From dioid algebra to p-time event graphs

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Abstract: The (max,+) algebra is usually used to model Timed event graph. In this paper, we show that P-time event graphs which extend Timed event graph, can be modelled using maximum, minimum and addition operations. The result is a new model called interval descriptor system where the time evolution is not strictly deterministic but belongs to intervals. The cycle-time vector allows us to to check its correct behaviour and to verify the existence of a state trajectory. Particularly, it detects the presence of token-deads which can generate a deadlock.

1 INTRODUCTION

Discrete Event Dynamic Systems can represent a great number of processes characterized as being concurrent, asynchronous, distributed or parallel, such as flexible manufacturing systems, multiprocessor systems or transportation networks. In such systems the behaviour depends on complex interactions of the timing of various discrete events. The topical algebra is an important field of mathematical and analysis techniques of these models. The (max,+) algebra makes it possible to analyse the Timed Event Graphs and many results are available like spectral theory and control synthesis. In this paper, a new class of systems is studied for which the time evolution is not strictly deterministic but belongs to intervals. At each step, the lower and upper bounds depends on the maximization, minimization and the addition operations simultaneously. The symbol $\oplus$ stands for the maximum operation while $\wedge$ corresponds to the minimum operation. The operation $\oplus$ has the neutral element $\varepsilon = -\infty$ whereas $\wedge$ has the neutral element $T = +\infty$. The notations $\otimes$ and $\odot$ corresponds to the usual addition with the following convention: $T \otimes \varepsilon = \varepsilon$ and $T \odot \varepsilon = T$. The expression $a \otimes b$ and $a \odot b$ are identical if at least either a or b is a finite scalar.

We propose to analyse the following implicit model called interval descriptor system. The evolution of the system is described by the following equations where f^+ and f^- are (min, max, +) functions. The interpretation of each variable is as follows: like the "dater" type in (max,+) algebra, each variable $x_i(k)$ represents the date of the kth firing of transition x_i .

$$\left\{ \begin{array}{c} x(k) = x(k) \wedge f^+(x(k), ..., x(k-m), u(k) \\ , ..., u(k-m)) \\ x(k) = x(k) \oplus f^-(x(k), ..., x(k-m), u(k) \\ , ..., u(k-m)) \\ \text{with } x(k) = \epsilon \text{ for } k \leq 0 \end{array} \right. \quad (1)$$

The vector u is the input and m is the horizon. We can also introduce the output y by $y(k) = C \otimes x(k)$. To simplify the writing, the matrix C will be chosen to make a direct correspondence between some component of $x_i(k)$ and a component of output $y_i(k)$ in a natural manner. Consequently, some columns can be null but each row contains only one element $e = 0$ and ε elsewhere. This equality will not be used in this paper. The functions $f^+()$ and $f^-()$ represent respectively an upper and lower bound of x whose trajectory is between these bounds. Particularly, if the lower bound defined by f^- is a (max, +) function and the upper bound is infinite, the classical (max, +) systems can be obtained after some classical manipulations.
In this paper, we give in a first part some preliminaries and important fixed-point theorems. We then introduce a class of interval descriptor system and show that P-time event graphs can be modelled under this form. Finally, we analyze the correct behaviour of p-

time event graphs and particulary the synchronization of transitions.

2 PRELIMINARIES

The partial order denoted $\leqslant$ is defined as follows: $x \leqslant y \Longleftrightarrow x \oplus y = y \Longleftrightarrow x \wedge y = x \Longleftrightarrow x_i \leqslant y_i$, for i from 1 to n in $\Re^n$. Notation $x < y$ means that $x \leqslant y$ and $x \neq y$.

Definition 2.1 A dioid D is complete if it is closed for infinite sums and the distributivity of the multiplication with respect to addition extends to infinite sums : $(\forall\, c \in D\,)\ (\forall\, A \subseteq D)\ c \otimes (\bigoplus_{x\in A} x) = \bigoplus_{x\in A} c \otimes x$

For example, $\bar{\Re}_{max}$ = $(\Re \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$ is complete.

The set of $n.n$ matrices with entries in a complete dioid D endowed with the two operations $\oplus$ and $\otimes$ is also a complete dioid which is noted $D^{n.n}$. The elements of the matrices in the (max, +) expressions (respectively (min, +) expressions) are either finite or ε ((respectively T). We can deal with nonsquare matrices if we complete by rows or columns with entries equals to ε (respectively T). The different operations operate as in the usual algebra: The notation $\odot$ refers to the multiplication of two matrices in which the $\wedge$ operation is used instead of $\oplus$.

$$(\mathrm{A} \oplus \mathrm{B})_{ij} = \mathrm{A}_{ij} \oplus \mathrm{B}_{ij}\ ,$$
$$(\mathrm{A} \wedge \mathrm{B})_{ij} = \mathrm{A}_{ij} \wedge \mathrm{B}_{ij}\ ,$$
$$(\mathrm{A} \otimes \mathrm{B})_{ij} = \bigoplus_{k=1}^{n} \mathrm{A}_{ik} \otimes \mathrm{B}_{kj}$$
$$(\mathrm{A} \odot \mathrm{B})_{ij} = \bigwedge_{k=1}^{n} \mathrm{A}_{ik} \odot \mathrm{B}_{kj}$$

In $(\oplus,\ \otimes)$ algebra, Kleene's star is defined by: $A^* = \bigoplus_{i=0}^{+\infty} A^i$. Respectively, in $(\wedge,\ \odot)$ algebra, Kleene's star is defined by: $A_* = \bigwedge_{i=0}^{+\infty} A^i$

Noted as $G(A)$, an induced graph of a square matrix A is deduced from this matrix by associating

- a node i to the column i and line i

- an arc from the node j towards the node i if $A_{ij} \neq \varepsilon$.

Theorem 2.2 (F. Baccelli and Quadrat, 1992) Given A and B in a complete dioid and consider the equation $x = A \otimes x \oplus B$ and the inequality $x \geqslant A \otimes x \oplus B$. Then, for these expressions : $A^* \otimes B$ is the least solution ; every solution x verifies $x = A^* \otimes x$; T is the greatest solution of the inequality.

Theorem 2.3 Given A and B in a complete dioid and consider the equation $x = A \odot x \wedge B$ and the inequality $x \leqslant A \odot x \wedge B$. Then, for these expressions : $A_* \odot B$ is the greatest solution ; every solution x verifies $x = A_* \odot x$; ε is the least solution of the inequality.

The left $\otimes$ residuation of b by a is defined by: $a\backslash b = \max\{x \in D$ such that $a \otimes x \leqslant b\}$. Respectively, in $(\wedge,\ \odot)$ algebra, the left $\odot$ residuation of b by a is defined by: $a\backslash' b = \min\{x \in D$ such that $a \odot x \geqslant b\}$.

Given A and B two matrices in a complete dioid, the residuation of B (dimensions $n.q$) by A (dimensions $n.p$) is clearly expressed in the other dioid:

$A\backslash B = (-A)^t \odot B$ and $A\backslash' B = (-A)^t \otimes B$ with t: transpose.

Lemma 2.4 part1 (F. Baccelli and Quadrat, 1992) We have the following equivalences: $x \geq ax \Leftrightarrow x = a^*x \Leftrightarrow x \leq a \setminus x \Leftrightarrow x = a^* \setminus x$

3 INTERVAL DESCRIPTOR SYSTEM AND COMPATIBILITY

3.1 Cycle time and compatibility

Now, we introduce the definitions of cycle time, eigen-vector, eigen-value and ultimately affine regime (Cheng and Zheng, 2002)(Gaubert and Gunawardena, 1998). These notions are relevant to the (min, max, +) functions but not always to the topical functions. Some connections can be established between these concepts. Addition $+$ is defined by: $\forall\lambda \in \Re, \forall x \in \Re^n$, $\lambda + x = (\lambda + x_1, ..., \lambda + x_n)^t$ (t: transpose)

Definition 3.1 A min-max function of type $(n, 1)$ is any function $f : \Re^n \rightarrow \Re^1$, which can be written as a term in the following grammar:
$f = x_1, x_2, \cdots x_n \mid f + a \mid f \wedge f \mid f \oplus f$ where a is an arbitrary real number $(a \in \Re)$
We consider dynamics of the form:

$x(k) = f(x(k-1))$, $\forall k \geqslant 1$

$x(0) = \xi \in \Re^n$

where f is a (min, max, +) function of type (n, n) $\Re^n \rightarrow \Re^n$. The set of $\min - \max$ function of type (n, m) is noticed $D^*(n, m)$.

Definition 3.2 The cycle time vector is defined by $\chi(f) = \lim_{k\to\infty} x(k)/k$ if it exists. It does not depend on ξ.

Definition 3.3 An eigen-vector x and its associated eigen-value $\lambda \in \Re$, if they exists, verify $f(x) = \lambda + x$

Definition 3.4 The pair $(\eta, v) \in (\Re^n)^2$ is an ultimately affine regime of f if there exists an integer K such that $\forall k \geqslant K$, $f(v + k\eta) = v + (k+1)\eta$.

Corollary 3.5 (J. Cochet-Terrasson and Gunawardena, 1999) Any function in D^* has a cycle time. Moreover, $\chi(f)$= η, for all ultimately regimes (η, v) $\in (\Re^n)^2$ of f.

In the following theorems, the notion of cycle time which always exists in D^* makes it possible to check the existence of a solution of different inequalities and equalities.

Theorem 3.6 (Gaubert and Gunawardena, 1998) Let $f \in D^*$. The two following conditions are equivalent:

(i) It exists a finite x such that $x \leqslant f(x)$

(ii) $\chi(f) \geqslant 0$

Theorem 3.7 Let $f \in D^*$. The two following conditions are equivalent:

(i) It exists a finite x such that $x \geqslant f(x)$

(ii) $\chi(f) \leqslant 0$

From the two previous theorems 3.6 and 3.7, we deduce directly the following result.

Theorem 3.8 Let $f \in D^*$. The two following conditions are equivalent:

(i) It exists a finite x such that $x = f(x)$

(ii) $\chi(f) = 0$

A set S of min-max functions is rectangular if for all G, $G' \in S$, and for all $i = 1, ..., n$ the function obtained by replacing the i-th component of G by the i-th component of G' belongs to S. We denote by $rec(S)$ the rectangular closure of a set S, which is finite when S is finite. Let S, T be rectangular max and min representations, respectively, of f. Since min-max functions are monotone, $\bigoplus_{g \in T} \chi(g) \leqslant \chi(f) \leqslant \bigwedge_{h \in S} \chi(h)$

The duality conjecture states that the extreme sides coincide. It was proved in (Gaubert and Gunawardena, 1998)

$$\chi(f) = \bigwedge_{h \in S} \chi(h) = \bigoplus_{g \in T} \chi(g)$$

Theorem 3.9 (J. Cochet-Terrasson and Gunawardena, 1999) Let $f \in D^*$and suppose that $S, T \in D^*$ are rectangular and, respectively, a max-representation and a min-representation of f. The following conditions are equivalent.

1. f has a fixed point with $f(x) = x + h$.
2. $\bigwedge_{h \epsilon S} \chi(h) = h$
3. $\bigoplus_{g \epsilon T} \chi(g) = h$

Remark : The theorem 3.8 can be considered as a corollary of the theorem 3.9 when h equals 0.

3.2 Generalized Upper Bound Constraint and Lower Bound Constraint forms.

In the aim of reducing the size of the expressions, the system 1 can classically be transformed in reduced form by increasing the vector state. With an abuse of notation, we keep the same notation for x, f^- and f^+ to alleviate the notation. From the system 1, we deduce:

$$x(k) \leqslant f^+(x(k), x(k-1), u(k))$$
$$x(k) \geqslant f^-(x(k), x(k-1), u(k))$$

As f^+ and f^- are (min, max, +) functions, the above form is more general that the "UBC" (Upper Bound Constraint) where f^+ is a max-only function (see (Walkup and Borriello, 1998) for more details). We can call the above forms respectively, "GUBC" (Generalized Upper Bound Constraint) and "GLBC" (Generalized Lower Bound Constraint). As the above theorems 3.6, 3.7 and 3.8 can only be applied to the forms $x \leqslant f(x)$, $x \geqslant f(x)$ or $x = f(x)$ where $f \in D^*$, we must consider special cases. As the type of the system 1 is defined by the types of the functions f^+ and f^-, we can characterize the model by the couple (type of f^-, type of f^+). The type ((min, max, +), (min, max, +)) represents the more general case for the system 1 . Under the assumption of the existence of a solution, they define corresponding classes of compatible interval descriptor systems. In the next sections, we will only consider the ((max, +), (min, +)) type and show that the p-time event graphs can be modelled under this form.

3.3 ((max, +), (min, +)) type

We consider the following system 3.3 of ((max, +), (min, +)) type :

$$f^-(z(k)) = \bigoplus_{i=0}^{1} A_i^- \otimes x(k-i) \oplus B^- \otimes u(k) \text{ and}$$
$$f^+(z(k)) = \bigwedge_{i=0}^{1} A_i^+ \odot x(k-i) \wedge B^+ \odot u(k). \quad (3.3)$$

Theorem 3.10 The system 3.3 is compatible in the horizon $[k-1, k]$ if and only if the cycle time of the following function g^+ is greater than or equals zero.

$$g^+(z(k) = \begin{pmatrix} f^+(z(k)) \wedge A_0^- \setminus x(k) \\ A_1^- \setminus x(k) \\ B^- \setminus x(k) \end{pmatrix}$$

with $z(k) = \begin{pmatrix} x(k) \\ x(k-1) \\ u(k) \end{pmatrix}$

Theorem 3.11 The system 3.3 is compatible in the horizon $[k-1, k]$ if and only if the cycle time of the following function g^- is lower than or equals zero.

$$g^-(z(k) = \begin{pmatrix} f^-(z(k)) \oplus A_0^+ \setminus' x(k) \\ A_1^+ \setminus' x(k) \\ B^+ \setminus' x(k) \end{pmatrix}$$

Corollary 3.12 If the system 3.3 is compatible in the horizon $[k-1, k]$, then the following are equivalent.

1. The cycle time of the following function g^+ is greater than or equals zero.
2. the cycle time of the following function g^- is lower than or equals zero.

Finally, the final inequality set presents the form $g^-(z(k)) \leq z(k) \leq g^+(z(k))$. The system of ((max, +), (min, +)) type is reduced to a $(-\infty, (min, +))$ (respectively $((max, +), +\infty))$) type and can be analyzed by the theorem 3.10 (respectively the theorem 3.11). If the cycle time verifies the corresponding condition of existence, it describes a compatible interval descriptor system.

We propose now a generalization of the preceding theorems on a wider horizon $[k, k+h]$.

Theorem 3.13 The system 3.3 is compatible in the horizon $[k, k+h]$ if and only if the cycle time of the function $g_h^+(z(k))$ is greater than or equals zero.

$$z(k) = \begin{pmatrix} x(k) \\ \vdots \\ x(k+h-1) \\ x(k+h) \\ u(k) \\ \vdots \\ u(k+h-1) \\ u(k+h) \end{pmatrix}, \; g_h^+(z(k)) =$$

$$\begin{pmatrix} f^+(x(k), x(k-1), u(k)) \\ \vdots \\ f^+(x(k+h-1), x(k+h-2), u(k+h-1)) \\ f^+(x(k+h), x(k+h-1), u(k+h)) \\ T \\ \vdots \\ T \\ T \end{pmatrix}$$

$\wedge M^t \setminus z(k) \wedge z(k+1)$

with $M = \left(\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right)$,

$$M_{11} = \begin{pmatrix} A_0^- & A_1^- & \cdots & T \\ T & \ddots & \ddots & \vdots \\ \vdots & & A_0^- & A_1^- \\ T & & \cdots & A_0^- \end{pmatrix}, \quad M_{21} = \begin{pmatrix} B^- & T & \cdots & T \\ T & \ddots & & \vdots \\ \vdots & & B^- & \\ T & & \cdots & B^- \end{pmatrix}$$

and $M_{12} = M_{22} = T$

Proof

For $0 \leq j \leq h$ we have :

$$\begin{cases} x(k+j) \leq f^+(x(k+j), x(k+j-1), u(k+j)) \\ x(k+j-1) \leq x(k+j) \\ u(k+j-1) \leq u(k+j) \end{cases}$$

and $A_0^- \otimes x(k+j) \oplus A_1^- \otimes x(k+j-1) \oplus B^- \otimes u(k+j) \leq x(k+j)$

we use the lemma 2.4, we arrive to :

$$\Longleftrightarrow \begin{cases} x(k+j) \leq A_0^- \setminus x(k+j) \\ x(k+j-1) \leq A_1^- \setminus x(k+j) \\ u(k+j) \leq B^- \setminus x(k+j) \end{cases}$$

let $x(k) = \epsilon$ for $k \leq 0$

A concatenation of the two last systems gives the following form : $\forall\, 0 \leq j \leq h$

$$\begin{cases} x(k+j) \leq \\ f^+(x(k+j), x(k+j-1), u(k+j)) \\ \wedge A_0^- \setminus x(k+j) \wedge \\ A_1^- \setminus x(k+j+1) \wedge x(k+j+1) \\ u(k+j) \leq B^- \setminus x(k+j) \wedge u(k+j+1) \end{cases}$$

$$\text{and} \begin{cases} x(k+h) \\ \leq f^+(x(k+h), x(k+h-1), u(k+h)) \\ \wedge A_0^- \setminus x(k+h) \\ u(k+h) \leq B^- \setminus x(k+h) \end{cases}$$

Lastly, the above system can be reduced to the following form where the function g_h^+ is described in the body of the theorem.

$$z(k) \leq g_h^+(z(k))$$

4 P-TIME PETRI NETS AND ACCEPTABLE FUNCTIONING

4.1 Modelling

The p-time Petri nets make it possible to model the discrete event dynamic systems with time constraints of stay of the tokens inside the places. We associate for each place a temporal interval.

Definition 4.1 (p-time Petri nets) The formal definition of p-Time PN is given by a pair $< R, IS >$ where R is a marked Petri nets

$IS : P \longrightarrow (Q^+ \cup \{0\}) \times (Q^+ \cup \{\infty\})$

$p_i \longrightarrow IS_i = [a_i, b_i]$ with $0 \leq a_i \leq b_i$

IS_i is the static interval of residence time or duration of a token in place p_i. The value a_i is the minimum residence duration that the token must stay in the place p_i. Before this duration, the token is in state of unavailability to firing the transition t_j. The value b_i is a maximum residence duration after which the token must thus leave the place p_i. If not, the system is found in a token-dead state. We conclude that the token is available to firing the transition t_j in the interval time $[a_i, b_i]$.

We will express the interval of shooting of each transition from the system which will guarantee an acceptable functioning. The assumption of functioning FIFO of the transition x_i guarantees the condition of non overtaking of the tokens between them. We consider S the set of all input places to transition x_i. For the p-time PNs, the evolution is described by the following inequations :

$$x_i(k) \geq \bigoplus_{j \in S} (x_j(k - m_j) + a_j)$$

with a_i the lower bound of an upstream place of x_i and m the number of tokens present in an upstream place of x_i and

$$x_i(k) \leq \bigwedge_{j \in S} (x_j(k - m_j) + b_j)$$

with b_i the upper bound of an upstream place of x_i. In this part, we study p-time event graphs which is an example of $((max, +), (min, +))$ type of interval descriptor system.

Remarks : - If one of the m tokens of a place p_l dies before firing transition x_i, this death is translated in the state equations. The new model becomes :

$$\bigoplus_{j \in S - \{p_l\}} (x_j(k - m_j) + a_j) \oplus (x_j(k - m_j - 1) + a_l) \leq x_i(k) \leq \bigwedge_{j \in S - \{p_l\}} (x_j(k - m_j) + b_j) \wedge (x_j(k - m_j - 1) + b_l)$$

- If we divide up each place which contains m tokens in m places, with one token by place, we can obtain the equations on a shorter horizon. Only the upstream place of x_i has temporization $[a, b]$. For the others, they have all the null time interval $[0, 0]$.

4.2 Analysis of transition synchronization in an horizon

The Petri nets make it possible to analyze several behavioral or structural properties related to the systems which they model. We consider one of these behavioral properties, the liveness which ensures the system not to reach a state of blocking. This property depends on initial marking. A state of blocking in PNs occurs when we reach a marking which does not allow the firing of any transition. Now we give the formal definition of liveness.

Definition 4.2 (liveness of a transition) A transition x_i is live for an initial marking M_0 if, for any marking M_j accessible since M_0 there is a sequence of firing S starting from M_j which includes the transition x_i

Definition 4.3 (liveness of a petri net) For a given initial marking, a PN is live if for any accessible

$\forall\, M \in E(M_0), \forall\, p \in P, \exists\, S \,/\, M \xrightarrow{S} M'$ and $t \in S$

Classically, one of the methods which allow to check liveness is analysis by enumeration. This approach consists in building the coverability graph if the number of markings is finished, or in building the coverability tree if the number of markings is infinite. For temporal PNs, checking and making study of the liveness property becomes more difficult since the latter depends not only on initial marking but also on the intervals of times related to the graph. It thus proves that the use of the method by enumeration is very difficult. Indeed, the passage of a state to another is related either to the firing of a transition or to the evolution from time. Thus, a consequence is combinative explosion of the coverability graph.

As p-time event graphs can be modelled under ((max, +),(min, +)) interval descriptor system, we propose to apply the results presented in the part 3 to detect any non-synchronization of transitions in an horizon. The following definition of acceptable functioning on an horizon will allow us to express easier the approach.

In a practical point of view, the deaths of token represent the lost of ressources and must naturally be avoid. Consequently, if we check the acceptable functioning, we guarantee a correct behaviour. Moreover, we can deduce that the corresponding system is live on the same horizon. Let us notice that the reverse is not true: the existence of a non-synchronization of a transition entails the death of at least one token but the liveness of the petri net can be ensured by the other tokens.

Definition 4.4 (acceptable functioning) We call an acceptable functioning of a p-time PN any dynamic evolution of the system without leading to a mark-dead state or a blocking state.

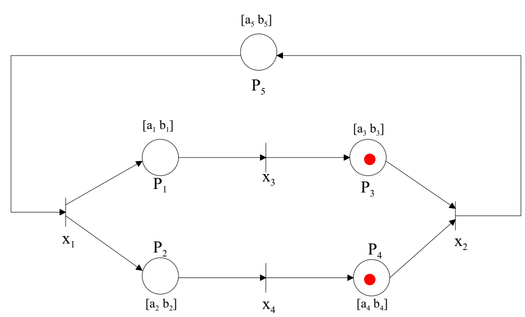

Figure 1: a p-time event graph (autonomous case)

Example

We consider the example of the figure 1 which will enable us to illustrate our approach. Initially we can check easily that the logical graph (without taking account of temporizations) is quite live. By considering temporizations related to each place, we can note that in spite of an initial marking which ensures the liveness of the logical graph, the temporal graph can be in a state of total blocking. Showing these behaviours, several cases can arise while acting on the bounds of the intervals related to the places.

The first step of our approach is to model the system by recurring state equations in the following form:

$$\begin{cases} x_2(k) + a_5 \leq x_1(k) \leq x_2(k) + b_5 \\ (x_3(k-1) + a_3) \oplus (x_4(k-1) + a_4) \leq x_2(k) \\ \leq (x_3(k-1) + b_3) \wedge (x_4(k-1) + b_4) \\ x_1(k) + a_1 \leq x_3(k) \leq x_1(k) + b_1 \\ x_1(k) + a_2 \leq x_4(k) \leq x_1(k) + b_2 \end{cases} \tag{2}$$

The second step consists to divide up the system 2, and to put it in the form $x \leq f(x)$. Thus we arrive at the following system:

$$\begin{cases} x_1(k) \leq (x_2(k) + b_5) \wedge (x_4(k) - a_2) \\ \wedge (x_3(k) - a_1) \\ x_1(k+1) \leq (x_2(k+1) + b_5) \wedge (x_4(k+1) \\ -a_2) \wedge (x_3(k+1) - a_1) \\ \vdots \\ x_4(k) \leq (x_2(k) - a_4) \wedge (x_1(k) + b_3) \\ x_4(k+1) \leq (x_1(k+1) + b_3) \end{cases}$$

Case 1:

Now we present the first case where we fix the bounds of the intervals as follows : $[a_1\ b_1] = [0, 1]$, $[a_2\ b_2] = [5, 6]$, $[a_3\ b_3] = [0, 1]$, $[a_4\ b_4] = [0, 1]$ and $[a_5\ b_5] = [3, 4]$.

We calculate the spectral vector of f, and we apply the theorem 3.13. We arrive at the following results:

$\chi(x_2(1)) = \frac{1}{2} \quad \chi(x_1(1)) = \frac{1}{2}$
$\chi(x_2(2)) = -\frac{3}{4} \quad \chi(x_1(2)) = -\frac{3}{4}$

We notice that $\chi \begin{pmatrix} x_2(1) \\ x_1(1) \end{pmatrix} \geq 0$ and $\chi \begin{pmatrix} x_2(2) \\ x_1(2) \end{pmatrix} < 0$

The system is live for the first step $(k = 1)$. It after loses its tokens (dead tokens) and its liveness property is not assured any more.

Case 2:

A second case is to consider these intervals such as $[a_3\ b_3] \cap [a_4\ b_4] = \varnothing$. The calculation of the spectral vector will enable us to show the non-liveness of the system in this case. We consider the following temporizations : $[a_1\ b_1] = [3, 4]$, $[a_2\ b_2] = [3, 4]$, $[a_3\ b_3] = [1, 2]$, $[a_4\ b_4] = [6, 7]$ and $[a_5\ b_5] = [4, 5]$. We obtain the following results :

$\chi(x_2(1)) = -\frac{12}{5} \chi(x_1(1)) = -\frac{12}{5}$

Then, in this case, the synchronization cannot be make to firing transition x_2 for the first time. The two tokens will die, the system is in state of blocking from the beginning because $\chi \begin{pmatrix} x_2(1) \\ x_1(1) \end{pmatrix} < 0$.

5 Conclusion

In this paper, we have introduced a new model, the interval descriptor system based on (min, max, +) functions and we have shown that p-time event graphs can be modelled in this form. The analysis of the spectral vector makes it possible to study the correct synchronization of the transitions. We have applied our approach to an example in section 4. A perspective will be to specify the token-deads and then to analyse the complete liveness of the model. We will show that each token-dead produces a variation in the model which contains the conditions of its own evolutions.

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