ANALYTICAL REDUNDANCY IN NON LINEAR INTERCONNECTED SYSTEMS BY MEANS OF STRUCTURAL ANALYSIS.

M. Staroswiecki , P. Declerck

Centre d'Automatique de Lille , Université des Sciences et Techniques de Lille Flandres-Artois , 59655 Villeneuve d'Ascq Cedex , France

Abstract . The parity space approach gives a means to generate static coherence models for fault detection , when the system is equipped with a number of sensors large enough . When few sensors are implemented , the generalized parity space approach has to be used and gives dynamic coherence models . Both approaches are based on an analytical expression of the knowledge we have about the system , under the form of linear state and measurement equations .
However, such a representation is not often available for large scale complex industrial systems . Those systems are characterized by the great number of variables which are necessary for their description , and by the great variety of the types of relationships which link these variables : qualitative or quantitative , statical or dynamical , linear or non linear .
The paper presents an approach based on structural analysis in order to exhibit coherence models for fault detection of large scale systems . The initial knowledge upon the normal operation of the system is given by its representation under the form of a network of elementary activities , issued from its functionnal analysis .

Keywords . Analytical redundancy ; coherence model ; large scale systems ; failure detection ; structural analysis .

INTRODUCTION

The basic principle of fault detection is the comparison of the actual behaviour of the system to a reference behaviour describing its normal operation . The reference behaviour is issued from the knowledge which is available upon the system , this knowledge being expressed under more or less precise terms , and under formalisms which may be very different (knowledge base , analytical models , ...). One of the most frequently used approach is based on the use of Analytical Redundancy Relationships (ARR) : the knowledge available upon the system leads to express its normal operation by a set of invariants : the residuals of the ARR (coherence model) . The fault detection resumes thus to a decision problem : is the variance of the residuals the effect of noise , of normal deviations and errors or the effect of a failure .

The implementation of a physical redundancy of sensors is an immediate means to generate redundancy relationships . This method allows the detection of sensors'failures but it is rather costly and moreover , sensible to common mode failures . The parity space approach (Potter 1977) based on the measurement equation , allows to use different sensors in order to generate ARR . However , the number of available sensors has to be larger than the order of the system . When few sensors are available , a fictitious redundancy has to be generated by observing the system on a larger time interval . One has then to use the measurement and the state equations , and obtains dynamical ARR whose residuals will be considered by the decision procedure (generalized parity space (Chow 1980 ; Chow 1984)) .

Both the parity space and the generalized parity space approaches are based on an analytical expression of the knowledge we have about the system : state and measurement equations . Moreover , these equations have to be linear : in fact , the residuals are obtained using a projection operator in the state space .

However , it is the most frequent case that such a representation is not available for large scale complex industrial systems . Those systems are characterized by the great number of variables which are necessary for their description , and by the great variety of the types of relationships which link these variables : qualitative or quantitative , statical or dynamical , linear or non linear . The functional analysis of such complex processes leads to represent their operation by means of a number of elementary activities , which are interconnected by product , energy or information flows . In that network , each of the elementary activities gives rise to a relationship linking two or more variables of the process . Among the set of all the variables , only some of them are known (computed by elementary activities) or measured (a sensor performs also an elementary activity) .

The present work uses such a representation in order to generate ARR for fault detection . The ARR are the result of a systematic approach which can be decomposed into two steps :

* <u>Qualitative step</u> . The structural analysis of the process (Richetin 1975 ; Harary 1962) gives subsets of non independant known or measured variables . It gives also subsets of elementary activities (or process functions) which link these variables . Each of those subsets will give rise to one or more ARR.

* <u>Quantitative step</u> . This step consists in the computation of the ARR corresponding to each of the previously mentionned subsets . This computation is made possible by the ordering of the process functions which is performed , within each subset , by the structural analysis .

The obtained ARR are the result of the overdetermination , within the system , of one or more variables .

We present an algorithm which systematically exhibits all the overdeterminations . The approach is based on the introduction of real or fictitious variables : the pretext for resolution variables (PRV) . Structural analysis is used to compute the values of the PRV (which are known on another hand) . For each of them , it gives the subset of process functions as well as the ordering of the computations which allow its evaluation . Each PRV is thus overdetermined , and provides an ARR , whose residual will be used by the decision procedure for fault detection .

MODEL OF A COMPLEX SYSTEM

The functional analysis of large scale complex industrial systems is a means for creating a comprehensive model of their operation , in a given operating mode . Structured analysis methods such as SADT (Lissandre 1982) have become quite popular and lead to such models . The basic representation of a process function is given by the diagram on fig. 1 .

The hierarchical decomposition of process functions gives descriptions of the process at different levels . The lowest level is composed of process , actuators , sensors and regulators functions , which form an interconnected net since the outputs of some functions are the inputs of others ... Such a representation forms the structural model of the system , at a given level . (Staroswiecki 1988)
Let us now suppose that , for each functional block of the structure , is

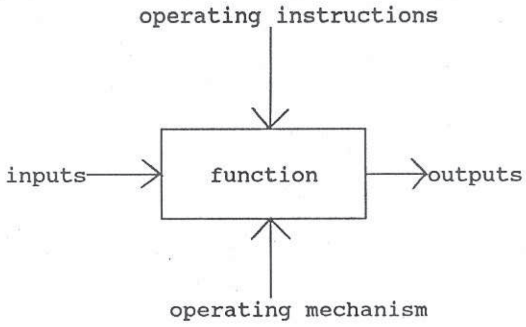


fig. 1 : basic representation of a process function .

known the comportmental model which describes the input/output relation of the block . Such a model may be static or dynamic , linear or non linear . The fig. 2 gives an example of such a representation , for a two inputs , single output system :

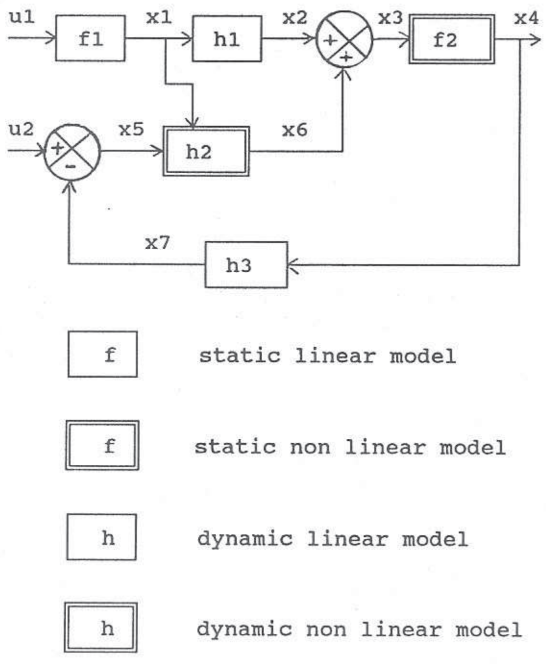


fig. 2 Model of a complex system .

STRUCTURAL REPRESENTATION OF THE MODEL .

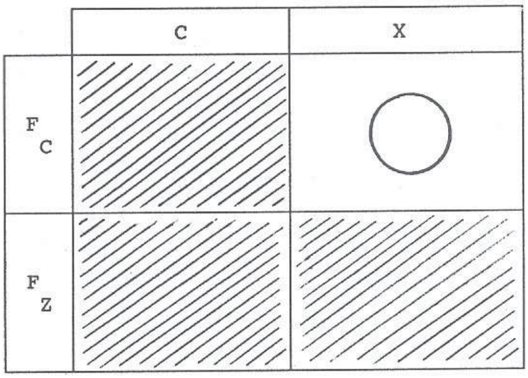
The model of the process is viewed as a collection of boxes (process functions) relating a collection of variables .
Let F be the set of functions which are used for the description of the system , and Z the set of the variables . This set will be decomposed into three subsets : Y , U and X , with :
Y the measured variables , issued from sensors
U the control variables , issued from operators or control computers
X the remaining variables .
Let us note that the variables in Y and U are known (possibly through noisy or

faulty instruments) , while the variables in X are unknown (but could eventally be computed or estimated using Y and U) .
The stucture of the process is represented by the following binary relation (Richetin 1975) .

$$S : F \times Z \longrightarrow \{0,1\}$$
$$(f , z) \longrightarrow S (f , z)$$

whith $S (f , z) = 1$ iff the variable z appears in the relation f .
$S (f , z) = 0$ otherwise .

So , the structure of the system can be represented by a table whose rows are the process functions and whose columns are the process variables . Such a table can be partitioned in the following way :
Let $Z = C \cup X$ be a partition of the columns (C represents the set of known variables) and $F_c \cup F_z = F$ such that F_c is the subset of those process functions which relate only known variables . The structural table of the system can be presented under the following form (fig. 3) .



fi. 3 Structural representation of the system .

Let us now consider the subsets of rows F_c and F_z , having in mind that we want to generate Analytical Redundancy Relations , in order to compute the residuals which will be used for fault detection (Gertler 1988) .
Obviously , each row in F_c can be used as an ARR since it represents a relation in which appear only known variables .
The rows in F_z are relations in which known as well as unknown variables appear . Creating ARR is equivalent to associate together some of these rows , so that the unknown variables are eliminated . This is a classification problem on the set F_z , each class being characterized by the overdetermination of a given subset of the unknown variables .

OVERDETERMINATION OF A SUBSET OF UNKNOWN VARIABLES .

Let $X' \subset X$ and $F (X') \subset F_Z$ be two subsets satisfying :

$$\left.\begin{array}{lr} \forall x \in X' \ \exists f \in F (X') \text{ such that } S (f,x) =1 & (1-1) \\ \forall f \in F (X') \ \forall x \notin X' \ S (f,x) =0 & (1-2) \\ F (X') \text{ is the maximal subset satisfying (1-1) and (1-2)} & (1-3) \end{array}\right\} \quad (1)$$

In other words , the class F (X') contains those process functions which relate only the variables C and X' , are insensitive with respect to the other variables , and such that no other process function has this property .
Taking the process functions in F (X') , a set of equations of the following form can be written :

$$\varphi (X' , C) = 0 \qquad (2)$$

Let us now introduce the independancy concepts associated with such a set of N process functions with Q unknown process variables . Those concepts are precisely formulated in (Murota 1987) , we only present under a simple form those which will be used .

Structural Independance of Variables .

Let $M_{X'}$, M_C be the sets of the possible values taken by the variables X' and C , let $\mathcal{F}$ be a given set of functions .
Let $y \subset C$, $x1 \subset X'$ and $x2 \subset X'$ be subsets of variables . The subsets x1 and x2 are said to be structurally independant on $\mathcal{F}$ iff :

$\nexists f \in \mathcal{F}$ s.t. $f (y,x1,x2) =0$ almost everywhere on $M_C \times M^2_{X'}$

When $\mathcal{F}$ is the set of linear forms on $M_C \times M^2_{X'}$, structural independance is equivalent to linear independance .
When $\mathcal{F}$ is a set of non trivial polynomials on a ring K , structural independance is named algebraic independance (Rech 1988) . In the structural representation of the subset $F(X')$, we take $\mathcal{F}$ as the set of all functions which are compatible with the physical laws governing the system and which are independant of the process functions in F(X') .

Structural Independance of Process Functions .

Let $\varphi 1$ and $\varphi 2$ be two subsets of process functions from (2) . $\varphi 2$ is said to be structurally dependant of $\varphi 1$ iff :

$\varphi 1 (y , x) =0 \Longrightarrow \varphi 2 (y , x) =0$
almost everywhere on $M_C \times M_{X'}$

Let $\{X' , F(X')\}$ be a pair satisfying conditions (1) and let us analyze the kind of model provided by (2) for the subset of variables X' (a subsystem of the complex system (Toro 1982)) .

a) When all the variables in X' and all the functions in F(X') are structurally independant (s.i) , (2) gives a complete and minimal model for the subsystem X' .

- <u>The model is complete</u> :

$\forall x1 , x2 \subset X'$, $\forall f$ such that
$f (C,x1,x2) =0$ almost everywhere ,
$\exists \bar{F} \subset F(X')$ such that :

$$g (C , X') =0 \text{ a.e } \forall g \in \bar{F} \implies f (C,x1,x2) =0 \text{ almost everywhere}$$

This is a consequence of the s.i of the variables in X'.

- <u>The model is minimal</u> :

$\forall f \in F (X')$, $\nexists \bar{F} \subset F (X') \setminus \{f\}$ such that :

$$g (C , X') =0 \text{ a.e } \forall g \in \bar{F} \implies f (C, X') =0 \text{ almost everywhere}$$

This is a consequence of the s.i. of the functions in F (X') .
Taking into account the fact that X' is a subsystem of a physical system , whose inputs are known (in C) , the operating point (or the trajectory , for some initial conditions) is unique . The system $\{X' , F (X')\}$ is thus just determined , with N = Q .

b) Let now F (X') be a set of s.i functions , and X' be a set of non s.i. variables : (2) gives an incomplete model of the subsystem X' . In fact , other physical relations than those in F (X') (or deduced from them) are existing between the variables of X' . The model (2) is undetermined , since no unique value of X' satisfy it . (N<Q)

c) Let X' be a set of s.i. variables and F (X') a set of non s.i. functions : (2) gives a complete but non minimal model of the subsystem X' , which is then overdetermined . In this case , the pair $\{X' , F (X')\}$ gives rise to N-Q redundancy relationships which can be used for residual generation .

So , the problem of generating ARR from the model of a complex system described by the interconnexion of static or dynamic , linear or non linear blocks , can be split into two parts :

<u>Qualitative step</u> . Exhibit all the pairs $\{X' , F(X')\}$

<u>Quantitative step</u> . For each pair , check if it is overdetermined and compute the redundancy relationships .

Such a procedure could be very long and time consuming ; we propose an algorithm for the direct computation of a set of possible ARR ,using " pretext for resolution variables " (PRV).

THE ALGORITHM .

It is based on a special case of overdetermination : we create pairs $\{X' , F (X')\}$ by the computation , via two different means , of variables which are named PRV . Under the hypothesis of structural independance , those PRV are introduced in order to obtain a pair $\{X \cup \{PRV\} , F \}$ which is just determined . The application of structural analysis methods for the resolution of systems of equations (Harary 1962 ; Richetin 1975) , based on the concept of coupling on a bigraph , gives a first means for their calculation . Their second determination is very simple as far as the PRV are taken among the known variables or are given by advance known values .
The fig. 4 shows the structure of the coupling obtained in the first case :

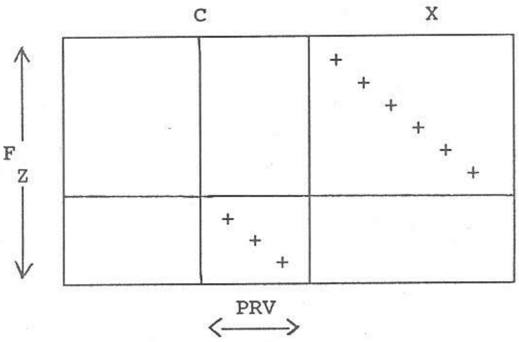


fig. 4 : coupling structure obtained with $\{PRV\} \subset C$.

The fig. 5 shows the structure of the coupling obtained via the introduction of PRV which are given by advance known values .

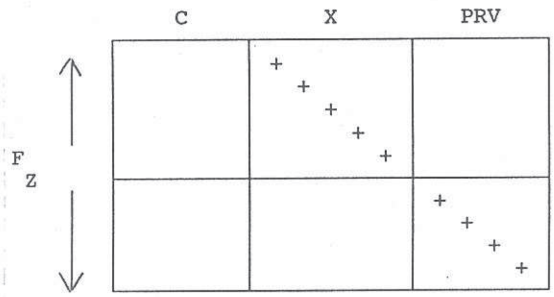


fig. 5 : coupling structure obtained with given PRV .

Those known values allow the introduction of the PRV into the process equations without any modification of those equations , by the choice of adequate values , for example

$f (C , X) =0$ will be written as
$f (C , X) + PRV =0$

the known value of PRV being 0 .
For each PRV , the structural analysis approach gives :
- the subset of the process.functions which are necessary for the computation of that PRV , the associated subset of the unknown variables which can be determined , and the associated subset of the known variables used for the determination .
- the ordering of the computations which allows its evaluation .

Each PRV gives thus rise to one redundancy relationship , whose residual will be tested by the fault detection procedure . The structure of the redundancy relationship is characterized by the associated process functions and the associated known variables . The knowledge of that structure will be used by the fault isolation procedure .

CONCLUSION

Model based failure detection and isolation methods rest on the use of decision procedures which test the value of residuals generated by ARR . The extraction of those ARR from the system's model is relatively easy for linear systems , but much more difficult in the non linear case . The approach we propose is based on the representation of complex industrial process by means of a network of interconnected blocks , each block performs linear or non linear , static or dynamic transformations . The structural analysis methods , together with the introduction of PRV variables (which are double-determinated) gives a means for the systematic extraction of the ARR from the model of the process (supposed to be complete) .

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