place p_2 belong to $[T_2^-, T_2^+] = [5, 7]$ for $k \in \{1, 2, 3, 4\}$ and $k \in \{8, 9, 10\}$. We suppose that the task associated to p_2 is stopped before its end: Place p_2 has a temporization $T_2(k) = 2$ for $k \in \{5, 6, 7\}$. Clearly, this provisional *decrease* of $T_2(k)$ produces a temporary increase in the production rate.

We assume that input transitions (control input) may be delayed from firing until some arbitrary time provided by a supervisor. Input $u_1(k)$ is such that $u_1(k) = k$ for $k \geq 1$. Following these conditions, a possible trajectory, which is not extremal (not the earliest or the latest trajectory) is as follows:

k	0	1	2	3	4	5	6	7	8	9	10
u_1	$-$	1	2	3	4	5	6	7	8	9	10
x_1	0	6	8	14	16	18	20.5	22.5	29	31	37
x_2	0	2	8	10	16	18.5	20.5	23	25	31.5	33.5
y	4	10	12	18	20	22	24.5	26.5	33	35	41

The fault detection procedure gives the following number of incoherent relations (9) on horizon $\{k-2, \ldots, k\}$. This number is denoted $nc(k)$.

k	0	1	2	3	4	5	6	7	8	9	10
$T_2(k)$	$-$	$[5,7]$	$[5,7]$	$[5,7]$	$[5,7]$	2	2	2	$[5,7]$	$[5,7]$	$[5,7]$
$nc(k)$	0	0	0	0	0	1	1	1	0	0	0

So, the procedure detects the fault on horizon $\{k-2, \ldots, k\}$ for $k = 5$, 6 and 7. We consider below the horizons $\{8, 9, 10\}$ and $\{3, 4, 5\}$.

Horizon $\{8, 9, 10\}$(normal behavior).

As the calculations of the observer are independent of the initial condition $x(0)$, the estimation can be made on any horizon if the process follows its normal mode on this horizon. Therefore, the estimation can be made after a past perturbation or an unknown change in the system. Using known data on the horizon $\{8, 9, 10\}$, the observer (8) gives the following the optimal greatest estimate:

k	8	9	10
$\widehat{x_1}$	29	31	37
$\widehat{x_2}$	26	32	34

.

The verification of constraints (9) shows that each inequality for $k \in \{8, 9, 10\}$ is satisfied: The system composed of (8) and (9) is consistent and no change is detected.

Horizon $\{3, 4, 5\}$ (faulty behavior)

Using known data on the horizon $\{3, 4, 5\}$, the observer (8) gives the following table:

k	3	4	5
$\widehat{x_1}$	11	16	18
$\widehat{x_2}$	11	13	19

.

The verification of constraints (9) shows that there is a unique non-satisfied inequality: Relation $y_1(3) = 18 \leq \widehat{x_1}(3) + T_3 = 11 + 4 = 15$ is not satisfied. Therefore, the system composed of (8) and (9) is inconsistent on horizon $\{3, 4, 5\}$ and a change is detected. The estimates of the observer are not guaranteed: We can note that $x(k) \nleq \widehat{x}(k)$ for $k = 3$, 4 and 5 as $x_1(3) = 14 \nleq \widehat{x_1}(3) = 11$ and $x_2(4) = 16 \nleq \widehat{x_2}(4) = 13$.

A deeper analysis of the relations (8) shows that the following subset of inequalities is incoherent for the considered data:

$$\begin{cases} y_1(k_s) \leq \widehat{x_1}(k_s) + T_3 \\ \widehat{x_1}(k_s) \leq \widehat{x_2}(k_s + 1) - T_4^- \\ \widehat{x_2}(k_s + 1) \leq \widehat{x_1}(k_s + 2) - T_2^- \\ \widehat{x_1}(k_s + 2) \leq y(k_s + 2) - T_3 \end{cases} \quad \text{which gives} \tag{10}$$

$$\begin{cases} 18 \nleq 11 + 4 \\ 11 = 13 - 2 \\ 13 = 18 - 5 \\ 18 = 22 - 4 \end{cases} . \tag{11}$$

Indeed, the observer considers the normal behavior for $k_s + 2 = 5$ which is expressed by inequality $\widehat{x_2}(k_s + 1) \leq \widehat{x_1}(k_s + 2) - T_2^-$ with $T_2^- = 5$ while the faulty behavior is described by the relation $x_1(k_s + 2) = x_2(k_s + 1) + T_2(k_s + 2)$ with $T_2(k_s + 2) = 2$.

2) Simulation 2: We consider below another simulation for example 2 where the change is an *increase* of $T_2(k)$.

Scenario of the simulation 2: T_2

The normal values of the temporization of place P_2 belong to $[T_2^-, T_2^+] = [5, 7]$ for $k \in \{1, 2, 3, 4\}$ and $k \in \{8, 9, 10\}$. We consider the following change in the system: place P_2 has a temporization $T_2(k) = 8.1$ for $k \in \{5, 6, 7\}$. Clearly, this change produces a temporary *decrease* in the production rate: The process shows some signs of wear.

We assume that input transitions (control input) may be delayed from firing until some arbitrary time provided by a supervisor. Input $u_1(k)$ is such that $u_1(k) = k$ for $k \geq 1$.
Following these conditions, a possible trajectory, which is not extremal is as follows.

k	0	1	2	3	4	5	6	7	8	9	10
u_1	$-$	1	2	3	4	5	6	7	8	9	10
x_1	0	6	8	14	16	24.1	26.6	34.6	35	43	44
x_2	0	2	8	10	16	18.5	26.5	29	37	37.5	45.5
y	4	10	12	18	20	28.1	30.6	38.6	39	47	48

The fault detection procedure gives the following number of incoherent relations (9) on horizon $\{k-2, \ldots, k\}$ which is denoted $nc(k)$.

k	0	1	2	3	4	5	6	7	8	9	10
$T_2(k)$	$-$	$[5, 7]$	$[5, 7]$	$[5, 7]$	$[5, 7]$	8.1	8.1	8.1	$[5, 7]$	$[5, 7]$	$[5, 7]$
$nc(k)$	0	0	0	0	0	1	1	1	0	0	0

So, the procedure detects the fault on horizon $\{k-2, \ldots, k\}$ for $k = 5$, 6 and 7. We consider below the horizons $\{8, 9, 10\}$ and $\{3, 4, 5\}$.

Horizon $\{8, 9, 10\}$(normal behavior).

As the calculations of the observer are independent of the initial condition $x(0)$, the estimation can be made on any horizon if the process follows its normal mode on this horizon. Therefore, the estimation can be made after a past perturbation or an unknown change in the system. Using known data on the horizon $\{8, 9, 10\}$, the observer (8) gives the following sub-optimal lower bound and greatest estimate:

k	8	9	10
x_1^-	35	43	44
x_2^-	$-\infty$	37	45

and

k	8	9	10
$\widehat{x}_1$	35	43	44
$\widehat{x}_2$	38	38	46

.

The comparison with the simulation shows that the observer gives the exact state x_1 and not x_2 (note that $x(k) \leq \widehat{x}$). The comparison with the lower bound detects that this estimate is accurate for x_1^-.

Moreover, the verification of constraints (9) shows that each inequality for $k \in \{8, 9, 10\}$ is satisfied: The system composed of (8) and (9) is consistent and no change is detected.

Horizon $\{3, 4, 5\}$ (faulty behavior)

Using known data on the horizon $\{3, 4, 5\}$, the observer (8) gives the following table:

k	3	4	5
$\widehat{x_1}$	14	16	24
$\widehat{x_2}$	11	17	19

.

The verification of constraints (9) shows that there is a unique non-satisfied inequality: The relation $y_1(5) = 28.1 \leq \widehat{x_1}(5) + T_3 = 24 + 4 = 28$ is not satisfied. Therefore, the system composed of (8) and (9) is inconsistent on horizon $\{3, 4, 5\}$ and the change of T_2 is detected. The simulation of the scenario shows that non-drastic changes in the model such as a small variation of the value of a temporization can be detected. As a change is detected, the estimates of the observer are not guaranteed and we can note that $x(k) \nleq \widehat{x}(k)$ for $k = 5$ as $x_1(5) = 24.1 \nleq \widehat{x_1}(5) = 24$.

A deeper analysis of the relations (8) shows that the following subset of inequalities is incoherent for the considered data:

$$\begin{cases} y_1(k_f) \leq \widehat{x_1}(k_f) + T_3 & \\ \widehat{x_1}(k_f) \leq \widehat{x_2}(k_f - 1) + T_2^+ & \text{with } \widehat{x_2}(k_f - 1) + T_2^+ \geq \widehat{x_1}(k_f - 1) \text{ and} \\ \widehat{x_2}(k_f - 1) + T_2^+ \geq u_1(k_f) + T_1^+ & \\ \widehat{x_2}(k_f - 1) \leq \widehat{x_1}(k_f - 2) + T_4^+ & \text{with } \widehat{x_1}(k_f - 2) + T_4^+ \geq \widehat{x_2}(k_f - 2) \\ \widehat{x_1}(k_f - 2) \leq y_1(k_f - 2) - T_3 & \end{cases} \tag{12}$$

So,

$$\begin{cases} 28.1 \nleq 24 + 4 & \\ 24 = 17 + 7 & \text{with } 17 + 7 \geq 16 \text{ and } 17 + 7 \geq 5 + 3 \\ 17 = 14 + 3 & \text{with } 14 + 3 \geq 11 \\ 14 = 18 - 4 & \end{cases} \tag{13}$$

Indeed, the observer considers the normal behavior for $k_f = 5$ which is expressed by inequality $\widehat{x_1}(k_f) \leq \widehat{x_2}(k_f - 1) + T_2^+$ with $T_2^+ = 7$ while the faulty behavior is described by the relation

$x_1(k_f) = x_2(k_f - 1) + T_2(k_f)$ with $T_2(k_f) = 8.1$.

VII. Conclusion

In this paper, we propose an on-line approach composed of two steps: An observer estimates the earliest state trajectory; using the state estimate, a change detection checks on-line the coherence of the model and the known data. Based on a sliding horizon, the change detection can start at any number of events and detect the appearance of change.

The places of the Event Graph can be unbounded and the synchronizations of the transitions can follow complex semantics. Contrary to numerous approaches in fault detection and model checking, our approach considers that the values of the temporizations are defined in $\mathbb{R}^+$. A direct consequence is that small changes in the process can be detected. As this situation can occur when the process shows some signs of wear, the detected changes bring indications which can be used in future maintenance operations.

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