State Estimation in Time Stream Event Graphs. Application to fault detection

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Abstract - Based on topical algebra, this paper presents a new modelling of Time Petri nets whose time evolution belongs to intervals. The lower and upper bounds depend on maximization, minimization and addition operations. P-time Petri nets and Time Stream Petri Nets are examples which generalize the semantic of synchronization of Timed Petri Nets. This model allows us to apply the algebraic tools of the (max,+) or more generally of the topical algebra. In this paper, we consider the problem of supervision of Time Petri nets and particularly of estimation of the greatest state. The proposed approach presents similarities to the techniques of observers and parity space used in continuous systems for fault detection.

Keywords— **Time Stream Petri Nets, P-time Petri Nets, Estimation, Fault Detection, (min, max, +) Functions.**

I. INTRODUCTION

Discrete event dynamics systems involving synchronization can be modeled by several types of Petri nets (PNs). Among these PNs, we can quote P-time Petri nets (P-time PNs), Time stream Petri nets (Time Stream PNs),.... which extend the application field of Timed Event Graphs. Time Stream Event Graphs for example, allow to specify the synchronization requirements of multimedia applications [DIA 97] and can describe complex synchronizations. The Time Stream Petri Nets present different types of semantic as "And", "Weak-And", "Strong-Or", "Or", "Master"and theirs variations [COU 96] which correspond to different temporal evolutions. In [DID 04] and [DEC 05], we show that P-time Event Graphs and Stream Flow Event Graphs can be modeled by a new class of systems called interval descriptor system for which the time evolution is not strictly deterministic but belongs to intervals. The general model is as follows. The symbol $\oplus$ stands for the maximum operation while $\wedge$ corresponds to the minimum operation. Variable $x_i(k)$ is the date of the kth firing of transition i.

$$\begin{cases} x(k) = x(k) \wedge f^-(x(k), ..., x(k-m)) \\ x(k) = x(k) \oplus f^+(x(k), ..., x(k-m)) \end{cases}$$

In the general form, the lower and upper bounds depends on the maximization, minimization and addition operations. The liveness of P-time Event graphs can be studied in the topical algebra through the spectral vector [DID 04]
An important objective is to make control synthesis of systems described by an interval descriptor system, so that the system fulfil its production target : given a desired behavior of some transitions (output) of the interval descriptor system such as a sequence of execution times, we wish to slow down the system as much as possible without causing any event to occur later than this sequence. In Timed Event Graphs, optimal approaches can be found in [BAC 92] [COH 93].The state equations and the "backward" recursive equations provide the earliest and the latest times of the tasks respectively. The differences between the co-state and the state represent the "spare time" or the "margin" which is available for the firing of the transitions. The existence of a negative difference prevents the future deadlines from being achieved.
Thus, checking the control requires the knowledge of the vector state values which is not always available from the information system if the process has for example a human aspect. Moreover, discrete event systems undergo perturbations such as failures that disrupt the usual behavior of the process and reduce the capacities of anticipation of the control system. We must consequently, consider unavoidable model errors produced by the following "internal" perturbations:
- the physical but also human nature of the process entails a variation of the holding times;
- this situation also occurs when the process undergoes a failure and must be recovered;
- to prevent breakdowns, the pre-emptive maintenance produces necessarily perturbations in production.

These perturbations can be described as "internal perturbations" in opposition to "external perturbations" like variations of desired outputs or supplying of products and parts. These internal perturbations produce variations of the model or even ruptures of the description of the model. A consequence can be a wrong assessment of the state vector. In this context, different problems are:
- to compute the latest firing dates of the input transitions in such a way that the output events occur at the latest before the desired dates;
- to estimate on-line the past values of the state from the known values of the input and output;
- to predict the future evolution of the output and to check the optimality of the calculated solution of the control synthesis.

Let us notice that a variation of the model like a failure can produce an incoherence in the data as in continuous systems: for instance, in Parity Space [DEC 91] [DEC 92], a residue different from zero detects this variation; in observers, two different estimated values of the state show this incoherence . Let us recall the approach of parity space using redundancy relations corresponds to an observer of a particular type, whose poles are in the origin [PAT 91]. Similarly, it can be shown in Interval Descriptor Systems, that the estimated state space is nonempty in the normal case and empty in the faulty case which makes it possible to detect a variation in the model.

II. Preliminaries

A monoid is a couple $(S, \oplus)$ where the operation $\oplus$ is associative and presents a neutral element. A semi-ring S is a triplet $(S, \oplus, \otimes)$ where $(S, \oplus)$ and $(S, \otimes)$ are monoids, $\oplus$ is commutative, $\otimes$ is distributive relatively to $\oplus$ and the zero element ε of $\oplus$ is the absorbing element of $\otimes$ ($\varepsilon \otimes a = a \otimes \varepsilon = \varepsilon$) . A dioid D is an idempotent semi-ring (the operation $\oplus$ is idempotent, that is $a \oplus a = a$). Let us notice that contrary to the structures of group and ring, monoid and semi-ring do not have a property of symmetry on S. The unit $R \cup \{-\infty\}$ provided with the maximum operation denoted $\oplus$ and the addition denoted $\otimes$ is an example of dioid. We have : $R_{max} = (R \cup \{-\infty\}, \oplus, \otimes)$. The neutral elements of $\oplus$ and $\otimes$ are represented by $\varepsilon = -\infty$ and $e = 0$ respectively. The absorbing element of $\otimes$ is ε. Isomorphic to the previous one by the bijection: $x \longmapsto -x$, another dioid is $R \cup \{+\infty\}$ provided with the minimum operation denoted $\wedge$ and the addition denoted $\odot$. The neutral elements of $\wedge$ and $\odot$ are represented by $T = +\infty$ and $e = 0$ respectively. The absorbing element of $\odot$ is ε. The following convention is taken: $T \otimes \varepsilon = \varepsilon$ and $T \odot \varepsilon = T$. The expression $a \otimes b$ and $a \odot b$ are identical if at least either a or b is a finite scalar. The partial order denoted $\leq$ is defined as follows: $x \leq y \iff x \oplus y = y \iff x \wedge y = x \iff x_i \leq y_i$, for i from 1 to n in R^n. Notation $x < y$ means that $x \leq y$ and $x \neq y$. A dioid D is complete if it is closed for infinite sums and the distributivity of the multiplication with respect to addition extends to infinite sums : ($\forall$ $c \in D$) ($\forall$ $A \subseteq D$) $c \otimes (\bigoplus_{x \in A} x) = \bigoplus_{x \in A} c \otimes x$. For example, $\bar{R}_{max} = (R \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$ is complete. The set of $n.n$ matrices with entries in a complete dioid D endowed with the two operations $\oplus$ and $\otimes$ is also a complete dioid which is denoted $D^{n.n}$. The elements of the matrices in the (max,+) expressions (respectively (min,+) expressions) are either finite or ε ((respectively T). We can deal with nonsquare matrices if we complete by rows or columns with entries equals to ε (respectively T). The different operations operate as in the usual algebra: The notation $\odot$ refers to the multiplication of two matrices in which the $\wedge$-operation is used instead of $\oplus$. The mapping f is said residuated if for all $y \in D$, the least upper bound of the subset $\{x \in D \mid f(x) \leq y\}$ exists and lies in this subset. The mapping x$\in (\bar{R}_{max})^n \mapsto A \otimes x$ defined over $\bar{R}_{max}$is residuated [BAC 92] and the left $\otimes-$ residuation of B by A is denoted by: $A \backslash B = \max\{x \in (\bar{R}_{max})^n$ such that $A \otimes x \leq B\}$.

In ($\oplus$, $\otimes$) algebra, Kleene's star is defined by: $A^* = \bigoplus_{i=0}^{+\infty} A^i$. Denoted $G(A)$, an induced graph of a square matrix A is deduced from this matrix by associating: a node i to the column i and line i ; an arc from the node j towards the node i if $A_{ij} \neq \varepsilon$. The weight of a path p, $\mid p \mid_w$ is the sum of the labels on the edges in the path. The length of a path p, $\mid p \mid_l$ is the number of edges in the path. A circuit is a path which starts and ends at the same node.

Theorem 1 [BAC 92] For matrix A with induced graph $G(A)$, if the cycle weights in $G(A)$ are all strictly negative, then there is a unique solution to the equation $x = A \otimes x \oplus B$ which is given by $A^* \otimes B$.

Definition 2 [COC 99] A (min, max, +) function of type $(n, 1)$ is any function $f : R^n \longrightarrow R^1$, which can be written as a term in the following grammar: $f = x_1, x_2, \ldots, x_n \mid f \otimes a \mid f \wedge f \mid f \oplus f$ where a is an arbitrary real number ($a \in R$). The vertical bars separate the different ways in which terms can be recursively constructed. A (min,max,+) function of type (n, m) is any function f : $R^n \longrightarrow R^m$, such that each component f_i is a (min, max, +) function of type $(n, 1)$. The set of (min, max, +) function of type (n, m) is noticed $F(n, m)$ and is a special class of topical functions which are homogeneous, monotonic and nonexpansive. Only homogeneity ($\forall \lambda \in R, \forall x \in R^n$ $f(\lambda \otimes x) = \lambda \otimes f(x)$ in the usual vector-scalar convention: $(\lambda \otimes x)_i = \lambda \otimes x_i$) will be used. They include (max, +) linear maps and (min, +) linear maps which can be written respectively as: $g(x)_i = \bigoplus_{1 \leq j \leq n} (A_{ij} \otimes x_j)$ where A is a $n \times n$ matrix with entries in $R \cup \{-\infty\}$; $h(x)_i = \bigwedge_{1 \leq j \leq n} (B_{ij} \otimes x_j)$ where B is a $n \times n$ matrix with entries in $R \cup \{+\infty\}$

Let $f \in F(n, n)$. A subset $S \subset F(n, n)$ is said to be a max-representation of f if S is a finite set of (max,+) functions such that $f = \bigwedge_{h \in S} h$. A subset $T \subset F(n, n)$ is said to be a min-representation of f if T is a finite set of (min,+) functions such that $f = \bigwedge_{h \in T} h$. The mutual distributivity of $\otimes$ and $\wedge$ ($(x \oplus y) \wedge z = (x \wedge z) \oplus (y \wedge z)$ and $(x \wedge y) \oplus z = (x \oplus z) \wedge (y \oplus z)$) entails that every (min, max, +) function have a max-representation and a min-representation.

The set of (min, max, +) functions $F(n, n)$ has a natural representation as an n-fold cartesian product: $F(n, n)$ =$F(n, 1)$x...$F(n, 1)$. Let R_i the set $\{h_i$ such that $h \in S\}$. The rectangularisation of S, denoted $rec(S)$, is defined by $rec(S)$ =R_1xR_2x...xR_n. In other words, a set S of min-max functions is rectangular if for all h, $h' \in S$, and for all $i = 1, ..., n$ the function obtained by replacing the i-th component of h by the i-th component of h' belongs to S. So, $rec(S)$ is finite when S is finite and $S \subset rec(S)$.

Dynamics of the form are considered: $x(k) = f(x(k-1))$, $\forall k \geq 1$ and $x(0) = \xi \in R^n$ where f is a (min, max, +)

function of type (n, n) $R^n \rightarrow R^n$. The cycle time vector is defined by $\chi(f) = \lim_{k\rightarrow\infty} x(k)/k$ if it exists. It does not depend on ξ. In the following theorem, the notion of cycle time which always exists in $F(n, n)$ makes it possible to check the existence of a solution of different inequalities and equalities.

Theorem 3 [GAU 98] [COC 99] [COC 01] Let $f \in F(n, n)$. The two following conditions are equivalent: (i) It exists a finite x such that $x \leq f(x)$ (respectively, $x \geq f(x)$) (ii) $\chi(f) \geq 0$ (respectively, $\chi(f) \leq 0$)

The calculation of the spectral vector can be realized as follows. If c is a circuit, its cycle mean, denoted $m(c)$ is defined by $m(c) =\mid c \mid_w / \mid c \mid_l$(the notation / represents the classical division). A node j is upstream from i, denoted $i \Longleftarrow j$, if either $i = j$ or there is a path in $G(A)$ from j to i. Vector $\mu(A) \in R^n$ is defined by $\mu_i(A) = \max\{m(c) \mid i \Longleftarrow c\}$. If $f \in F(n, n)$ is $\max$-only and A is the associated matrix over R_{max}, then $\chi(f)$ exits and $\chi(f) = \mu(A)$. The result is identical for min-only function. If S and T are rectangular max and min-representations, respectively, of $f \in F(n, n)$, then $\chi(f) = \bigwedge_{h\in S} \chi(h) = \bigoplus_{g\in T} \chi(g)$.

III. Models of Time Stream Event Graphs

Time Stream PNs are an extension of Petri nets which allows to represent complex synchronizations and temporal compositions of the tasks or processes which are carried out [COU 96] [DIA 97]. Time Stream PNs directly extend P-time PNs. We consider Event Graphs which constitute a subclass of Petri nets of which each place has exactly one upstream and one downstream transition.

A. Description of Time Stream Event Graphs

Definition 4 (Time Stream Event Graph) Let I_j a set of upstream arcs of a transition j and P_j the corresponding set of upstream places. A Time Stream Event Graph is an Event-Graph such as: an interval $[\alpha_i, \beta_i]$ $(Q^+ \cup 0) \times (Q^+ \cup +\infty)$ is associated to each $a_i \in I_j$; defined below, a special semantic of firing is associated to each transition.

Considering one outgoing arc from a given place, when a token is received by that place at time x , the token should remain in the place during an amount of time defined by a value inside the range $[x + \alpha, x + \beta]$ associated with the arc. As the firing time of a transition depends on the nature of the processes which will be synchronized, different semantics of firing may be associated to a transition. In this paper, we consider two types of semantics, And and Weak-And, which we will use later. They are defined by a couple $[x + \alpha_i, x + \beta_i]$ associated to each ingoing arc.

Definition 5 For a transition i , let I_i denote a set of upstream arcs and P_i the corresponding set of upstream places. A transition i of the type "And" and "Weak-And" is firing at absolute time x_i if and only if the two following conditions are satisfied:

1) transition i is enabled for the current marking: every upstream place j of P_i contains at least one token. Let x_j the entrance date of the token which is also the date of firing of the upstream transition of this place.

2) For the **semantic And,** the value of x_i is such as: $(x_j + \alpha_j) \leq x_i \leq (x_j + \beta_j)$ for every upstream place $p_j \in P_i$ and arc $a_j \in I_i$ (every time condition has to be fulfilled).

For the **semantic Weak-And,** the value of x_i is such as: $(x_i + \alpha_i) \leq x_i$ for every upstream place $p_j \in P_i$ and arc $a_j \in I_i$ and $\exists j \in P_{i,}\ x_i \leq (x_j + \beta_i)$ (the firing may wait until the last time interval).

Now, let us consider the variable $x_i(k)$ as the date of the kth firing of transition i and P_i the set of the upstream places of this transition. If we take the assumption of functioning FIFO of the transition i which guarantees the condition of non overtaking of the tokens between them, a numbering of the events can be carried out and the model can be written as follows: Given n_j the number of the present tokens in each place p_j at the instant $t = 0$ (initial marking), for each transition , $\bigoplus_{j\in P_i} (x_j(k - n_j) + \alpha_j) \leq x_i(k) \leq \bigwedge_{j\in P_i} (x_j(k - n_j) + \beta_j)$ if the semantic is And; $\bigoplus_{j\in P_i} (x_j(k - n_j) + \alpha_j) \leq x_i(k) \leq \bigoplus_{j\in P_i} (x_j(k - n_j) + \beta_j)$ if the semantic is Weak-And.

Let us notice that the inequalities of P-time Event Graph correspond to semantic And.

B. (min, max, +) algebraic models

One can represent the date $x(k)$ by the following formal power series in γ: $x(\gamma) = \bigoplus_{k\in Z} x(k)\gamma^k$. Variable may also be regarded as the backward shift operator in event domain (formally, $\gamma x(k) = x(k - 1)$) and γ-transforms of functions can express this effect. Reciprocally, the dater algebraic function $\Phi_{f,l}(X)$ associated to a formal $f(x(\gamma))$ on a horizon l is a function obtained by developing $f(x(\gamma))$ algebraically with dater variables over the appropriate dimensions. It describes every connection which links the different variables which composed $X = (x(k), x(k + 1), ..., x(k + l))^t$ with (min, max, +) functions. The evolution of the system is described by the following equations where f^- and f^+ are formal (min, max, +) functions on the set of sequences over $R \cup \{\pm\infty\}$

$$f^-(x(\gamma), u(\gamma)) \leq x(\gamma) \leq f^+(x(\gamma), u(\gamma))$$

The vectors x and u are respectively the state and the input. We can also introduce the output y by $y(\gamma) = C(\gamma) \otimes x(\gamma)$ without reduction of generality. As the type of the system is defined by the types of the functions f^- and f^+, we can characterize the model by the following couple (type of f^-, type of f^+). The type ((min,max,+), (min,max,+)) represents naturally the more general mathematical case. Under the assumption of the existence of a solution, they define corresponding classes of interval descriptor systems.

B.1 Timed Event Graphs

If the lower bound defined by f^- is a (max,+) function and the upper bound is infinite, the (max,+) inequation can be obtained. In this case, equality arises from the assumption that there is no extra delay for firing transitions whenever tokens are all available.
$x(\gamma) = A\gamma \otimes x(\gamma) \bigoplus B \otimes u(\gamma)$
$y(\gamma) = C(\gamma) \otimes x(\gamma)$
avec $k \in [k_s\ k_f]$, $\dim(x) = n, \dim(u) = q, \dim(y) = \dim(z) = m$,
$\dim(A) = n \times n, \dim(B) = n \times q, \dim(C) = m \times n$
Consequently, $f^-(x(\gamma), u(\gamma)) = f^+(x(\gamma), u(\gamma)) = A\gamma \otimes x(\gamma) \bigoplus B \otimes u(\gamma)$

B.2 Time Stream Event Graphs

For stream Petri nets for semantics And and Weak-And, $f^-(x(\gamma), u(\gamma))$ can be a (max, +) function and $f^+(x(\gamma), u(\gamma))$ a (min, max, +) function. We can write: $f^-(x(\gamma), u(\gamma)) = A^- \otimes x(\gamma) \oplus B^- u(\gamma)$ and

$$f^+(x(\gamma), u(\gamma)) = \bigwedge_{i=1}^{j_1} A_i^+ \otimes x(\gamma) \oplus B_i^+ u(\gamma)$$

So, this form generalizes the form of Timed Event Graphs if we take: $j_1 = 1$; $A^- = A_1^+ = A\gamma$; $B^- = B_1^+ = B$;

IV. Estimation

The objective is to find the least upper bound of $x(k)$ knowing the values of the input $u(k)$ and the output $y(k)$ for k from k_s to k_f with k_s and k_f the numbers of start and final events. The model is supposed to be known on the same horizon of observation. One can notice that this problem of estimation is thus different from the control synthesis which considers that the control and the output are the unknown data. The upper bound of the estimate and the co-state have a similar type but meet two different aims.
As functions f^- is residuated , the determination of the greatest solution $x(\gamma)$ of the following inequality set, will give the greatest estimate.
theorem 6 The problem of the greatest estimate can be written as follows: search the greatest state of the following inequality $x(\gamma) \leq h\,(x(\gamma))$ with

$$h\,(x(\gamma)) = \begin{pmatrix} (A^- \backslash x(\gamma)) \wedge (C \backslash y(\gamma)) \wedge \\ \bigwedge_{i=1}^{j_1} (A_i^+ \otimes x(\gamma) \oplus B_i^+ u(\gamma)) \end{pmatrix} \quad (1)$$

with the constraints $u(\gamma) \leq B^- \backslash x(\gamma)$ and $y(\gamma) \leq Cx(\gamma)$

Proof

$$A^- \otimes x(\gamma) \oplus B^- u(\gamma) \leq x(\gamma) \leq \bigwedge_{i=1}^{j_1} A_i^+ \otimes x(\gamma) \oplus B_i^+ u(\gamma)$$

$y(\gamma) = C(\gamma) \otimes x(\gamma)$

So, $x(\gamma) \leq (A^- \backslash x(\gamma)) \wedge \bigwedge_{i=1}^{j_1} A_i^+ \otimes x(\gamma) \oplus B_i^+ u(\gamma)$

$u(\gamma) \leq B^- \backslash x(\gamma)$
$y(\gamma) \leq C(\gamma) \otimes x(\gamma)$
and $C(\gamma) \otimes x(\gamma) \leq y(\gamma)$

In short, $x(\gamma) \leq A^- \backslash x(\gamma) \wedge C \backslash y(\gamma) \wedge \bigwedge_{i=1}^{j_1} A_i^+ \otimes x(\gamma) \oplus$

$B_i^+ u(\gamma)$
with $u(\gamma) \leq B^- \backslash x(\gamma)$ and $y(\gamma) \leq C(\gamma) \otimes x(\gamma)$ □
Clearly, this set contains (min,max,+) functions. Notice, that the first expression presents an usual backward part $(A^- \backslash x(\gamma)) \wedge (C \backslash y(\gamma))$ but also, in the case where A_i^+

and B_i^+ have positive exponents a forward part ($\bigwedge_{i=1}^{j_1} (A_i^+ \otimes$

$x(\gamma) \oplus B_i^+ u(\gamma)))$ which increases the complexity of the problem and forbids the writing of simple equations on a short horizon as the classical backward equations in control. In other words, we must solve a (min, max, +) fixed-point problem of type $x \leq f(x)$ over the horizon of the known values of the control u and the output y. Let us notice that h is not a (min, max, +) function of type (n,m) which is homogeneous. The form of our practical problem is to find a greatest x (if x exists) such that

$$x \leq f(x) \quad (2)$$

with f a non-homogeneous (min, max, +) function which can be defined by the following grammar: $f = b, x_1, x_2, \ldots, x_n \mid f \otimes a \mid f \wedge f \mid f \oplus f$ where a, b are arbitrary reals.

A. Analysis by spectral theory

In the aim of applying the spectral theory about these functions, we will realize a relaxation by associating in the above definition a variable x_0 to b such that b is replaced by $b \otimes x_0$. So, the problem is to find a greatest $y = (x_0, x_1, ..., x_n)^t$ (if y exists) such that $x_0 = 0$ and $x_i \leq f_i(x_0, x_1, ..., x_n)^t)$ for $i \neq 0$. If we introduce the obvious inequality $x_0 \leq x_0$, the general problem becomes: find a greatest $y = (x_0, x_1, ..., x_n)^t$ (if y exists) such that $x_0 \leq x_0$ and $x_i \leq f_i(x_0, x_1, ..., x_n)^t)$ for $i \neq 0$ with $x_0 = 0$. In other terms, we have to solve the new system

$$y \leq g(y) \quad (3)$$

with g an homogeneous function of type $(n+1, n+1)$. The followin theorem makes it possible to apply the spectral theory to the estimation problem.

Theorem 7 The system of ((max, +), (min, max, +)) type can be defined formally by

$$\begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix} \leq h_r \begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix} \qquad (4)$$

with

$$h_r \begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix} = \begin{pmatrix} x_0 \\ (A^{-} \backslash x(\gamma)) \wedge (C \backslash (y(\gamma) \otimes x_0)) \wedge \\ \bigwedge_{i=1}^{j_1} (A_i^{+} \otimes x(\gamma) \oplus B_i^{+} u(\gamma) \otimes x_0) \end{pmatrix}$$

Remark

As the resolution is now applied to system (3), the constraints $u(\gamma) \leq B^{-} \backslash x(\gamma)$ and $y(\gamma) \leq Cx(\gamma)$ are replaced by $x_0 = 0$ which must be verified. As the approach is based on a minimization of x_0 and x , a fault is detected when $x_0 < 0$

Theorem 8

It exists a solution verifying the system (4) on horizon l if and only if $\chi(\Phi_{h,l}(.)) \geq 0$

Proof The final inequality set presents the general form $x(\gamma) \leq \varphi(x(\gamma))$ and is associated to the algebraic inequality $X \leq \Phi_{h,l}(X)$. The spectral vector is here $\chi(\Phi_{h,l}(X))$. The system of ((max, +), (min, max, +)) type is reduced to a ($-\infty$, (min, max, +)) type and can be analyzed by the relevant theorem (3). If the cycle time verifies the corresponding condition of existence, it describes a compatible interval descriptor system. □

Now, we consider the expression of the spectral vector and its underlined structure of h.

In the function h, we can consider the classical "backward" part. $\begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix} \leq h_1 \begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix}$ with $h_1 \begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix} = \begin{pmatrix} x_0 \\ (A^{-} \backslash x(\gamma)) \wedge (C \backslash (y(\gamma) \otimes x_0)) \end{pmatrix}$

The structural observability (respectively controllability) gives a condition to observe an effect in the output (resp. transition) whose origin comes from at least one internal transition (resp. input) and allows us to introduce the following propositions.

Definition 9 [BAC 92] An event graph is structurally controllable if, every internal transition can be reached by a path from at least one input transition. An event graph is structurally observable if, from every internal transition, there exists a path to at least one output transition.

Theorem 10 A structurally observable event graph verifies $\chi(\Phi_{h_1,l}(.)) = 0$ and the greatest solution of "backward" part h_1 verifies $h(x) \leq x$ and gives an upper bound of the solutions of $x \leq h(x)$ and $x = h(x)$

Proof

Given $h(x) = h_1(x) \wedge h_2(x) \wedge h_3(x) \wedge ....$ with h_i a max only function.

- A circuit whose weight equals zero, is associated to x_0 which is associated to every known values particularly the outputs. If we assume that the system is observable, from every internal transition, there exists a path to at least one output transition in the event graph. In the associated graph, the direction of the paths are opposite and for every vertex corresponding to a transition at k, it exists a path going from x_0. The definition of the spectral vector shows that $\chi(h_1) = 0$. Consequently, it exists x_1 such that $x_1 = h_1(x_1)$.

As $h(x_1) = h_1(x_1) \wedge h_2(x_1) \wedge h_3(x_1) \wedge .... \leq h_1(x_1) = x_1$, the greatest solution of "backward" part verifies $h(x) \leq x$.

- It can be proved that the greatest solution of the equality $x = h_1(x)$, denoted x_2 is also the greatest solution satisfying $x \leq h_1(x)$. Moreover, as $x \leq h(x)$ is equivalent to the set of inequalities $x \leq h_i(x)$, every variable x must satisfy each inequality and particularly $x \leq h_1(x)$. Consequently, each solution x de $x \leq h(x)$ is lower than or equal to x_2. Finally, this set includes the solution set satisfying $x = h(x)$ □.

Theorem 11 In a structurally observable event graph, there exists a solution verifying the equality $\begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix} = h \begin{pmatrix} x_0 \\ x(\gamma) \end{pmatrix}$ or there exists no solution in inequality (4)

Proof The spectral vector of the complete system is lower than or equals zero. The terms $[Ax(k-1) \oplus Bu(k) \otimes x_0]$ and $u(k) \backslash (B \backslash x(k)) \wedge y(k) \backslash (Cx(k))$ can create strictly negative circuits and in this case, there is no solution. □

B. Algorithm

The notation $M(i,.)$ represents the line i of matrix M, and $M(.,j)$ represents the column j of matrix M. Now we specify the model of Time Stream Event Graphs as follows:

$$f_i^{+}(x(\gamma), u(\gamma)) = \bigwedge_{l=1}^{j(i)} h_l(x(\gamma), u(\gamma))$$

with $j(i)$ is the number of the functions max-plus only h_l corresponding to component (transition) x_i and

$h_l(x(\gamma), u(\gamma)) = A_l^{+}(i,.) \otimes x(\gamma) \oplus B_l^{+}(i,.) \otimes u(\gamma)$

Element $A_l^{+}(i,j)$ equals ε (respectively $B_l^{+}(i,j) = \varepsilon$) if there are not places connecting internal transition directly from x_j to x_i (respectively from input transition u_j to internal transition x_i). $A_l^{+}(i,.) = (T, T, \ldots, T)$ and $B_l^{+}(i,.) = (T, T, \ldots, T)$ for each component x_i such that $l > j(i)$.

Remark The matrices A_l^+ and B_l^+ can comprise at the same time ε and T elements. The manipulation of these matrices is possible since we considered a complete dioid.

The effective calculation of the greatest control can be made by a classical iterative algorithm [HAS 99]. The resolution of $x \leq f(x)$ is given by the iterations of $x_{i+1} \leftarrow f(x_i) \wedge x_i$ if the starting point is greater than the final solution. Number i represents here the number of iteration and not the number of component of the vector x.
Following this framework, we give below an algorithm specific to the estimation of the greatest state for Time Stream Event Graph. It can also be applied to Timed and P-time Event Graphs.

Step 0 : $\mu_i(k_f) \leftarrow T; \lambda_i(k_f) \leftarrow T$
Repeat until $\lambda_i(k) = \mu_i(k)$ for $1 \leq i \leq n$ and $k_s \leq k \leq k_f$,
Step 1 : $\lambda_i(k) \leftarrow \mu_i(k) \wedge [A^-(.,i)\backslash\lambda(k+1)] \wedge [C(.,i)\backslash y(k)]$
Step 2 : $\mu_i(ks) \leftarrow \lambda_i(ks)$

$$\mu_i(k) \leftarrow \lambda_i(k) \wedge \bigwedge_{l=1}^{j(i)} [A_l^+(i,.) \otimes \mu(k-1) \oplus B_l^+(i,.) \otimes u(k)]$$

for $k > k_s$
As the general algorithm is known to be pseudo-polynomial, the above algorithm converges to the greatest state for Time Stream Event Graphs (with semantics And and Weak-And) in a finished number of iterations.
Proposition 12
For Timed Event Graph , the algorithm converges to the greatest state in one iteration.
proof
- If f is residuated, then it exists h such that $foh \leq Id$ and $hof \geq Id$ [BAC 92]
So, $A \otimes (A\backslash x) \leq Id$ and $A\backslash[A \otimes x] \geq Id$
Consequently, $A\backslash[A \otimes \mu(k)] \geq \mu(k)$ and $A\backslash[A \otimes \mu(k) \oplus B \otimes u(k+1)] \geq \mu(k)$ by isotonie of $\backslash$
- moreover, step 3 entails $\mu(k) \leq \lambda(k)$
In short, $\mu(k) \leq \lambda(k) \wedge A\backslash[A \otimes \mu(k) \oplus B \otimes u(k+1)]$
Moreover,
$A\backslash\mu(k+1) \wedge C\backslash y(k) = A\backslash[\lambda(k+1) \wedge [A\mu(k) \oplus Bu(k+1)]] \wedge C\backslash y(k) =$
$A\backslash\lambda(k+1) \wedge A\backslash[A\mu(k) \oplus Bu(k+1)] \wedge C\backslash y(k) =$
$A\backslash\lambda(k+1) \wedge C\backslash y(k) \wedge A\backslash[A\mu(k) \oplus Bu(k+1)] =$
$\lambda(k) \wedge A\backslash[A\mu(k) \oplus Bu(k+1)]$ because $\mu(k) = T$ for the first step.
Finally, as $\mu(k) \leq [A\backslash\mu(k+1)] \wedge [C\backslash y(k)]$, the algorithm does not need a new minimisation by step 1. □

C. Application to fault detection

Parity Space
As a fault is not modelled in the nominal model, it entails an incoherency between the nominal model and the change of the process. So, the fault detection approach is as follows.

For a given horizon l, check $\chi(\Phi_{h_1,l}(.)) = 0$ in a structurally observable event graph or check $\chi(\Phi_{h_1,l}(.)) \geq 0$ otherwise. In any cases, the case ($\chi(\Phi_{h_1,l}(.))$ is not greatest than or equals zero) corresponds to the existence of a fault. Consequently, the spectral vector in a structurally observable event graph, corresponds to the vector of parity space. Moreover, the structural analysis of the spectral vector shows that equalities linking only known values can be generated. This fact shows that a strong analogie can be realized between substructure generated by the spectral vector and analytical redundancy relations of parity space.

Observer
If the resolution is applied to system (1), the constraints $u(\gamma) \leq B^-\backslash\widehat{x}(\gamma)$ and $y(\gamma) \leq C\widehat{x}(\gamma)$ must be verified when the system follows its normal or nominal model and $\widehat{y}(k) = y(k)$. Otherwise an evolution of the process is detected.

V. EXAMPLE

In the aim of clearly illustrating the approach, we consider a tutorial example. Calculations has been realised with Scilab.

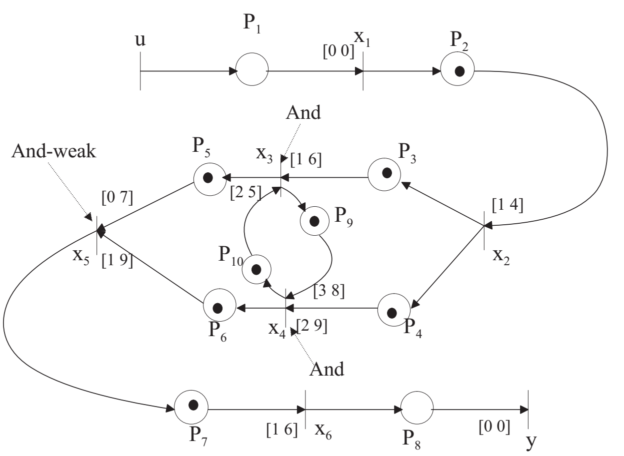

Fig. 1. Time Stream Event Graphs

Estimation
We consider a simple Time Stream Event Graph (see Figure 1) whose nominal model $M_1$is as follows:

$$A^- = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 1 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon & 2 & \varepsilon & \varepsilon \\ \varepsilon & 2 & 3 & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & e & 1 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 1 & \varepsilon \end{pmatrix},$$

$$A_1^+ = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 6 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 9 & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 7 & 9 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon & \varepsilon & 6 & \varepsilon \end{pmatrix},$$

$$A_2^+ = \begin{pmatrix} T & T & T & T & T & T \\ T & T & T & T & T & T \\ \varepsilon & \varepsilon & \varepsilon & 5 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & 8 & \varepsilon & \varepsilon & \varepsilon \\ T & T & T & T & T & T \\ T & T & T & T & T & T \end{pmatrix} B^- = \begin{pmatrix} e \\ \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix},$$

$$B_1^+ = \begin{pmatrix} e \\ \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix}, B_2^+ = \begin{pmatrix} T \\ T \\ \varepsilon \\ \varepsilon \\ T \\ T \end{pmatrix} \text{and}$$

$$C = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & e \end{pmatrix}.$$

First, we calculate an acceptable trajectory which satisfies the algebraic model of the system in the horizon $[1, 10]$.

k	1	2	3	4	5	6	7	8	9	10
u	3	5	7	9	11	13	17	19	22	24
x_1	2	5	7	9	11	13	17	19	22	24
x_2	1	3	6	8	10	12	14	18	20	23
x_3	2	5	7	10	12	15	17	20	22	25
x_4	3	5	8	10	13	15	18	20	23	25
x_5	1	4	6	9	11	14	16	19	21	24
x_6	1	5	8	11	13	16	18	20	25	27
y	1	5	8	11	13	16	18	20	25	27

Then, the observer uses u and y for estimate the greatest state $\widehat{x}$.

k	1	2	3	4	5	6	7	8	9	10
$\widehat{x_1}$	3	5	7	9	11	13	17	19	22	24
$\widehat{x_2}$	6	9	9	11	13	15	17	21	23	26
$\widehat{x_3}$	5	8	10	13	15	18	21	23	27	29
$\widehat{x_4}$	6	8	11	13	16	18	21	25	27	32
$\widehat{x_5}$	4	7	10	12	15	17	19	24	26	36
$\widehat{x_6}$	1	5	8	11	13	16	18	20	25	27
$C \otimes \widehat{x}$	1	5	8	11	13	16	18	20	25	27
$B^- \backslash \widehat{x}$	3	5	7	9	11	13	17	19	22	24

The constraints $y \leq C \otimes \widehat{x} = \widehat{y}$ and $B^- \otimes u \leq \widehat{x}$ are verified on the horizon $[1, 10]$.

Fault detection

Now, a failure is considered. The temporization for the arc connecting the place p_6 with the transition x_5 is $[7, 9]$. We have a new model M_2 after failure, $A(5, 4) = 7$. The new model M_2 was used to calculate a new output trajectory y. The observer uses the nominal model M_1 and the new data to estimate a new trajectory $\widehat{x}$. The results of a new calculations is given by the following table

k	1	2	3	4	5	6	7	8	9	10
$\widehat{x_1}$	3	5	7	9	11	12	16	18	21	23
$\widehat{x_2}$	7	9	9	11	13	15	17	21	23	26
$\widehat{x_3}$	6	8	11	13	16	18	21	23	27	29
$\widehat{x_4}$	6	9	11	14	16	19	21	25	27	32
$\widehat{x_5}$	7	10	18	20	23	25	28	30	34	36
$\widehat{x_6}$	1	8	11	24	26	29	31	34	36	40
y	1	8	11	28	31	33	35	37	39	41
$C \otimes \widehat{x}$	1	8	11	24	26	29	31	34	36	40
$B^- \backslash \widehat{x}$	3	5	7	9	11	12	16	18	21	23

Knowing the input and the outpout from $k = 1$ to 10, the procedure detects the fault : the constraints $y \leq C \otimes \widehat{x} = \widehat{y}$ and $B^- \otimes u \leq \widehat{x}$ are not verified on the horizon $[4, 10]$ ($y(k)$ is not lower than or equals to $\widehat{y}(k)$ for $k = 4$ to 10 and $B^- \otimes u(k)$ is not lower than or equals to $\widehat{x}(k)$ for k=6 to 10)

VI. CONCLUSION

In this paper, we propose an approach based on an constraint propagation (Declerck P. and Didi Alaoui K. 2003) which estimates the greatest solution in Stream Event Graphs. The particular case of the classical state equation of Timed Event Graphs can be reformulated under the form $x \leq f(x)$ with $f(x)$ a special (min, max,+) function. This formulation is similar to the form of observer in classical automatic control and in usual algebra. A "backward/forward" resolution calculates the greatest estimate of the state. Moreover, the checking of the estimate make it possible to detect the possible incoherencies in the model as variations in the values of temporizations. At last, the spectral vector in a structurally observable event graph, show a strong analogie to the vector of parity space.

REFERENCES

[BAC 92] BACCELLI F., COHEN G., OLSDER G., QUADRAT J., *Synchronization and linearity. An Algebra for Discrete Event Systems*, Wiley, New York, 1992.

[COC 99] COCHET-TERRASSON J., GAUBERT S., GUNAWARDENA J., A constructive fixed point theorem for min-max functions, vol. 14, 1999.

[COC 01] COCHET-TERRASSON J., Algorithme d'itration sur les politiques pour les applications monotones contractantes, Thèse, Ecole Nationale Supérieure des Mines de Paris, Décembre 2001.

[COH 93] COHEN G., GAUBERT S., QUADRAT J., From first to second-order theory of linear discrete event systems, *12th IFAC*, Sydney, Jul. 1993.

[COU 96] COURTIAT J., DIAZ M., DE OLIVIERA R., SNAC P., *Formal models for the description of timed behaviors of multimedia and hyper media distributed systems*, vol. 19, p. 1134–1150, Computer Communications, 1996.

[DEC 91] DECLERCK P., Analyse structurale et fonctionnelle des grands systèmes complexes. Application à une centrale PWR 900MW, Ph.D., Villeneuve d'Ascq, Décembre 1991.

[DEC 92] DECLERCK P., STAROSWIECKI M., *Generation of analytical redundancy relations for fault detection*, p. 18–21, International conference Sicici '92, Singapore, IEEE, 1992.

[DEC 05] DECLERCK P., DIDI-ALAOUI M., *Optimal Control synthesis in Interval Descriptor Systems*, 16th IFAC World Congress, 2005.

[DIA 97] DIAZ M., OWEZARSKI P., *From multimedia models to multimedia transport protocols*, vol. 29, p. 745–758, Computer Networks and ISDN Systems, august 1997.

[DID 04] DIDI-ALAOUI M., DECLERCK P., *From dioid algebra to P-time Event graphs*, International Conference on Informatics in Control, Automation and Robotics, ICINCO, 2004.

[GAU 98] GAUBERT S., GUNAWARDENA J., *The duality theorem for min-max functions*, *IEEE Trans. on Automatic Control*, vol. t. 326 Srie I, p. 43–48, C. R Académie des Sciences/ Elsevier, Paris, 1998.

[HAS 99] HASHTRUDI-ZAD S., KWONG R., WONHAM W., *Supremum operators and computation of supremal elements in system theory*, *Siam J. Control Optm*, vol. Vol 37, No.3, p. 695–709, 1999.

[PAT 91] PATTON R., J. C., *A Parity Space Approach to Robust Fault Detection, using Eigenstructure Assignment*, *ECC '91*, 1991.