GENERATION OF ANALYTICAL REDUNDANCY RELATIONS FOR FAULT DETECTION

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ABSTRACT

In this paper, we apply the embedding procedure to quantitative models based on state equations or block diagram representation. The result is a unique formulation which contains the generation of ARR and the least square estimate of the state vector. Particularly, the three classical relations which define the parity space are generalized to partially observable processes.

KEYWORDS

Observability ; fault detection and isolation ; embedding procedure ; large scale systems.

I - INTRODUCTION

The basic principle of fault detection is the comparison of the actual behaviour of the system to a reference behaviour describing its normal operation. The reference behaviour is issued from the knowledge which is available upon the system, this knowledge being expressed under more or less precise terms, and under formalisms which may be very different (knowledge base, analytical models,...). One of the most frequently used approach is based on the use of Analytical Redundancy Relationships (ARR) : the knowledge available upon the system leads to express its normal operation by a set of invariants : the residuals of the ARR (coherence model). The fault detection resumes thus to a decision problem : is the variance of the residuals the effect of noise, of normal deviations and errors or the effect of a failure.

However, it is the most frequent case that such an analytical representation is not directly available for large scale complex industrial systems. Those systems are characterized by the great number of variables which are necessary for their description, and by the great variety of the types of relationships which link these variables : qualitative or quantitative, statical or dynamical, linear or non linear. Moreover, in practical situations, some models are not known precisely (class of the model, values of its parameters,...) although their structure , i.e the different relationships and the variables which intervene, is known. The system may thus be represented by a network of elementary activities, each of them processing a subset of variables. Among the set of all the variables, only some of them are known (computed by elementary activities) or measured (a sensor performs also an elementary activity).

So, an approach is to use such a representation in order to identify a possible candidate ARR for fault detection, based on the overdetermination, within the system of one or more variables [12]. We define extended graphs which include the initial structures by an embedding procedure to analyze the system [3] [4]. The new graphs contain more vertices and arcs than the original ones.

However, this approach is not limited to qualitative models. The embedding procedure may also be applied to the following quantitative representations of the process :

1) An interconnection of transfer functions (block diagram representation)

The large scale system under consideration is represented by a network of elementary activities. This description, as shown in [13] [14], can be directly used to generate a set of ARR.

2) The state and measurement equations

The most commonly used approaches are based on identification [7] or estimation procedures. The more practical technique is probably the parity space which is a special observer (dead beat observer [9]). The early contributions to the parity space approach were made by [10] [5] and the group around Willsky [1] [8].

The key idea is to check the parity (consistency) of the mathematical equations of the system by using the actual measurements. A fault is declared to have occured once preassigned error bounds are surpassed. The analytical redundancy relations can be static or dynamic.

For these different representations, we propose to apply the embedding procedure to partially observable complex system.

II - STATE SPACE AND MEASUREMENT EQUATIONS

Linear dicrete time processes can be described by a set of equations called state-space and measurement equations :

$$x(k+1) = A\,x(k) + B\,u(k) + E\,e(k) + F\,f(k) + w(k) \qquad (1)$$
$$y(k) = C\,x(k) + G\,g(k) + H\,h(k) + v(k) \qquad (2)$$

where :
$x(k)$, $u(k)$, $y(k)$ are respectively the state, control and output vectors at time k.

E is the distribution matrix of the modelization errors $e(k)$.
F is the distribution matrix of the components and actuators faults $f(k)$.
G is the distribution matrix of the modelization errors $g(k)$.
H is the distribution matrix of the sensors faults $h(k)$.
$w(k)$ and $v(k)$ are respectively the process and measurement noises.

For any time k, these equations can be re-written as

$$y(k,p) = \phi(k,p)\,x(k) + C(k,p)\,u(k,p) + E(k,p)\,e(k,p) + F(k,p)\,f(k,p) + W(k,p)\,w(k,p) + G\,g(k,p) + H\,h(k,p) + v(k,p) \qquad (3)$$

where :

$$z(k,p) = (z^t(k),\ z^t(k+1),\ \dots\ z^t(k+p))^t$$
$$z \in \{y, u, e, f, w, g, h, v\}$$

$$\phi(k,p) = \begin{bmatrix} C \\ CA \\ : \\ CA^p \end{bmatrix}$$ the observability matrix on p periods

$$C(k,p) = \begin{bmatrix} 0 & & & \\ CB & 0 & 0 & \\ CAB & CB & 0 \ddots & \\ \vdots & & \ddots & \\ CA^{P-1}B & \cdots & CB & 0 \end{bmatrix}$$

$$E(k,p) = \begin{bmatrix} 0 & & & \\ CE & 0 & 0 & \\ CAE & CE & 0 \ddots & \\ \vdots & & \ddots & \\ CA^{P-1}E & \cdots & CE & 0 \end{bmatrix}$$

$$F(k,p) = \begin{bmatrix} 0 & & & \\ CF & 0 & 0 & \\ CAF & CF & 0 \ddots & \\ \vdots & & \ddots & \\ CA^{P-1}F & \cdots & CF & 0 \end{bmatrix}$$

$$W(k,p) = \begin{bmatrix} 0 & & & \\ C & 0 & 0 & \\ CA & C & 0 \ddots & \\ \vdots & & \ddots & \\ CA^{P-1} & \cdots & C & 0 \end{bmatrix}$$

We can write :

$$y(k,p) = \phi(k,p)\,x(k) + C(k,p)\,u(k,p) + \varepsilon(k,p)$$

with $\varepsilon(k,p) = E(k,p)\,e(k,p) + F(k,p)\,f(k,p) + W(k,p)\,w(k,p) + G\,g(k,p) + H\,h(k,p) + v(k,p)$

So,

$$\phi(k,p)\,x(k) = z(k,p) - \varepsilon(k,p) \qquad (4)$$

with $z(k,p) = y(k,p) - C(k,p)\,u(k,p)$

Note than $\varepsilon(k,p)$ contains all the informations on the deviations between the actual behaviour of the system and the expected one on the time interval [k, k+p] (model errors, process and measurement noises, process and sensors or actuators faults).

For simplicity, we write :

$$\phi\,.\,x = z - \varepsilon$$

III - THE EMBEDDING PROCEDURE

Since the value of $x(k)$ in equation (4) is unknown, two approaches are classicaly encountered :

- eliminate $x(k)$ by projecting equation (4) on a space orthogonal to the space spanned by the columns of $\phi(k,p)$: this is the Parity Space approach,

- estimate $x(k)$ through an estimation procedure. A least square estimate may for instance be used (since the system $\phi x = z$ has generally no solution in real cases).

The embedding procedure, which includes the actual system in a larger one (over-system), will be used in order to present both approaches simultaneously. Moreover, we will generalize our presentation to partially observable systems.

For such systems, the matrix ϕ, of dimensions m.n, is of rank r with r<m and r<n. We generate an over-matrix in the following manner :

Let R (row) be an n-r.n matrix such that :

$$R \,.\, \phi^t = 0$$
$$R \,.\, R^t = I$$

So, $(\phi \,,\, R)^t$ spans a space of dimension n.

Symetrically, let K (column) be an m. m-r matrix such that :

$$K^t \,.\, \phi = 0$$
$$K^t \,.\, K = I$$

So, $(K \,,\, \phi)$ spans a space of dimension m.

The over-matrix and the over-system are defined by :

$$\phi^+ = \begin{bmatrix} K & \phi \\ 0 & R \end{bmatrix}$$

and

$$\phi^+ x^+ = z^+$$

with

$x^+ = (v^t, x'^t)^t$ and $z^+ = (z^t, d^t)^t$

v (respectively d) is a new m-r (resp. n-r) vector of variables (data).

x and x' have the same dimension.

The equations which represent the system and the over-system are thus respectively :

$\phi x = z - \varepsilon$ (equation 4)

$$\begin{cases} Kv + \phi x' = z \\ Rx' = d \end{cases} \qquad (5)$$

Note that the over-system is more general than the original one since the latter is a special case of the former, obtained for $Kv = \varepsilon$. In that sense, it can be seen that v introduces m-r degrees of freedom in the over-system. Symetrically, as $Rx' = d$ doesn't introduce new constraints, d constitutes a vector of n-r degrees of freedom.

The observability matrix ϕ is included in a "overmatrix" which has special characteristics: it contains more columns and more rows and the system of equations contains more variables and data. The aim of the construction of the overmatrix is in the following properties.

Property 1 :

ϕ^+ is invertible and its inverse is given by :

$$(\phi^+)^{-1} = \begin{bmatrix} K^t & 0 \\ (\phi^t\phi + R^tR)^{-1}\phi^t & (\phi^t\phi + R^tR)^{-1} R^t \end{bmatrix} \qquad (6)$$

Property 2 :

$\phi^t \phi + R^t R$ as well as its inverse, have n-1 unit eigenvalues, and the corresponding eigenvectors are the columns of R^t.

Proof : $(\phi^t \phi + R^t R)\, R^t = \phi^t \phi R^t + R^t R\, R^t$

The right-hand member is equal to R^t, since $\phi R^t = 0$ and $RR^t = I$ by the definition of R. The result concerning the inverse is obtained by premultiplying both members by $(\phi^t \phi + R^t R)^{-1}$.

Taking into account the property 2, the inverse of ϕ^+ becomes :

$$(\phi^+)^{-1} = \begin{bmatrix} K^t & 0 \\ (\phi^t \phi + R^t R)^{-1} \phi^t & R^t \end{bmatrix} \qquad (7)$$

As $\phi^+ \,.\, (\phi^+)^{-1} = I$ and $(\phi^+)^{-1} \,.\, \phi^+ = I$, we obtain the four following relations.

$$K K^t + \phi\, (\phi^t \phi + R^t R)^{-1} \phi^t = I \qquad (8\text{-a})$$
$$R^t R + (\phi^t \phi + R^t R)^{-1} \phi^t \phi = I \qquad (8\text{-b})$$
$$(\phi^t \phi + R^t R)^{-1} \phi^t K = 0 \qquad (8\text{-c})$$
$$R\, (\phi^t \phi + R^t R)^{-1} \phi^t = 0 \qquad (8\text{-d})$$

The first relation defines usually the parity space however it is only a consequence of the orthonormalization. This result is generally not very well explained in the litterature.

Property 3 :

The solutions of the over system are :

v : the parity vector of the system (4)

x' : a least square estimate of x in (4)

Proof : Using the form of $(\phi^+)^{-1}$, the solution is given by :

$$v = K^t z \qquad (9)$$
$$x' = (\phi^t \phi + R^t R)^{-1} \phi^t z + R^t d \qquad (10)$$

From (9) and (4) one has :

$v = K^t (\phi x + \varepsilon) = K^t \varepsilon$

which shows that v is the parity vector.

On another hand, let us premultiply (10) by $\phi^t \phi$ (a square matrix, of dimensions nxn and of rank r). We obtain :

$\phi^t \phi\, x' = \phi^t \phi\, (\phi^t \phi + R^t R)^{-1} \phi^t z + \phi^t \phi\, R^t d$

Using (8-a) and the fact that $\phi R^t = 0$, this gives :

$\phi^t \phi x' = \phi^t (I - KK^t) z$

and finally

$\phi^t \phi x' = \phi^t z$ (since $\phi^t K = 0$)

which shows that x' satisfies the stationarity condition required by the minimization of $\varepsilon^t \varepsilon$.

IV - BLOCK DIAGRAM REPRESENTATION

To keep close to the physical structure of the system we can use a graphical description based on the interconnexion of sub-systems. The graph contains variables and functions and so is bipartite. The sub-models can be transfer functions or z-transforms in the linear case. For a single-input single-output block, the equation can be:

$$s(z)\,D(z) - e(z)\,N(z) = 0$$

with s and e the ouput and input variables.

If c(z) and x(z) are respectively the known and unknown variables, the system can be described by:

$$J(z)\,c(z) + K(z)\,x(z) = 0$$

where J(z) and K(z) are polynomial matrices in z.

As above, we can define an over-matrix $K^+(z)$ and use the same approach, with the only difference that the computations are carried out on polynomial matrices.

V - CONCLUSION

The generation of ARR and the least squares estimate of vector space are simultaneous made in an unique formulation. Moreover , this approach permits to deduce easily many results, as a new expression of the pseudo-inverse. The three classical relations which define the parity space are generalized to partially observable processes. So, the embedding procedure is not limited to structural models but is also an efficient tool for the handling of quantitative representations in fault detection.

REFERENCES

[1] E.Y. Chow, A.S. Willsky
Analytical redundancy and the design of robust failure detection systems.
IEEE Trans on Automatic Control, AC-29 n° 7, July, 1984.

[2] P. Declerck
Analyse structurale et fonctionnelle des grands systèmes. Application à une centrale PWR 900 MW.
Thèse de doctorat, Université des Sciences et Technologies de Lille, Villeneuve d'Ascq, France, 20 décembre 1991.

[3] P. Declerck, M. Staroswiecki
Identification of structurally solvable subsystems for the design of fault detection and isolation schemes, using the embedding procedure.
9th IFAC/IFORS Symposium on identification and system parameter estimation. Budapest, July 8-13, 1991.

[4] P. Declerck, M. Staroswiecki
Characterization of the canonical components of a structural graph for fault detection in large scale industrial plants.
European control conference. Grenoble July 2-5, 1991

[5] M. Desai, A. Ray
A fault detection an isolation methodology.
Proc. 20th Conf. on Decision and Control, 1981, 1363-1369.

[6] J.J. Gertler
Survey of model-based failure detection and isolation in complex plants.
IEEE Trans on Automatic Control, december 1988.

[7] R. Iserman
Process fault detection based on modeling and estimation methods - A survey.
Automatica, Vol. 20, 387-404, 1984.

[8] X.C. Lou, A.S Willsky, G.L. Verghese
Optimally robust redundancy relations for failure detection in uncertain systems.
Automatica. 1986. Vol. 22, 333-344.

[9] R.J. Patton, P.M. Frank, R.N Clark
Fault diagnosis in dynamical systems, theory and application.
Prentice Hall, 1989.

[10] J.E. Potter, M.C. Suman
Thresholdeless redundancy management with array of skewed instruments. Integrity in Electronic Flight Control Systems.
Agardograph-224, 15-25. 1977

[11] M. Staroswiecki, P. Declerck
Analytical redundancy in non linear interconnected systems by means of structural analysis.
IFAC Symposium on Advanced Information Processing in Automatic Control, Nancy, France, 3-5 july, 1989.

[12] M. Staroswiecki, J.Ph. Cassar, P.Declerck
AutomaticAnalytical Redundancy relationships generation in complex interconnected system based on a structural approach.
INCARF, New Delhi, 1989.

[13] M. Staroswiecki, V. Cocquempot, J.Ph. Cassar
Generation of relations for the system supervision using the transfer function representation.
Proceeding of the ninth IASTED international conference on "modelisation, identification and control" pp 378-381. Innsbrück, Austria, february 18-21, 1990.

[14] M. Staroswiecki, V. Cocquempot, J.Ph. Cassar
Generation of analytical redundancy relations in a linear interconnected system.
Proceedings of IMACS-IFAC International Symposium. MIM-S^2, Brussels, September 3-7, 1990.