Modelling and analysis of P-time event graphs in the (min, max, +) algebra*

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Abstract – *P-Time petri nets can represent the dynamic behaviour of discrete event systems for which the time evolution of the state is not strictly deterministic but belongs to dynamic intervals. After introducing the modelling of p-Time event graphs, we show that the corresponding algebraic model is a subclass of a special model called the interval descriptor system which uses only maximization, minimization and the addition operations. The following aim is to check the behaviour of the model and to study the existence of a state trajectory. Using the cycle-time vector, we give an approach which makes it possible to detect the non- synchronization of the transitions and consequently the presence of dead-marks.*

Keywords: (min,max,+) functions, cycle-time vector, fixed point, P-Time Petri Nets.

1 Introduction

Discrete Event Dynamic Systems can describe many systems characterized as being concurrent, asynchronous, distributed or parallel, such as flexible manufacturing systems, multiprocessor systems or transportation networks. In such systems the behaviour depends on complex interactions of processes. In this field, (max,+) algebra makes it possible to analyse Timed Event Graphs and many results are available like spectral theory and control synthesis. More generally, topical algebra is an important field of mathematical and analysis techniques which includes particularly (max, +), (min, +) and (min, max, +) algebras. In this paper, a new class of systems is studied for which the time evolution is not strictly deterministic but belongs to intervals. At each step, the lower and upper bounds depends on the maximization, minimization and the addition operations. Instead considering only two operations, maximum and addition, we integrate a new operation which is the minimum operation. The symbol $\oplus$ stands for the maximum operation while $\wedge$ corresponds to the minimum operation. The operation $\oplus$ has the neutral element $\varepsilon = -\infty$ whereas $\wedge$ has the neutral element $T = +\infty$. The notations $\otimes$ and $\odot$ corresponds to the usual addition with the following convention: $T \otimes \varepsilon = \varepsilon$ and $T \odot \varepsilon = T$. The expression $a \otimes b$ and $a \odot b$ are identical if at least either a or b is a finite scalar.

We propose to analyse the following implicit model called interval descriptor system. The evolution of the system is described by the following equations where f^+ and f^- are (min, max, +) functions. The interpretation of each variable is as follows: like the "dater" type in (max,+) algebra, each variable $x_i(k)$ represents the date of the kth firing of transition x_i .

$$\begin{cases} x(k) = x(k) \wedge f^+(x(k), ..., x(k-m), u(k) \\ \qquad , ..., u(k-m)) \\ x(k) = x(k) \oplus f^-(x(k), ..., x(k-m), u(k) \\ \qquad , ..., u(k-m)) \end{cases} \tag{1}$$

The vector u is the input and m is the horizon. The functions $f^+()$ and $f^-()$ represent respectively an upper and lower bound of x whose trajectory is between these bounds. On the other hand, discrete event dynamics systems involving synchronization can be modelled by several types of Petri nets (PNs). Among these PNs, we can quote P-time Petri nets [6][5],.... Time stream PNs directly extend P-time Petri nets and can describe complex synchronization. In this paper, we will show that P-time Petri nets when the Petri net is an Event Graph can be modelled by an interval descriptor system.

After modelling, an important problem is to know if the obtained model is coherent and represents the dynamic evolution of a system. For the Petri nets, a blockade state, i.e. that no transition is validated starting from an initial marking, shows the absence of evolution. The token death in the places of a P-time event graph, can occur and introduces another difficulty. The dead marks in this case, cannot take part for any more in firing of the downstream transitions. The study of liveness for PNs is usually treated by the enumerative analysis based on the construction of the coverability graph. Without considering time, this combinatorial approach turns out to be complicated because the number of states becomes important for any size of the system. Moreover, the temporal character of P-time event graphs increases the complexity of the problem. The study of liveness by the method based on the markings graph is not always feasible. In this article, we propose an algebraic approach which makes it possible to check the correct behaviour of the synchronization of the transitions. Every non-synchronization entails the death of at least one token.

The paper is organized as follows: in section 2, we recall the algebraic elements needed in the paper, order properties, and dioid algebra. In section 4, we present some definitions of P-Time petri nets and give the modelling of p-Time Event Graph in the (min, max, +) equations form. We present the problem of synchronization and liveness of transitions and dead-marks. In the following section, we analyse the interval (min, max, +) systems. We lastly show that we can detect the non-synchronisations of transitions on some examples.

2 Algebraic tool

We briefly recall the algebraic results needed here. More on dioid and residuation theory details can be found in [7][2].

A monoid is a couple $(D, \oplus)$ where the operation $\oplus$ is associative and presents a neutral element. A semi-ring D is a triplet $(S, \oplus, \otimes)$ where $(D, \oplus)$ and $(D, \otimes)$ are monoids, $\oplus$ is commutative, $\otimes$ is distributive relatively to $\oplus$ and the zero element of $\oplus$ is the absorbing element of $\otimes$. A dioid D is an idempotent semi-ring. Let us notice that contrary to the structures of group and ring, monoid and semi-ring do not have a property of symmetry on D.

The unit $\Re \cup \{-\infty\}$ provided with the maximum operation denoted $\oplus$ and the addition denoted $\otimes$ is usually called (max, +) algebra and is an example of dioid.

$\Re_{\min} = (\Re \cup \{+\infty\}, \wedge, \odot)$ is an another dioid which is isomorphic to the previous one by the bijection: $x \longmapsto -x$. The neutral element of $\wedge$ is T which is absorbing for $\odot$.

The partial order denoted $\leq$ is defined as follows: $x \leq y \Longleftrightarrow x \oplus y = y \Longleftrightarrow x \wedge y = x \Longleftrightarrow x_i \leq y_i$, for i from 1 to n in $\Re^n$. Notation $x < y$ means that $x \leq y$ and $x \neq y$.

Definition 2.1 A dioid D is complete if it is closed for infinite sums and the distributivity of the multiplication with respect to addition extends to infinite sums : $(\forall\ c \in D\ )\ (\forall\ A \subseteq D)\ c \otimes (\bigoplus_{x \in A} x) = \bigoplus_{x \in A} c \otimes x$

For example, $\bar{\Re}_{max} = (\Re \cup \{-\infty\} \cup \{+\infty\}, \oplus, \otimes)$ is complete.

The set of $n.n$ matrices with entries in a complete dioid D endowed with the two operations $\oplus$ and $\otimes$ is also a complete dioid which is noted $D^{n.n}$. The elements of the matrices in the (max, +) expressions (respectively (min, +) expressions) are either finite or ϵ ((respectively T). We can deal with nonsquare matrices if we complete by rows or columns with entries equals to ε (respectively T). The different operations operate as in the usual algebra: The notation $\odot$ refers to the multiplication of two matrices in which the $\wedge$ operation is used instead of $\oplus$.

$$(\mathrm{A} \oplus \mathrm{B})_{ij} = \mathrm{A}_{ij} \oplus \mathrm{B}_{ij}\ ,$$
$$(\mathrm{A} \wedge \mathrm{B})_{ij} = \mathrm{A}_{ij} \wedge \mathrm{B}_{ij}\ ,$$
$$(\mathrm{A} \otimes \mathrm{B})_{ij} = \bigoplus_{k=1}^{n} \mathrm{A}_{ik} \otimes \mathrm{B}_{kj}$$
$$(\mathrm{A} \odot \mathrm{B})_{ij} = \bigwedge_{k=1}^{n} \mathrm{A}_{ik} \odot \mathrm{B}_{kj}$$

The left $\otimes$ residuation of b by a is defined by: $a \backslash b = \max\{x \in D \text{ such that } a \otimes x \leq b\}$. Respectively, in $(\wedge, \odot)$ algebra, the left $\odot$ residuation of b by a is defined by: $a \backslash' b = \min\{x \in D \text{ such that } a \odot x \geq b\}$.

Given A and B two matrices in a complete dioid, the residuation of B (dimensions $n.q$) by A (dimensions $n.p$) is clearly expressed in the other dioid:

In (max, +) algebra $A \backslash B = (-A)^t \odot B$ and in (min, +) algebra $A \backslash' B = (-A)^t \otimes B$ with t: transpose.

Lemma 2.2 (part1 of lemma 4.77 in [2]) We have the following equivalences: $x \geq ax \Leftrightarrow x = a^* x \Leftrightarrow x \leq a \setminus x \Leftrightarrow x = a^* \setminus x$

3 P-time Petri nets

Our approach is based on a mathematical description which represents the dynamic evolution of the P-time event graphs. In this paper, we introduce the new modelling which is made in the form of a linear equations system of the type (min, max, +).

3.1 Modelling of P-time Petri nets

The P-time Petri nets makes it possible to model the discrete event dynamic systems with time constraints of stay of the tokens inside the places. We associate for each place a temporal interval.

Definition 3.1 (p-time Petri nets) The formal definition of P-time Petri net is given by a pair $< R, IS >$ where R is a marked Petri nets

$IS : P \longrightarrow (Q^+ \cup \{0\}) \times (Q^+ \cup \{\infty\})$

$p_i \longrightarrow IS_i = [a_i, b_i]$ with $0 \leq a_i \leq b_i$

IS_i is the static interval of residence time or duration of a token in place p_i. The value a_i is the minimum residence duration that the token must stay in the place p_i. Before this duration, the token is in state of unavailability to firing the transition t_j. The value b_i is a maximum residence duration after which the token must thus leave the place p_i. If not, the system is found in a token-dead state. We conclude that the token is available to firing the transition t_j in the interval time $[a_i, b_i]$.

Let us consider the variable $x_i(k)$ as the date of the kth firing of transition x_i and S_i the set of the upstream places of this transition. For each place p_j, we associate an interval $[a_j, b_j]$ of which a_j the lower bound and b_j the upper bound. The assumption of functioning FIFO of the transition x_i guarantees the condition of non overtaking of the tokens between them. We will express the interval of shooting of each transition from the system. Under the FIFO assumption, we obtain the following interval descriptor system : $\bigoplus_{j \in S_i} (x_j(k-m_j)+a_j) \leq x_i(k) \leq \bigwedge_{j \in S_i} (x_j(k-m_j)+b_j)$ with m_j the number of the present tokens in each place p_j at the instant $t = 0$ (initial marking). The lower bound (respectively upper bound) f^- (respectively f^+) is a (max, +) function (respectively (min, +) function).

Remark 3.2 : If we divide up each place which contains m tokens in m places, with one token by place, we can obtain the equations on a shorter horizon: for example, only the upstream place of x_i has temporization $[a, b]$ and the other places have all the null time interval $[0, 0]$.

3.2 Synchronization and liveness of transitions

The Petri nets make it possible to analyze several behavioral or structural properties related to the systems which they model. We consider one of these behavioral properties, the liveness which ensures the system not to reach a state of blocking. This property depends on initial marking. A state of blocking in PNs occurs when we reach a marking which does not allow the firing of any transition. Now we give some definitions of liveness.

Definition 3.3 (liveness of a transition) A transition x_i is live for an initial marking M_0 if, for any marking M_j accessible since M_0 there is a sequence of firing S starting from M_j which includes the transition x_i

Definition 3.4 (liveness of a petri net) For a given initial marking, a Petri net is live if for any accessible marking M, and for any transition t, there is a sequence of firing S which includes the transition t:

$\forall\, M \in E(M_0), \forall\, p \in P, \exists\, S \,/\, M \longrightarrow^{S} M'$ and $t \in S$

Classically, one of the methods which allow to check liveness is analysis by enumeration. This approach consists in building the coverability graph if the number of markings is finished, or in building the coverability tree if the number of markings is infinite. For temporal petri nets, checking and making study of the liveness property becomes more difficult since the latter depends not only on initial marking but also on the intervals of times related to the graph. It thus proves that the use of the method by enumeration is very difficult. Indeed, the passage of a state to another is related either to the firing of a transition or to the evolution from time. Thus, a consequence is combinative explosion of the coverability graph.

We define the acceptable functioning of a system by any functioning which guarantees non-dead tokens and which does not lead to any deadlock situation. In the following parts, we will verify if the process has an acceptable functioning.

4 Analysis

4.1 Cycle time and compatibility

Now, we introduce the definitions of cycle time, eigenvector, eigen-value and ultimately affine regime [7]. These notions are relevant to the (min, max, +) functions but not always to the topical functions. Some connections can be established between these concepts. Addition + is defined by: $\forall \lambda \in \Re$, $\forall x \in \Re^n$, $\lambda + x = (\lambda + x_1, ..., \lambda + x_n)^t$ (t: transpose)

Definition 4.1 A min-max function of type $(n, 1)$ is any function $f : \Re^n \to \Re^1$, which can be written as a term in the following grammar:
$f = x_1, x_2, \cdots x_n \mid f + a \mid f \wedge f \mid f \oplus f$ where a is an arbitrary real number ($a \in \Re$)
The set of $\min - \max$ function of type (n, m) is noticed $D^*(n, m)$.

Definition 4.2 The cycle time vector is defined by $\chi(f) = \lim_{k \to \infty} x(k)/k$ if it exists. It does not depend on ξ.

Definition 4.3 An eigen-vector x and its associated eigen-value $\lambda \in \Re$, if they exists, verify $f(x) = \lambda + x$

Definition 4.4 The pair $(\eta, v) \in (\Re^n)^2$ is an ultimately affine regime of f if there exists an integer K such that $\forall k \geq K$, $f(v + k\eta) = v + (k + 1)\eta$.

Corollary 4.5 [4] Any function in D^* has a cycle time. Moreover, $\chi(f)= \eta$, for all ultimately regimes $(\eta, v) \in (\Re^n)^2$ of f.

In the following theorems, the notion of cycle time which always exists in D^* makes it possible to check the existence of a solution of different inequalities and equalities.

Theorem 4.6 [7] Let $f \in D^*$. The two following conditions are equivalent:

(i) It exists a finite x such that $x \leq f(x)$

(ii) $\chi(f) \geq 0$

Theorem 4.7 Let $f \in D^*$. The two following conditions are equivalent:

(i) It exists a finite x such that $x \geq f(x)$

(ii) $\chi(f) \leq 0$

From the two previous theorems 4.6 and 4.7, we deduce directly the following result.

Theorem 4.8 Let $f \in D^*$. The two following conditions are equivalent:

(i) It exists a finite x such that $x = f(x)$

(ii) $\chi(f) = 0$

4.2 Different types of interval descriptor system

In the aim of reducing the size of the expressions, the system 1 can be easily transformed in reduced form by increasing the vector state. With an abuse of notation, we keep the same notation for x, f^- and f^+ to alleviate the notation. From the system 1, we deduce:

$x(k) \leq f^+(x(k), x(k-1), u(k))$

$x(k) \geq f^-(x(k), x(k-1), u(k))$

As f^+ and f^- are (min, max, +) functions, the above form is more general that the "UBC" (Upper Bound Constraint) where f^+ is a max-only function (see [10] for more details). We can call the above forms respectively, "GUBC" (Generalized Upper Bound Constraint) and "GLBC" (Generalized Lower Bound Constraint). As the above theorems 4.6, 4.7 and 4.8 can only be applied to the forms $x \leq f(x)$, $x \geq f(x)$ or $x = f(x)$ where $f \in D^*$, we must consider special cases. As the type of the system 1 is defined by the types of the functions f^+ and f^-, we can characterize the model by the couple (type of f^-, type of f^+). The type ((min, max, +), (min, max, +)) represents the more general case for the system 1 . In the next sections, we will respectively consider special cases. The type of the first one is ((max, +), (min, max, +)) whereas the type of the second one is ((min, max, +), (min, +)). In the third part we consider the ((max, +), (min, +)) type and show that the P-time event graphs can be modelled under this form. However, every interval descriptor system does not describe a trajectory on an horizon. We introduce the following useful definition

Definition 4.9 An interval descriptor system is compatible on the horizon $[k, k+h]$ if it exists a state trajectory $x(k)$ and an input trajectory $u(k)$ verifying the equalities of the interval descriptor system on the same horizon.

The following theorems analyse the compatibility of different types of system without calculating a specific trajectory.

4.2.1 ((max,+), (min,max,+)) type

Theorem 4.10 Given a system of ((max, +), (min, max, +)) type with

$$f^-(x(k)) = \bigoplus_{i=0}^{1} A_i^- \otimes x(k-i) \oplus B^- \otimes u(k). \qquad (2)$$

This system is compatible in the horizon $[k, k+h]$ if and only if it the cycle time of the function $g_h^+(z(k))$ is greater than or equals zero.

$$z(k) = \begin{pmatrix} x(k) \\ \vdots \\ x(k+h-1) \\ x(k+h) \\ u(k) \\ \vdots \\ u(k+h-1) \\ u(k+h) \end{pmatrix}, \; g_h^+(z(k)) =$$

$$\begin{pmatrix} f^+(x(k), x(k-1), u(k)) \\ \vdots \\ f^+(x(k+h-1), x(k+h-2), u(k+h-1)) \\ f^+(x(k+h), x(k+h-1), u(k+h)) \\ T \\ \vdots \\ T \\ T \end{pmatrix}$$

$\wedge M^t \setminus z(k) \wedge z(k+1)$

with $M = \left(\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right)$,

$$M_{11} = \begin{pmatrix} A_0^- & A_1^- & \cdots & T \\ T & \ddots & \ddots & \vdots \\ \vdots & & A_0^- & A_1^- \\ T & & \cdots & A_0^- \end{pmatrix}, \quad M_{21} = \begin{pmatrix} B^- & T & \cdots & T \\ T & \ddots & & \vdots \\ \vdots & & B^- & \\ T & & \cdots & B^- \end{pmatrix}$$

and $M_{12} = M_{22} = T$

proof

For $0 \leq j \leq h$ we have : $\begin{cases} x(k+j) \leq f^+(x(k+j), x(k+j-1), u(k+j)) \\ x(k+j-1) \leq x(k+j) \\ u(k+j-1) \leq u(k+j) \end{cases}$

and $A_0^- \otimes x(k+j) \oplus A_1^- \otimes x(k+j-1) \oplus B^- \otimes u(k+j) \leq x(k+j)$

we use the lemma 2.2, we arrive to :

$$\Longleftrightarrow \begin{cases} x(k+j) \leq A_0^- \setminus x(k+j) \\ x(k+j-1) \leq A_1^- \setminus x(k+j) \\ u(k+j) \leq B^- \setminus x(k+j) \end{cases}$$

A concatenation of the two last systems gives the following form : $\forall\ 0 \leq j \leq h$

$$\begin{cases} x(k+j) \leq \\ f^+(x(k+j), x(k+j-1), u(k+j)) \\ \wedge A_0^- \setminus x(k+j) \wedge \\ A_1^- \setminus x(k+j+1) \wedge x(k+j+1) \\ u(k+j) \leq B^- \setminus x(k+j) \wedge u(k+j+1) \end{cases}$$

and $\begin{cases} x(k+h) \\ \leq f^+(x(k+h), x(k+h-1), u(k+h)) \\ \wedge A_0^- \setminus x(k+h) \\ u(k+h) \leq B^- \setminus x(k+h) \end{cases}$

Lastly, the above system can be reduced to the following form where the function g_h^+ is described in the body of the theorem.

$$z(k) \leq g_h^+(z(k))$$

The system of ((max,+), (min,max,+)) type is reduced to a (-∞, (min,max,+)) type and can be analyzed by the relevant theorem 4.6. If the cycle time verifies the corresponding condition of existence, it describes a compatible interval descriptor system.

4.2.2 ((min,max,+), (min,+)) type

Theorem 4.11 Given a system (f^-, f^+) of ((min,max,+), (min,+)) type with

$$f^+(x(k)) = \bigwedge_{i=0}^{1} A_i^+ \odot x(k-i) \wedge B^+ \odot u(k). \tag{3}$$

This system is compatible in the horizon $[k, k+h]$ if and only if the cycle time of the following function $g_h^-(z(k))$ is lower than or equals zero.

$$g_h^-(z(k)) = \begin{pmatrix} f^-(x(k), x(k-1), u(k)) \\ \vdots \\ f^-(x(k+h-1), x(k+h-2), u(k+h-1)) \\ f^-(x(k+h), x(k+h-1), u(k+h)) \\ \epsilon \\ \vdots \\ \epsilon \\ \epsilon \end{pmatrix} \oplus N^t \setminus' z(k) \oplus z(k-1)$$

with $N = \left(\begin{array}{c|c} N_{11} & N_{12} \\ \hline N_{21} & N_{22} \end{array}\right)$, $N_{11} = \begin{pmatrix} A_0^+ & A_1^+ & \cdots & \epsilon \\ \epsilon & \ddots & \ddots & \vdots \\ \vdots & & A_0^+ & A_1^+ \\ \epsilon & \cdots & \epsilon & A_0^+ \end{pmatrix}$,

$$N_{21} = \begin{pmatrix} B^+ & \epsilon & \cdots & \epsilon \\ \epsilon & \ddots & \ddots & \vdots \\ \vdots & & B^+ & \epsilon \\ \epsilon & \cdots & \epsilon & B^+ \end{pmatrix}$$

and $N_{12=}N_{22} = \epsilon$

The system of ((min,max,+), (min,+)) type is reduced the following form :

$$z(k) \geq g_h^-(z(k))$$

Finally this system can be reduced to a ((min,max,+), +∞) type and can be analyzed by the relevant theorem 4.7. If the cycle time verifies the corresponding condition of existence, it describes a compatible interval descriptor system.

4.2.3 Application to P-time event graphs

We have shown in section 4 that the modelling of the P-time event graphs results in an interval descriptor system whose type is ((max, +), (min, +)). In this case, the system (f^-, f^+) is ((max, +), (min, +)) type, and the results given in the theorems 4.10 and 4.11 remain valid for this last system. Indeed, it is enough to replace f^+ in the theorem 4.12 by $f^+(z(k)) = \bigwedge_{i=0}^{1} A_i^+ \odot x(k-i) \wedge B^+ \odot u(k)$ and f^- in the theorem 4.13 by $f^-(z(k)) = \bigoplus_{i=0}^{1} A_i^- \otimes x(k-i) \oplus B^- \otimes u(k)$ Checking liveness for a horizon h, amounts to studying the sign of $\chi(x_i(h))$. Consequently, we can use this result to study and check the liveness of every transition x_i.

We add the following theorem specific to P-time event graphs :

Theorem 4.12 If the system (f^-, f^+) which models the p-time event graphs is compatible in the horizon $[k, k+h]$ with $k = 1$, then the following conditions are equivalent :
1. The cycle time of the function g_h^+ is greater than or equals zero.
2. The cycle time of the function g_h^- is lower than or equals zero.

5 Examples

Example 1

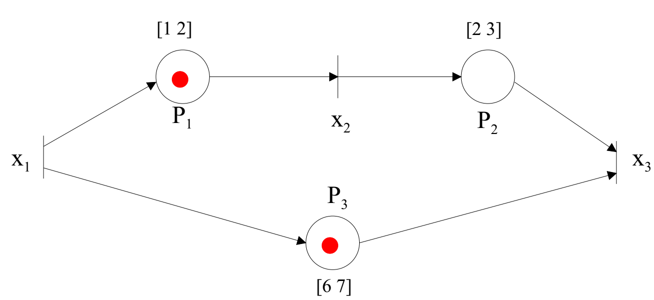

Figure 1: A p-time event graph without circuit

We consider the first example of the figure 1 which will enable us to illustrate our approach. Initially we can check easily that the logical graph (without taking account of temporizations) is quite live. The first step of our approach is to model the system by recurring state equations in the following form:

$$\begin{cases} x_1(k-1) + 1 \leq x_2(k) \leq x_1(k-1) + 2 \\ (x_2(k) + 2) \oplus (x_1(k-1) + 6) \leq x_3(k) \\ \leq (x_2(k) + 3) \wedge (x_1(k-1) + 7) \end{cases} \tag{4}$$

We divide up the system 4, and we put it in the form $x \leq f(x)$. Thus we arrive at the following system:

$$\begin{cases} x_1(k-1) \leq (x_2(k)-1) \wedge (x_3(k)-6) \\ x_2(k) \leq (x_1(k-1)+2) \wedge (x_3(k)-2) \\ x_3(k) \leq (x_2(k)+3) \wedge (x_1(k-1)+7) \end{cases}$$

In the second step for our approach, we calculate the spectral vector of f, and we apply the theorem 4.6. We arrive at the following results:
$\chi(x_3(1)) = -\frac{1}{3}$
The calculation of the spectral vector will enable us to show the non-liveness of the transition x_3.
Then, the synchronization cannot be make to firing transition x_3 for the first time. The two tokens will die, the system is in state of blocking from the beginning because $\chi\left(\ x_3(1)\ \right) < 0$.
By considering temporizations related to each place, we can note that in spite of an initial marking which ensures the liveness of the logical graph, the temporal graph can be in a state of total blocking.

Example 2

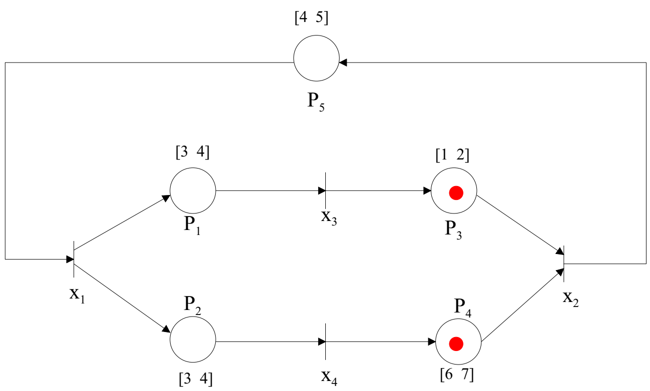

Figure 2: A p-time event graph with circuit

Now, we consider the second example of the figure 2. We apply the same steps followed in the preceding example, we obtain the following results :

$\chi(x_2(1)) = \frac{1}{2}$ $\quad \chi(x_1(1)) = \frac{1}{2}$
$\chi(x_2(2)) = -\frac{3}{4}$ $\quad \chi(x_1(2)) = -\frac{3}{4}$

We notice that $\chi\begin{pmatrix} x_2(1) \\ x_1(1) \end{pmatrix} \geq 0$ and $\chi\begin{pmatrix} x_2(2) \\ x_1(2) \end{pmatrix} < 0$

The system is live for the first step ($k = 1$). It after loses its tokens (dead tokens) and its liveness property is not assured any more.

6 Conclusion

In this paper, we have introduced a new model, the interval descriptor system which is based on (min, max, +) functions. By reason of their special type of synchronisation of the transitions, P-time event graphs can be modelled in this form. Future works will show that Time Stream Petri Net is another example where the approach can be applied for different types of synchronisation. On the other hand, the analysis of the spectral vector makes it possible to study the problem of synchronisation and liveness of the transitions. Particularly, the approach can precise the transitions which present a non-synchronization and the corresponding number of event. Let us notice that the method does not need to calculate a specific trajectory.

This work shows the practical interest of the spectral vector which is not limited to the production rate or to an only theoretical aspect. Developing some algorithms of calculation of this vector [4] will complete the approach and will make it possible to apply the approach on large scale Petri nets.

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