Sequence Estimation and Schedulability Analysis for Partially Observable Petri Nets (Part I)

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Plan

- Introduction
- Preliminaries
- Estimation/Diagnostic in P-timed Petri nets
- Schedulability Analysis : Generation and checking of count vectors (next talk !)
- Fault detection for non-modelled faults (next talk !)

Aim

- To make estimation in Petri nets with application to fault detection
- Let $TR = TR_{obs} \cup TR_{un}$ where TR_{obs} is the set of observable transitions while TR_{un} is the set of unobservable ones.
- For a sequence (word) ω (or a subsequence) observed, the aim is to compute a firing sequence (or some firing sequences) of unobservable transitions necessary to complete ω into a fireable sequence of the Petri net consistent with its evolution.

A difficulty

Ru, Y., and Hadjicostis, C. N. (2009). Bounds on the number of markings consistent with label observations in Petri nets. IEEE Transactions on Automation Science and Engineering, 6(2), 334-344.

- The number of consistent markings in a Petri net with nondeterministic transitions (unobservable transitions and/or transitions that share the same label) is at most polynomial in the length of the observation sequence.→ **Increasing** with the new observations in the worst case...
- The number of firing sequences can be **exponential** in the length of the observation sequence.

→ **Guiding thread**

Compromise between the accuracy of the interpretation and the numerical efficiency of the approach.

Principe of estimation

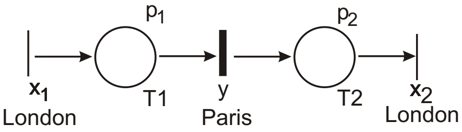

FIGURE: Example : Timed Event Graph.

$T = T1 = T2$ is the minimum travel time from London to Paris and conversely.
- Let us assume that a person in Paris observes that 10 planes coming from London have landed at time 100 minutes or before.
We can conclude that at least ten planes have taken off from London at time $100 - T$ minutes.
So, the least number of plane take-offs from London is 10 at time $100 - T$ minutes.
$\rightarrow$ If each plane has been checked before its take-off, we can estimate the minimum activity of the maintenance department at London.

- Let us assume that a person in Paris observes that at the most 10 planes have taken off at time 100 minutes or before at Paris. We can conclude that the greatest number of 10 planes have landed in London at time $100+T$ minutes. It could be lower : A pilot can decide to return or to land at another airport for technical reasons. The least number of landings is zero in the worst case.

$\rightarrow$ Maximum activity of the maintenance at London

Fault detection

No-modelled faults

The objective is the detection of variations of the model which are not described.

On-line comparison of consistency of informations with models or submodels

Ex : connection of three pipes in continuous systems

- Nominal model : $Q1 + Q2 = Q3$
- Real model (leak in a pipe) : $Q1 + Q2 \neq Q3$

Faults or changes in the process in Petri nets

- Variation of a temporisation (deterioration of a machine, repairing) ;
- Loss or addition of a token (loss of a ressource, addition of a part) ;
- Another graph (new schedule)

Principle

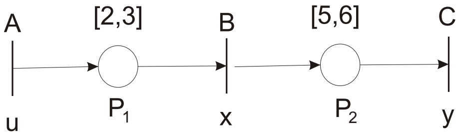

FIGURE: Example : P-Time Petri net.

Journey of a vehicle from the town A to B between 2 and 3 hours). Journey from B to C between 5 and 6 hours.
Observable transitions : u and y.
Unobservable : Time x
Time u known, $x \in [u+2, u+3]$.
Time y known, $x \in [y-6, y-5]$.
Therefore, $x \in [\max(u+2, y-6), \min(u+3, y-5)]$
otherwise, model$\neq$reality

Modelled faults

$$TR = TR_{obs} \cup TR_{un}$$

We assume that the faults occurring in the process are modeled by unobservable transitions and the notation TR_f represents the relevant set. The set of unobservable transitions describing a normal behavior is denoted TR_n . Therefore, $TR_f \subset TR_{un}$ and

$$TR_{un} = TR_n \cup TR_f \quad (1)$$

Determination of a fault state D

If $\min_{\mathbb{Z}}(c_{det}.\overline{x}) \geq 1$ with $c_{det} \geq 0$ a row-vector, then at least a fault is detected on the horizon (State D).

- It always exists as $\overline{x} \geq 0$.
- The minimum presents an interest if path from an unobservable transition x_i to an observable transition y

Example on minimum

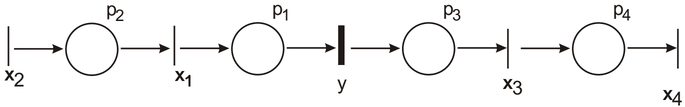

FIGURE: Example : Timed Petri net - chain x_4 and x_5.

If $y = 1$, then $x_1^- = 1$, $x_2^- = 1$ for the marking $M = 0 \rightarrow$ State D for x_1, x_2 . Also $x_3^+ = 0$ and $x_4^+ = 0 \rightarrow$ Cannot lead to the state D.

Now, if the marking in the place p_2 is $M_2 = 3$, then $x_1^- = 1$ but $x_2^- = 0$

Determination of a normal state N

If $\max_{\mathbb{Z}}(c_{\text{det}}.\overline{x}) = 0$, then no fault is detected (State N).

The maximum presents an interest if path from an observable transition y to an unobservable transition x_i

It can be equal to $+\infty$: source transition which is a perturbation, circuit with no input place →infinite marking.

Example on maximum

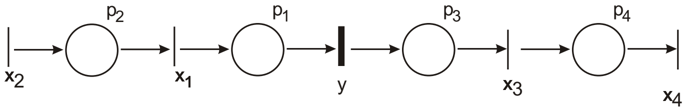

FIGURE: Example : Timed Petri net - chain.

If $y = 1$, then $x_2^+ = +\infty$ (perturbation), $x_1^+ = +\infty$ and also $x_3^+ = M_3 + 1$ and $x_4^+ = M_4 + M_3 + 1$

If $y = 0$, then $x_2^+ = +\infty$ (perturbation), $x_1^+ = +\infty$ and also $x_3^+ = M_3$ and $x_4^+ = M_4 + M_3 \rightarrow$ State N for x_3 , x_4 if $M_3 = M_4 = 0$.

Preliminaries

Petri nets

- A Place/Transition net (a P/TR net) is a structure $N = (P, TR, W^+, W^-)$, where P is a set of $|P|$ places and TR is a set of $|TR|$ transitions which are denoted by x
- Notation t corresponds to the current time, T_l to the temporization of place $p_l \in P$, and T to the transposition of a matrix.
- Matrices W^+ and W^- are $|P| \times |TR|$ post- and pre-incidence matrices over N where each row $l \in \{1, \ldots, |P|\}$ specifies the weight of the incoming and outgoing arcs of place $p_l \in P$ respectively. The incidence matrix is $W = W^+ - W^-$.
- Vector M_l is the marking of place p_l with $l \in \{1, \ldots, |P|\}$.

A net system (N, M^{init}) is a net N with an initial marking M^{init}.

Sequences

- Each transition and its corresponding variable is denoted with the same letter. Each transition is associated with the number of events which happen before or at time t. The number of events which are the firings of the transition is denoted by $x(t)$.
- Time is discrete ($t \in Z$)
- Time is defined by an external clock with a unique origin of time during the evolution of the system.
- Assuming that the events can only occur at $t \geq 1$, we have $x(t) = 0$ for $t \leq 0$.
- For any $t \in N^*$, it may be that no event takes place at t, a single event happens at t, or several events occur simultaneously at t.

Example

For a given transition x_i, the arrival of two events at times 3 and 5 repectively gives $x(t = 3) = 1$ and $x(t = 5) = 2$.

It implies that the sequence of numbers of events starting at $t = 0$ and finishing at $t = 7$ is

t	0	1	2	3	4	5	6	7
$x_i(t)$	0	0	0	1	1	2	2	2

So, $x(t = 4) = 1$ and $x(t = 7) = 2$.

- Partial order $\leq$ defined on set R^n is defined componentwise : $x \leq y$ if and only if $x_i \leq y_i$, $\forall i \in \{1, 2, ..., n\}$.$\rightarrow$Minimum/Maximum solution.
- Minimal (Maximal) element of a subset : an element of the subset which is not greater (less) than any other element of the subset ; x minimal (maximal) $\Leftrightarrow \nexists y \neq x$ such that $y \leq x$ $(x \leq y)$.

Example Subset $\{x_1, x_2\}$ with $x_1 = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $x_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$: x_1, x_2 are minimal and maximal.

Timed Petri nets

Each place $p_l \in P$ is associated with a temporization $T_l \in N$. Its initial marking is the entry l of the vector M^{init} which is denoted by M_l^{init}. A token remains in place p_l at least for time T_l.

$$\sum_{i \in {}^\bullet p_l} x_i(t - T_l) + M_l^{init} \geq \sum_{i \in p_l^\bullet} x_i(t) \tag{2}$$

with $x_i(t) \in N$.

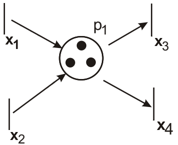

FIGURE: Example : A place of a Timed Petri net with time duration T1.

$$x_1(t - T_1) + x_2(t - T_1) + 3 \geq x_3(t) + x_4(t)$$

If we split each place p_l associated with a temporization $T_l > 1$ into T_l places, such that the temporization of each place is equal to one, the temporization of each place in the new graph is equal to zero or one :

$$G \quad \cdot \begin{pmatrix} x(t-1) \\ x(t) \end{pmatrix} \leq \quad M^{init}$$

where the l^{th} row of G contains the weights of the incoming and outgoing arcs of place p_l.

If all the time durations are equal to 1,

$$W^{+}.x(t-1) + M^{init} \geq W^{-}.x(t)$$

If all the time durations are equal to 0,

$$W.x(t) + M^{init} \geq 0$$

Estimation in P-timed Petri nets

Objective

The aim is the estimation of the sequence of numbers of transition firings and markings by considering the system for $\theta \in \{t-h+1, t-h+2, ..., t\}$ where $h \in N^*$ is the horizon of the sequence estimation.

Let $y(\theta)$ (respectively, $x_{un}(\theta)$) be the subvector of the state vector $x(\theta)$ such that the relevant transitions belong to the set of observable transitions TR_{obs} (respectively, unobservable transitions TR_{un}).

The objective for each time t is the estimation of optimal sequences $x_{un}(\theta)$ for $\theta \in \{t-h, t-h+1, ..., t\}$ knowing the observable state vector $y(\theta)$ in the same window.

Knowing this sequence, the relevant markings are directly deduced from the fundamental marking relation.

Under the condition of existence, an optimal sequence can be :
- a minimal (respectively, maximal) estimate sequence denoted by $x^{-}_{un}(\theta)$ (respectively, $x^{+}_{un}(\theta)$)
- or can be a sequence optimal for any linear criterion.

Sliding horizon

- We consider a sequence of observable events at each step of the estimation on a horizon which can be a sliding horizon.
- After the computation of the state estimate on a given horizon at each iteration, the horizon shifts to the next sample, and the estimation of the state estimate is restarted using known information of the new horizon.

→The interest of a sliding horizon stems from the possibility of dealing with a **limited amount of data**, instead of using all the information available from the beginning.

→ The interpretation is relevant to the considered horizon and not outside.

Assumptions

- The model of the timed Petri net and the initial marking are assumed to be known.
- The Timed Petri net is 'time live' or consistent, that is, it presents at least one time sequence during the application of the on-line approach and also after : After the last observed event, other observable events (provisionally unknown) can occur.
- The firing of the different observable transitions can be distinguished (see next paper).
- The firings of the transitions can be simultaneous (when time is considered).
- In general, no assumptions on the non-cyclicity and boundedness of the Petri net (when time is considered).

Solution space

Given a net $N = (P, TR, W^+, W^-)$, and a subset $TR' \subseteq TR$ of its transitions, the TR'-induced subnet of N is defined as the new net $N' = (P, TR', W^{+\prime}, W^{-\prime})$ where $W^{+\prime}$ (respectively, $W^{-\prime}$) is the restriction of W^+ (respectively, W^-) to $P \times TR'$. The net N' is obtained from N by removing all transitions in $TR \setminus TR'$.

The system for time $\theta \in \{t-h+1, t-h+2, ..., t\}$ can be rewritten as follows :

$$\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix} \cdot \begin{pmatrix} x_{un}(\theta-1) \\ x_{un}(\theta) \end{pmatrix} \leq M^{init} - \begin{pmatrix} G_{1,obs} & G_{0,obs} \end{pmatrix} \cdot \begin{pmatrix} y(\theta-1) \\ y(\theta) \end{pmatrix} \quad (3)$$

after an adequate permutation of the columns of matrix G with respect to the observable/unobservable transitions :

The columns of $\begin{pmatrix} G_{1,un} & G_{0,un} \end{pmatrix}$ (respectively, of $\begin{pmatrix} G_{1,obs} & G_{0,obs} \end{pmatrix}$) correspond to the unobservable transitions (respectively, to the observable transitions).

Polyhedron

The solution space of the Petri net is characterized by the following polyhedron

$$A \cdot \mathbf{x}_{un} \leq b \tag{4}$$

$$x_{un} = \begin{pmatrix} x_{un}(t-h) \\ x_{un}(t-h+1) \\ x_{un}(t-h+2) \\ \dots \\ x_{un}(t-1) \\ x_{un}(t) \end{pmatrix}, \; y = \begin{pmatrix} y(t-h) \\ y(t-h+1) \\ y(t-h+2) \\ \dots \\ y(t-1) \\ y(t) \end{pmatrix},$$

$$A = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \text{ and } b = \begin{pmatrix} C_1 - B_1 \cdot \mathbf{y} \\ 0_{h.|TR_{un}|x1} \\ 0_{nx1} \end{pmatrix}.$$

Description of $A \cdot x_{un} \leq b$

- The relations (3) of the time Petri net describing the set of trajectories on horizon h are :

$$A_1 \cdot \mathbf{x}_{un} \leq C_1 - B_1 \cdot \mathbf{x}_{obs} \qquad (5)$$

with $A_1 = \begin{pmatrix} G_{1,un} & G_{0,un} & 0 & \dots & 0 & 0 \\ 0 & G_{1,un} & G_{0,un} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & G_{0,un} & 0 \\ 0 & 0 & 0 & \dots & G_{1,un} & G_{0,un} \end{pmatrix}$

$B_1 = \begin{pmatrix} G_{1,obs} & G_{0,obs} & 0 & \dots & 0 & 0 \\ 0 & G_{1,obs} & G_{0,obs} & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & G_{0,obs} & 0 \\ 0 & 0 & 0 & \dots & G_{1,obs} & G_{0,obs} \end{pmatrix}$ and

$C_1 = \begin{pmatrix} M^{init} \\ M^{init} \\ \dots \\ M^{init} \\ M^{init} \end{pmatrix}.$

- Moreover,

$$A_2 \cdot x_{un} \leq 0_{h.|TR_{un}|x1} \qquad (6)$$

expresses that the trajectories are non-decreasing, that is, $x_{un}(\theta - 1) \leq x_{un}(\theta)$ for $\theta \in \{t - h + 1, t - h + 2, ..., t\}$.
- Finally,

$$A_3 \cdot \mathbf{x}_{un} \leq 0_{nx1} \qquad (7)$$

where $A_3 = -I_{nxn}$ (the trajectories are non-negative).

Example

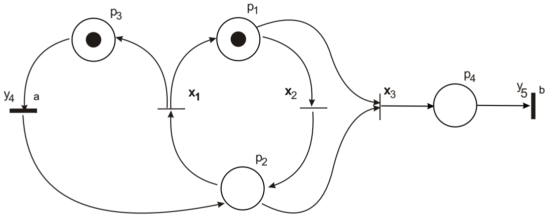

FIGURE: Example 2 : P-timed Petri net with observable transitions y_4 and y_5.

The TR_{un}-induced subnet is BCF (Backward Conflict Free) and presents a circuit. Each place is associated with a temporization equal to 1 second.
Simulation : The initial marking is $M^{init} = \begin{pmatrix} 1 & 0 & 1 & 0 \end{pmatrix}^{\mathrm{T}}$.
A possible evolution of the Petri net for $t \in \{0, 1, ..., 9\}$ is given in Table 3.

Time t	0	1	2	3	4	5	6	7	8	9
Events		y_4	x_1 x_2	x_3 y_4	y_5 x_1	y_4	x_1	y_4	x_3 x_2	y_5 x_1
$M(t)$	1	1	1	0	1	1	2	2	0	1
	0	1	1	1	0	1	0	1	1	0
	1	0	1	0	1	0	1	0	0	1
	0	0	0	1	0	0	0	0	1	0

Algebraic model

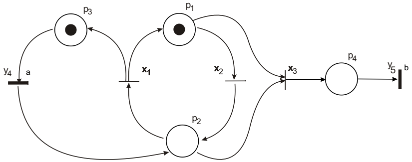

FIGURE: Example 2 : P-timed Petri net with observable transitions y_4 and y_5.

The matrices of the relevant matrix model $G_1 \cdot x(t-1) + G_0 \cdot x(t) \leq M^{init}$ are :

$$G_1 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix} \text{ and}$$

$$G_0 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Existence of extremum solutions for some structures

The TR_{un}-induced subnet is Backward Conflict Free (BCF), i.e., any two distinct unobservable transitions have no common output place.

The TR_{un}-induced subnet is Forward Conflict Free (FCF), i.e., any two distinct unobservable transitions have no common input place.

The system of linear inequalities $A.x \leq b$ is inf-monotone (respectively, sup-monotone) if each row of matrix A has one strictly negative (respectively positive) element at most. In fact,

$$\begin{array}{lcl} BCF & \leftrightarrow & \inf - monotone \\ FCF & \leftrightarrow & \sup - monotone \end{array}$$

Theorem Let us assume that the Timed Petri net is time live. In a BCF TR_{un}-induced subnet, the least estimate x_{un}^{-} exists over R. In a FCF TR_{un}-induced subnet, the greatest estimate x_{un}^{+} exists over R if x_{un} has a finite majorant.

Generalization to the integers

An inf-monotone (respectively, sup-monotone) system of linear inequalities $Ax \leq b$ is also 1-inf-monotone (respectively, 1-sup-monotone) if : A and b are integers ; the strictly negative (respectively positive) coefficients of A are equal to -1 (respectively, $+1$).

The TR_{un}-induced subnet is Unitary Backward Conflict Free or UBCF (respectively, Unitary Forward Conflict Free or UFCF) if : The subnet is BCF (respectively, FCF) ; the weight of each incoming (respectively, outgoing) arc of the subnet is unitary.

Theorem Let the TR_{un}-induced subnet of the considered Petri net be UBCF (respectively, UFCF).

The least sequences x_{un}^{-} (respectively, greatest sequences x_{un}^{+}) of system (4) in R^n and N^n are equal.

The relevant extremum sequence is given by the following linear programming problem : $\min\{c.x_{un}\}$ (respectively, $max\{c.x_{un}\}$) such that $A \cdot x_{un} \leq b$ for any $c > 0$.

Other results with totally unimodularity

Example continued
Observer

The labels a and b in the Petri net correspond to the events of the observable transitions x_4 and x_5 (i.e. $TR_{obs} = \{x_4, x_5\}$) while the label ε corresponds to the unobservable transitions x_1, x_2 and x_3 (i.e. $TR_{un} = \{x_1, x_2, x_3\}$).
So, we have $y = (\underline{y}_4,\ \underline{y}_5)^{\mathrm{T}}$ and $x_{un} = (x_1,\ x_2,\ x_3)^{\mathrm{T}}$. The events associated with label a (respectively, b) are observed at times 1, 3, 5 and 7 (respectively, 4 and 9).

$G_{1,un} \cdot x_{un}(\theta - 1) + G_{0,un} \cdot x_{un}(\theta) \leq$
$M^{init} - G_{1,obs} \cdot y(\theta - 1) - G_{0,obs} \cdot y(\theta)$ for $\theta \in \{t - h + 1, t\}$
where

$$G_{1,un} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},\ G_{0,un} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$G_{1,obs} = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } G_{0,obs} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Estimation

We make an estimation of x_{un} at $t = 9$ and we arbitrarily take $h = 3$. We estimate the firing numbers of the transitions based on the observations on the window $\{t - h, t - h + 1, ..., t\} = \{6, 7, 8, 9\}$.

Exact numbers of firing

Time t	6	7	8	9
x_1	3	3	3	4
x_2	1	1	2	2
x_3	1	1	2	2

Known data

θ	6	7	8	9
$\underline{y}_4$	3	4	4	4
$\underline{y}_5$	1	1	1	2

Least estimates for $t = 9$ and $h = 3$

θ	6	7	8	9
x_1^-	3	3	3	3
x_2^-	1	1	1	1
x_3^-	1	1	2	2

Diagnosis in P-timed Petri nets
State D

Let us define the relevant criterion $c_{\text{det}}.\overline{x}$ where the row-vector c_{det} is the concatenation of the submatrix of $k.n'$ zeros and the submatrix $\begin{pmatrix} c^{<t-h>} & c^{<t-h+1>} & c^{<t-h+2>} & \dots & c^{<t>} \end{pmatrix}$ relevant to TR_{obs} and TR_{un}, respectively. The components $(c^{<t-i>})_j$ for any fault transition $j \in TR_f \subset TR_{un}$ are equal to 1, while the other ones are null. As a result, $c^{<t-h>} = c^{<t-h+1>} = ... = c^{<t>}$.

$$\begin{cases} J^{-}_{\text{det}} = \min(\boldsymbol{c}_{\text{det}}.\overline{x}) \\ s.t. \quad \boldsymbol{A}.\overline{x} \leq \boldsymbol{b} \text{ with } \overline{x} \geq 0 \end{cases} \tag{8}$$

- By solving the optimization problem (8) in $\mathbb{Z}$, the computed criterion $\min_{\mathbb{Z}}(c_{\text{det}}.\overline{x})$ is a lower bound of the number of faults.
- If $\min_{\mathbb{Z}}(c_{\text{det}}.\overline{x}) \geq 1$, then at least a fault is detected on the horizon (State D).

- These detected faults can be the repetition of the firing of the same fault transition. The fault can also be transient.
- If the obtained vector is not an explanation vector, then there is an explanation vector as we assume that the LPN is time live :
It can only give the same value or a greater value as $c \geq 0$.

State N

- Symmetrically, the maximum number of faults cannot be greater than the obtained value $\max_{\mathbb{Z}}(c_{\text{det}}.\overline{x})$. So :

- If $\max_{\mathbb{Z}}(c_{\text{det}}.\overline{x}) = 0$, then no fault is detected (State N).

- If the obtained count vector is not an explanation vector, there will be no better count vector leading to a criterion greater than zero.

There is an explanation vector as we assume that the LPN is time live

which can only give the same value or a lower value as $c \geq 0$.

- The interpretation does not need an additional assumption as the acyclicity of the unobservable induced subnet.

- If $\min_{\mathbb{Z}}(c_{\text{det}}.\overline{x}) = 0$ and $\max_{\mathbb{Z}}(c_{\text{det}}.\overline{x}) \geq 1$, then we cannot conclude on the existence of a fault (State **U**). Nevertheless, we can always say that the number of detected faults is between $\min_{\mathbb{Z}}(c_{\text{det}}.\overline{x})$ and $\max_{\mathbb{Z}}(c_{\text{det}}.\overline{x})$ under the liveness condition of the Petri net.

Relaxations

The same reasoning holds if we relax the minimization and the maximization problems over R.

$$\min_{\mathbb{R}}(c_{\det}.\overline{x}) \leq \lceil \min_{\mathbb{R}}(c_{\det}.\overline{x}) \rceil \leq \min_{\mathbb{Z}}(c_{\det}.\overline{x}) \leq \dots \quad (9)$$

$$\min(c_{\det}.\overline{x'}) \ \leq c_{\det}.\overline{x'} \leq \max(c_{\det}.\overline{x'}) \quad (10)$$

on the space of explanation vectors $\overline{x'}$

$\dots \leq \max_{\mathbb{Z}}(c_{\det}.\overline{x}) \leq \lfloor \max_{\mathbb{R}}(c_{\det}.\overline{x}) \rfloor \leq \max_{\mathbb{R}}(c_{\det}.\overline{x})$

When the execution time for $\mathbb{Z}$ is too large, we can solve over R with the same interpretation (but less accurate) :

If $\lceil \min_{\mathbb{R}}(c_{\det}.\overline{x}) \rceil \geq 1$, then at least a fault is detected on the horizon (State D).

If $\lfloor \max_{\mathbb{R}}(c_{\det}.\overline{x}) \rfloor = 0$, then no fault is detected (State N).

If $\min_{\mathbb{R}}(c_{\det}.\overline{x}) = 0$ and $\max_{\mathbb{R}}(c_{\det}.\overline{x}) \geq 1$, then we cannot conclude on the existence of a fault (State U).

Fault isolation (localisation) of a fault

The criterion $c_{loc}(x_f)_i.\overline{x}$ enables the isolation, where the row-vector c_{loc} is the concatenation of a submatrix of $k.n'$ zeros relevant to TR_{obs} and the submatrix $\begin{pmatrix} c^{<t-h>} & c^{<t-h+1>} & c^{<t-h+2>} & \dots & c^{<t>} \end{pmatrix})$.

So, $c^{<t-i>} = 0$ except $(c^{<t-i>})_j = 1$ for a given fault transition j.

Following the same reasoning as the detection approach for fault isolation, we define two diagnostic indicators,

$J^{-}_{loc}((\overline{x_f})_j) = \min_{\mathbb{Z}}(c_{loc}.\overline{x})$ and

$J^{+}_{loc}((\overline{x_f})_j) = \max_{\mathbb{Z}}(c_{loc}.\overline{x})$, associated with the fault transition.

The fault of the transition $(x_f)_j$ presents an occurrence number between $\lceil J_{loc}^-((\overline{x_f})_j) \rceil$ and $\lfloor J_{loc}^+((\overline{x_f})_j) \rfloor$.
The isolation procedure guarantees the following specific interpretations :
- If $\lceil J_{loc}^-((\overline{x_f})_j) \rceil \geq 1$, then the relevant fault is detected (State D).
- If $\lfloor J_{loc}^+((\overline{x_f})_j) \rfloor = 0$, then no fault relevant to transition $(x_f)_j$ occurs (State N).
- If $J_{loc}^-((\overline{x_f})_j) = 0$ and $J_{loc}^+((\overline{x_f})_j) \geq 1$, then the available pieces of information do not lead to a conclusion on the presence of a fault relevant to the transition $(x_f)_j$ (State U).

Conclusion

- Only few assumptions (unique origin of time, cumulative sum of the count vectors).
- The approach can be adapted to the available CPU time as the horizon and the relaxation (No explosion of the number of estimated markings or count vectors).
- Can be completed by standard approaches providing starting markings (or basis markings) if some assumptions are added (acyclicity, boundedness of the marking).
- Can be generalized to unknown intial marking, indistinguishable observable events, fault classes, other criteria, etc...

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