From Linear Programming to Graph Theory: Standardization of the Algebraic Model of Timed Event Graphs

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Plan

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Motivations and Objectives

- Description of Timed Event Graphs with the form $Ax \leq b$.

$\longrightarrow$ analogeous to the state equation of automatic control in continuous systems.

- Development of a path theory but completely defined in the standard algebra.

$\longrightarrow$ Possible application of polyvalent algorithms of linear programming like the simplex

- Objective : **Standardization of the Algebraic Model of Timed Event Graphs**

$\longrightarrow$ Avoid the useless calculations in the calculation of the state trajectory

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Timed Event Graphs

Model

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Inequations

1. Dater : each variable $x_i(k)$ is the $k^{ème}$ firing date of transition x_i.

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3. $\forall p_l : T_l + x_j(k - m_l) \leq x_i(k) \Longleftrightarrow x_j(k - m_l) - x_i(k) \leq -T_l$.

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Example of Timed Event Graph

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Technique 1 using linear programming

Dual technique 2

Properties

Conclusion

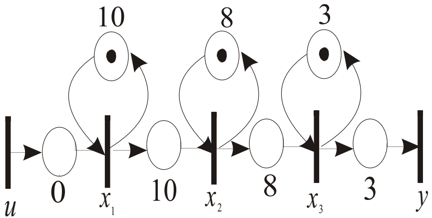

FIGURE: Timed Event Graph

Initial model

Internal inequalities.

$$\begin{pmatrix} A_{.,1} & A_{.,0} \end{pmatrix} \begin{pmatrix} x(k-1) \\ x(k) \end{pmatrix} \leq -T^A \qquad (1)$$

Input inequalities.

$$\begin{pmatrix} B_1 & B_0 \end{pmatrix} \begin{pmatrix} u(k) \\ x(k) \end{pmatrix} \leq -T^B \qquad (2)$$

Output inequalities.

$$\begin{pmatrix} C_1 & C_0 \end{pmatrix} \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} \leq -T^C \qquad (3)$$

Each row of matrices $A = \begin{pmatrix} A_{.,1} & A_{.,0} \end{pmatrix}$, $B = \begin{pmatrix} B_1 & B_0 \end{pmatrix}$ and $C = \begin{pmatrix} C_1 & C_0 \end{pmatrix}$, is null except two coefficients 1 and -1.

We assumes that the set of input and output places presents a null initial marking.

Example of Timed Event Graph.

The relevant matrices are as follows : $A_{.,1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

$A_{.,0} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $T^A = \begin{pmatrix} 10 \\ 8 \\ 10 \\ 8 \\ 3 \end{pmatrix}$.

$B_1 = 1$, $B_0 = \begin{pmatrix} -1 & 0 & 0 \end{pmatrix}$ and $T^B = 0$.

$C_1 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$ and $C_0 = -1$ and $T^C = 3$.

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Initial model

Internal inequalities.

$$\begin{pmatrix} A_{.,1} & A_{.,0} \end{pmatrix} \begin{pmatrix} x(k-1) \\ x(k) \end{pmatrix} \leq -T^A \tag{4}$$

Input inequalities.

$$\begin{pmatrix} B_1 & B_0 \end{pmatrix} \begin{pmatrix} u(k) \\ x(k) \end{pmatrix} \leq -T^B \tag{5}$$

Output inequalities.

$$\begin{pmatrix} C_1 & C_0 \end{pmatrix} \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} \leq -T^C \tag{6}$$

Each row of matrices $A = \begin{pmatrix} A_{.,1} & A_{.,0} \end{pmatrix}$, $B = \begin{pmatrix} B_1 & B_0 \end{pmatrix}$ and $C = \begin{pmatrix} C_1 & C_0 \end{pmatrix}$, is null except two coefficients 1 and -1.

We assumes that the set of input and output places presents a null initial marking.

Objective : Final model using incidence matrices

$$\begin{pmatrix} W^{+}_{x\to x} & -W^{-}_{x\to x} \end{pmatrix} \begin{pmatrix} x(k-1) \\ x(k) \end{pmatrix} \leq -T_{x\to x} \quad (7)$$

$$\begin{pmatrix} W^{+}_{u\to x} & -W^{-}_{u\to x} \end{pmatrix} \begin{pmatrix} u(k) \\ x(k) \end{pmatrix} \leq -T_{u\to x} \quad (8)$$

$$\begin{pmatrix} W^{+}_{x\to y} & -W^{-}_{x\to y} \end{pmatrix} \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} \leq -T_{x\to y} \quad (9)$$

The matrices of this new model are well-known ingoing/outgoing incidence matrices used in the fundamental relation of marking. A Petri net can directly be deduced from this model.

Initial model

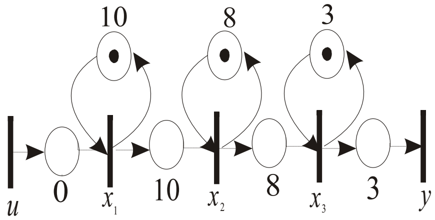
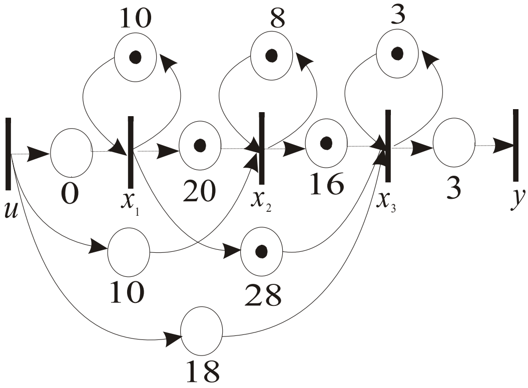


FIGURE: Timed Event Graph (initial)

Final model



Advantages

Connections with the incidence matrices of fundamental equation of marking

$$W^{+} = \begin{pmatrix} W^{+}_{u\to x} & 0 & 0 \\ 0 & W^{+}_{x\to x} & 0 \\ 0 & W^{+}_{x\to y} & 0 \end{pmatrix} \text{ and } W^{-} = \begin{pmatrix} 0 & W^{-}_{u\to x} & 0 \\ 0 & W^{-}_{x\to x} & 0 \\ 0 & 0 & W^{-}_{x\to y} \end{pmatrix}$$

, for a vector of transitions $\begin{pmatrix} u^t & x^t & y^t \end{pmatrix}^t$.

The temporisations are $T_{u\to x}$, $T_{x\to x}$ and $T_{x\to y}$.

Each internal place $\longrightarrow$ initial marking equal to one

Each input/output place $\longrightarrow$ initial marking equal to zero.

Advantages

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Connections with the incidence matrices of fundamental equation of marking

$$W^+ = \begin{pmatrix} W^+_{u\to x} & 0 & 0 \\ 0 & W^+_{x\to x} & 0 \\ 0 & W^+_{x\to y} & 0 \end{pmatrix} \text{ and } W^- = \begin{pmatrix} 0 & W^-_{u\to x} & 0 \\ 0 & W^-_{x\to x} & 0 \\ 0 & 0 & W^-_{x\to y} \end{pmatrix}$$

, for a vector of transitions $\begin{pmatrix} u^t & x^t & y^t \end{pmatrix}^t$.

The temporisations are $T_{u\to x}$, $T_{x\to x}$ and $T_{x\to y}$.

Each internal place $\longrightarrow$ initial marking equal to one

Each input/output place $\longrightarrow$ initial marking equal to zero.

Calculations

This model allows an efficient calculation of the state (knowing the past state and the control)$\longrightarrow$ it avoids the repetition of the useless calculations in the iterative calculation of the state.

Example : the initial Timed Event Graph.

The relevant matrices are as follows : $A_{.,1} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$,

$A_{.,0} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $T^A = \begin{pmatrix} 10 \\ 8 \\ 10 \\ 8 \\ 3 \end{pmatrix}$.

$B_1 = 1$, $B_0 = \begin{pmatrix} -1 & 0 & 0 \end{pmatrix}$ and $T^B = 0$.

$C_1 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$ and $C_0 = -1$ and $T^C = 3$.

Example : the final Timed Event Graph.

$$W^{+}_{u\to x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, -W^{-}_{u\to x} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$T_{u\to x} = \begin{pmatrix} 0 \\ 10 \\ 18 \end{pmatrix}$$

$$W^{+}_{x\to x} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} -W^{-}_{x\to x} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$(T_{x\to x})^t = \begin{pmatrix} 10 & 20 & 28 & 8 & 16 & 3 \end{pmatrix}$$

Earliest state trajectory.

k	0	1	2	3
x	0	10	20	35
	0	20	30	45
	0	28	38	53
u	-	0	0	35
y	3	31	41	56

Technical point

$$\begin{pmatrix} A_{.,1} & A_{.,0} \end{pmatrix} \begin{pmatrix} x(k-1) \\ x(k) \end{pmatrix} \leq -T^A \quad (10)$$

$$\begin{pmatrix} 0 & A_{0,0} \\ A_{1,1} & A_{1,0} \end{pmatrix} . \begin{pmatrix} x(k-1) \\ x(k) \end{pmatrix} \leq \begin{pmatrix} -T_0^A \\ -T_1^A \end{pmatrix} \quad (11)$$

Elimination of the relations connected the entries of state vector $x(k)$ for given k

$$A_{0,0}.x(k) \leq -T_0^A \quad (12)$$

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Technique 1 using linear programming.

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The possible effects on date $x_i(k)$ are produced by :
- the firing dates of a control transition $u_j(k)$ (case a) and also, produced by
- the firing dates of the upstream transitions of places whose initial marking is one $x_j(k-1)$ (case b).

- Case a) The minimal effect is the minimal difference $x_i(k) - u_j(k)$ or
  $min(c'x)$ for the following constraints
  $$\begin{pmatrix} 0 & A_{0,0} \\ B_1 & B_0 \end{pmatrix} \begin{pmatrix} u(k) \\ x(k) \end{pmatrix} \leq \begin{pmatrix} -T_0^A \\ -T^B \end{pmatrix}$$
  where c' is a null row-vector except $c'_i = 1$ et $c'_j = -1$

For each pair (x_i, u_j), the resolution of this problem gives the minimal difference ΔT
$x_i(k) - u_j(k) \geq \Delta T$ or $x_i(k) \geq u_j(k) + \Delta T$.

- Case b) The minimal effect is the minimal difference $x_i(k) - x_j(k-1)$ or $min(c'x)$ for the following constraints

$$\begin{pmatrix} 0 & A_{0,0} \\ A_{1,1} & A_{1,0} \end{pmatrix} \cdot \begin{pmatrix} x(k-1) \\ x(k) \end{pmatrix} \leq \begin{pmatrix} -T_0^A \\ -T_1^A \end{pmatrix} \quad (13)$$

  where c' is a null row-vector except $c'_i = 1$ and $c'_j = -1$

For each pair (x_i, x_j), the resolution of this problem gives the minimal difference ΔT $x_i(k) - x_j(k-1) \geq \Delta T$ or $x_i(k) \geq x_j(k-1) + \Delta T$

Example

a) The minimal difference $x_i(k) - u_j(k)$ is $\begin{pmatrix} 0 \\ 10 \\ 18 \end{pmatrix}$. So,

$W^+_{u \to x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $-W^-_{u \to x} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and

$-T_{u \to x} = \begin{pmatrix} 0 \\ -10 \\ -18 \end{pmatrix}$

b) The minimal difference $x_i(k) - x_j(k-1)$ is $\begin{pmatrix} 10 & -\infty & -\infty \\ 20 & 8 & -\infty \\ 28 & 16 & 3 \end{pmatrix}$

. So, $W^+_{x\to x} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $-W^-_{x\to x} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{pmatrix}$ and

$$-T_{x\to x} = \begin{pmatrix} -10 \\ -20 \\ -28 \\ -8 \\ -16 \\ -3 \end{pmatrix}$$

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Dual technique 2

Theorem of duality.
Primal problem (P) : $\min \; y.b$ with $y \in R^n_+$, $y.A = c$ and y real positive.
and
Dual problem (D) : $\max \; c.x$ with $x \in R^m$, $A.x \leq b$ and x real. ■

- Case b)
  $\max \; y. \begin{pmatrix} T_0^A \\ T_1^A \end{pmatrix}$ with $y \in R^m_+$ ($y \geq 0$) under constraints

$$y. \begin{pmatrix} 0 & A_{0,0} \\ A_{1,1} & A_{1,0} \end{pmatrix} = -c' \quad (14)$$

  and $y \geq 0$ where c' is a null row-vector except $c'_i = 1$ and $c'_j = -1$.
  i and j are respectively the indexes of outgoing transition x_i and ingoing transition x_j.

- Case a)

  $\max y.\begin{pmatrix} T_0^A \\ T^B \end{pmatrix}$ with $y \in R_+^m$ $(y \geq 0)$ under constraints

$$y.\begin{pmatrix} 0 & A_{0,0} \\ B_1 & B_0 \end{pmatrix} = -c' \qquad (15)$$

  where c' is a null row-vector except $c'_i = 1$ et $c'_j = -1$.

i and j are respectively the index of internal transition x_i and input transition u_j .

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Properties

- Approach 1. determination of the minimal time difference between the same vertices.
- Approach 2. determination of the greatest paths in graph theory. Integer vector y can only choose a unique path from transition x_s to transition x_e and its coefficients are zero or one.
- $y_{opt} \geq 0$
- Solution y_{opt} of the linear programming (P) is integer.

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Conclusion

- We generalize the technique of Roy (see book of Gondran and Minoux) to the modeling of Event Graphs
- Connections of our model using daters $Ax \leq b$, with incidence matrices of equation of marking
- Two dual approaches which solve the problem of standardisation in linear programming.
- In (max, +) algebra, an equivalent technique exists (Kleene star–problem P)
- A perspective is the application to the model checking (Calculation of the polyhedrons in state classes)