Liveness and acceptable trajectories in P-time Event Graphs

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Plan of the presentation

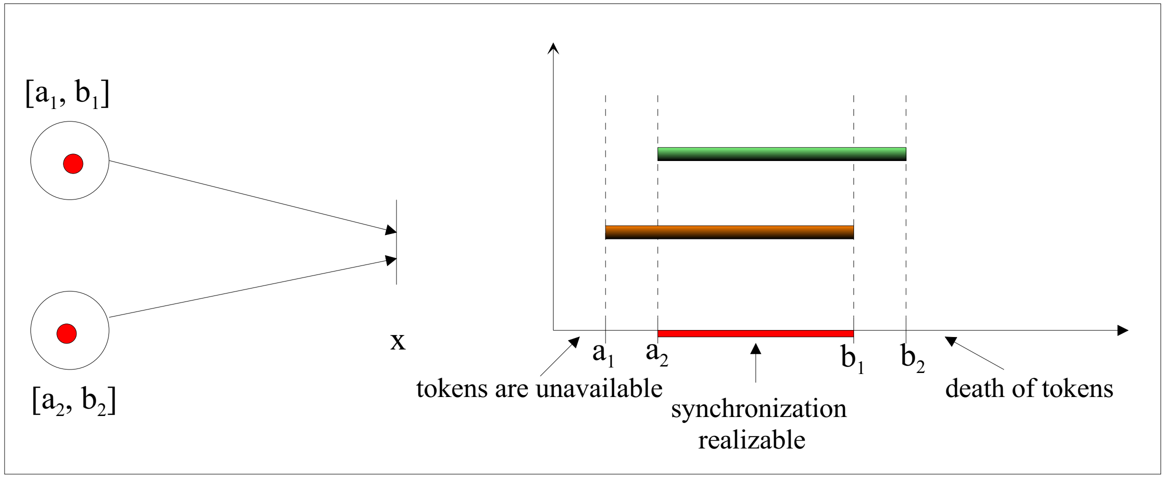
- Two objectives
- Modelling of P-time Event Graphs
- Existence of a state trajectory
- Calculation of the extremal trajectories
- Conclusion & Perspectives

Two objectives

- Modelling of Event Graphs:
  - Different Event Graphs: Timed Event Graphs, P-Time Event Graphs, Time Stream Event Graphs, ...
- Liveness and state trajectories
  - Existence of a state trajectory (temporal liveness)
  - Calculation of extremal acceptable trajectories

P-time event graphs

- A P-time Petri net is a pair $< R, IS >$ where R is a marked Petri nets
  $IS : P \longrightarrow \mathbb{R}^+ \times (\mathbb{R}^+ \cup \{+\infty\})$
  $p_i \longrightarrow IS_i = [a_i, b_i]$ with $0 \leq a_i \leq b_i$
  IS_i is the static interval of residence time or duration of a token in place p_i belonging to the set of places P.
- Synchronization phenomenon



- Problems:
  - ⇝ Death of tokens
  - ⇝ Deadlock situation

Interval model

- First model

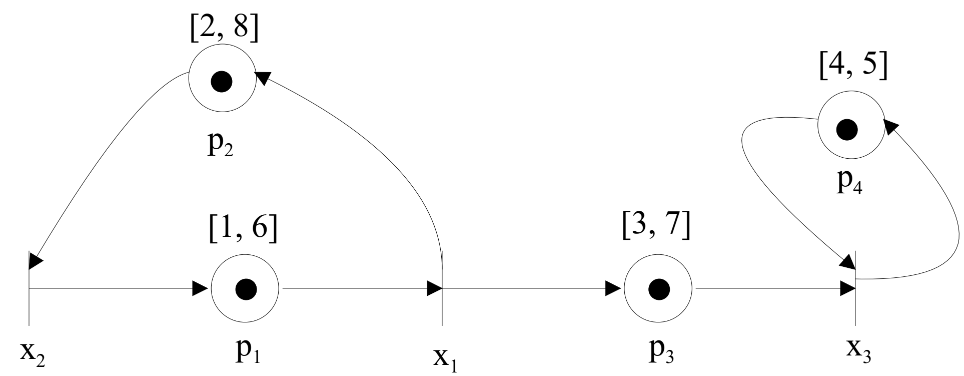
$$\bigoplus_{j \in s_i} (x_j(k - m_j) + a_j) \leq x_i(k) \leq \bigwedge_{j \in s_i} (x_j(k - m_j) + b_j)$$

- Second Model
  - State inequality:

$$\mathcal{X} \geq (\gamma^1 \mathcal{A}^- \oplus \gamma^{-1} \mathcal{A}^+)\mathcal{X}$$

Example

- p-time event graphs



- Modelling

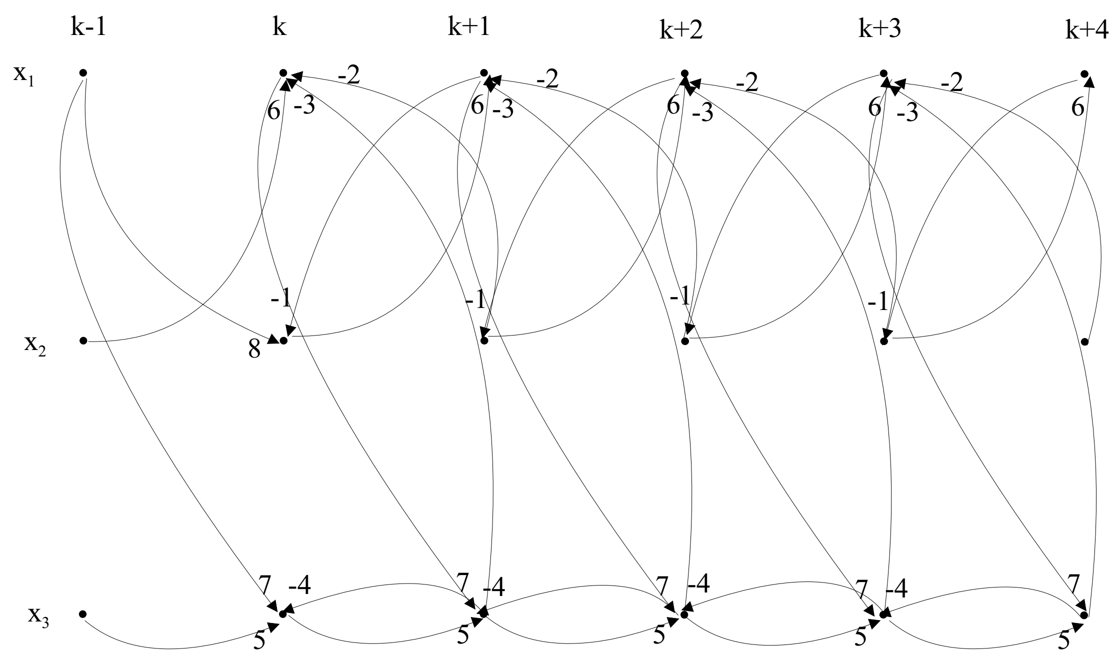
$$\mathcal{A}^- = \begin{pmatrix} \varepsilon & 1 & \varepsilon \\ 2 & \varepsilon & \varepsilon \\ 3 & \varepsilon & 4 \end{pmatrix} \mathcal{A}^+ = \begin{pmatrix} \varepsilon & -8 & -7 \\ -6 & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & -5 \end{pmatrix}$$

Existence of a state trajectory

- **Definition**

  An acceptable functioning of a system can be defined by any functioning which guarantees the liveness of tokens and which does not lead to any deadlock situation.

- Dynamic induced graph $G_h(\mathcal{A}^-, \mathcal{A}^+)$



A complex interconnexion of Backward/Forword links

Existence of a state trajectory

- The matrix $(w_h)^*_{ij}$ represents the maximum of all the paths from vertices j_h to vertices i_h in the dynamic induced graph $G_h(\mathcal{A}^-, \mathcal{A}^+)$ developed on the horizon h. Each element of the diagonal $(w_h)^*_{ii}$ represents the maximum between the greatest circuit and zero.

- **Serie of matrices**

  $w_0 = Id, w_1 = \mathcal{A}^- \otimes \mathcal{A}^+$

  $w_2 = \mathcal{A}^- \otimes (w_1)^* \otimes \mathcal{A}^+$

  $w_3 = \mathcal{A}^- \otimes (w_2)^* \otimes \mathcal{A}^+$

  $\ldots$

Existence of a state trajectory

- **Property**
  Given $w_k = \mathcal{A}^- \otimes (w_{k-1})^* \otimes \mathcal{A}^+$ with $w_1 = \mathcal{A}^- \otimes \mathcal{A}^+$, a necessary condition of existence in $\mathbb{R}$ of a state trajectory on an infinite horizon is that the matrices w_k have only negative or null circuits.

- **Property**
  The serie $w_0 = \varepsilon$ and $w_k = \mathcal{A}^- \otimes (w_{k-1})^* \otimes \mathcal{A}^+$ for $k \geq 1$ is nondecreasing.

Example (continuation)

- Calculation of the serie of matrices

$$w_1 = \begin{pmatrix} -5 & \varepsilon & \varepsilon \\ \varepsilon & -6 & -5 \\ \varepsilon & -5 & -1 \end{pmatrix} ; w_2 = \begin{pmatrix} -5 & \varepsilon & -9 \\ \varepsilon & -6 & -5 \\ -7 & -5 & -1 \end{pmatrix} ;$$

$$w_3 = \begin{pmatrix} -5 & -19 & -9 \\ -18 & -6 & -5 \\ -7 & -5 & -1 \end{pmatrix} \text{ and } w_4 = w_3$$

$\leadsto$ $w_4 = w_k$ for $k \geq 4$

- Existence of an arbitrary trajectory without tokens death on an infinite horizon

  $\leadsto$ The p-time event graph is live

Other example

- The following model is not live

$$A_0^- = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & 1 & \varepsilon \end{pmatrix}, A_0^+ = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon \\ \varepsilon & \varepsilon & -10 \\ \varepsilon & \varepsilon & \varepsilon \end{pmatrix},$$

$$A_1^- = \begin{pmatrix} \varepsilon & \varepsilon & 4 \\ \varepsilon & 1 & \varepsilon \\ 1 & \varepsilon & \varepsilon \end{pmatrix} \text{ and } A_1^+ = \begin{pmatrix} \varepsilon & \varepsilon & -3 \\ \varepsilon & -2.499 & \varepsilon \\ -4 & \varepsilon & \varepsilon \end{pmatrix}$$

$\leadsto$ $w_{9003} = w_k$ for $k \geq 9004$

- We replace -2.499 by -2.5
  - the system is live
  - the transient is reduced

  $\leadsto$ $w_3 = w_4$

Extremal acceptable trajectories

- **Initial condition**

$$\mathcal{X}(0) \in [\mathcal{X}_0^-, \mathcal{X}_0^+]$$

- **Problem**: Is there an acceptable trajectory starting from the interval $[\mathcal{X}_0^-, \mathcal{X}_0^+]$?

- **Assumption on horizon of calculation**

for $k = 0 \rightsquigarrow \mathcal{X}(0) \geq \mathcal{A}^+ \otimes \mathcal{X}(1) \oplus \mathcal{X}_0^-$

for $k = h \rightsquigarrow \mathcal{X}(h) \geq \mathcal{A}^- \otimes \mathcal{X}(h-1)$

Lowest state trajectory

- **Theorem**

  ▶ If the process operates on the horizon h and if the matrices w_k defined below have no positive circuit, the lowest state trajectory checking $\mathcal{X}(0) \geq \mathcal{X}_0^-$ is given by the following forward/backward algorithm.

- **Coefficients by forward iteration**

  Initialization: $w_0 = \varepsilon$ and $\beta_0^- = \mathcal{X}_0^-$

  for $k = 1$ to h, $\quad w_k = \mathcal{A}^- \otimes (w_{k-1})^* \otimes A^+$
  and $\beta_k^- = \mathcal{A}^- \otimes (w_{k-1})^* \otimes \beta_{k-1}^-$

- **Trajectory $\mathcal{X}^-$ by backward iteration**

  $\mathcal{X}^-(h) = (w_h)^* \otimes \beta_h^-$
  for $k = h - 1$ to 0, $\mathcal{X}^-(k) = (w_k)^* \otimes [\mathcal{A}^+ \otimes \mathcal{X}(k+1) \oplus \beta_k^-]$

Greatest state trajectory

- **Theorem**

  ▶ If the process operates on the horizon h and if the matrices w_k defined below have no positive circuit, the greatest state trajectory checking $\mathcal{X}(0) \leq \mathcal{X}_0^+$ is given by the following forward/backward algorithm.

- **Coefficients by forward iteration**

  Initialization: $w_0 = \varepsilon$ and $\beta_0^+ = \mathcal{X}_0^+$

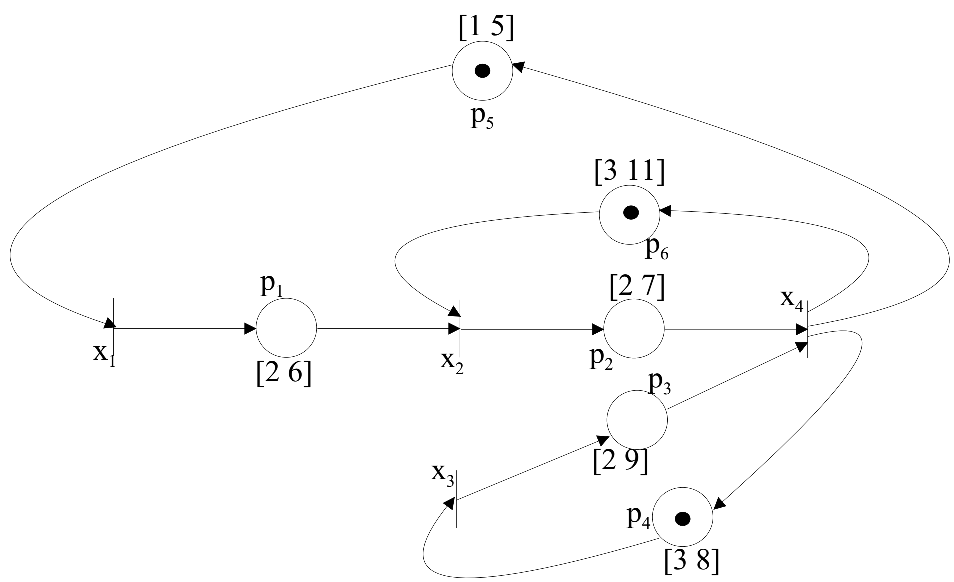
  for $k = 1$ to h, $\quad w_k = \mathcal{A}^- \otimes (w_{k-1})^* \otimes A^+$
  and $\beta_k^+ = ((w_{k-1})^* \otimes \mathcal{A}^+) \backslash \beta_{k-1}^+$

- **Trajectory $\mathcal{X}^+$ by backward iteration**

  $\mathcal{X}^+(h) = (w_h)^* \backslash \beta_h^+$
  for $k = h - 1$ to 0, $\mathcal{X}^+(k) = (w_k)^* \backslash [\mathcal{A}^- \backslash \mathcal{X}(k+1) \wedge \beta_k^+]$

Example

- P-time event graphs



- $\mathcal{A}^- = \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon & 1 \\ \varepsilon & \varepsilon & \varepsilon & 3 \\ \varepsilon & \varepsilon & \varepsilon & 3 \\ \varepsilon & \varepsilon & \varepsilon & 5 \end{pmatrix} \mathcal{A}^+ = \begin{pmatrix} -18 & -24 & -21 & \varepsilon \\ -12 & -18 & -15 & \varepsilon \\ -14 & -20 & -17 & \varepsilon \\ -5 & -11 & -8 & \varepsilon \end{pmatrix}$

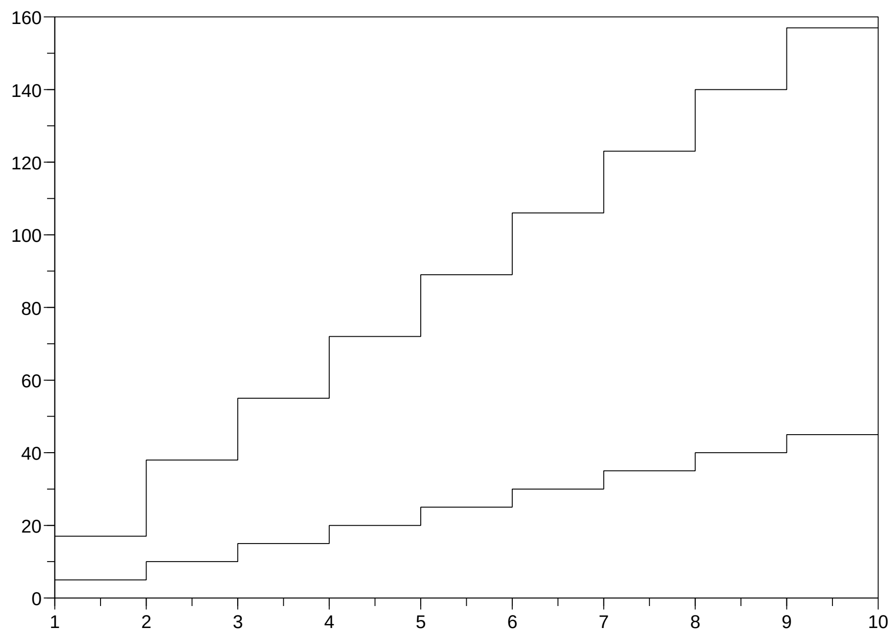
Example (continuation)

- Calculation of the matrices w_k

  $\rightsquigarrow$ $w_k = w_2$ for $w \geq 2$

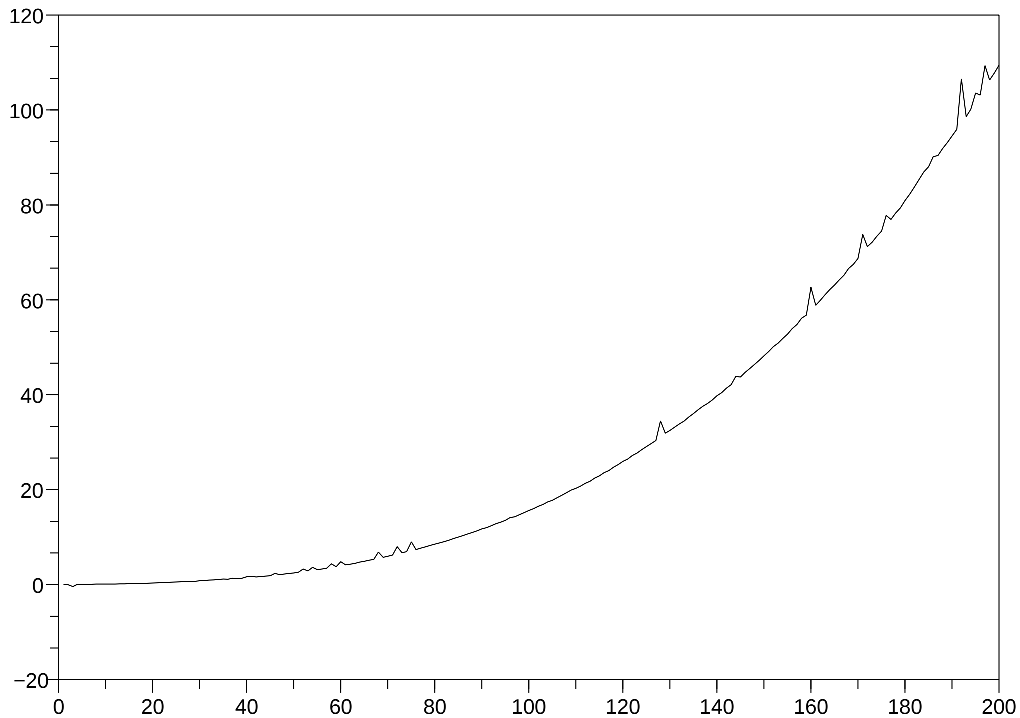
- Initial conditions:
  - ▶ $\mathcal{X}^-(0) = (1,\ 3,\ 3,\ 5)^t$
  - ▶ $\mathcal{X}^+(0) = (5,\ 11,\ 8,\ 17)^t$

- Extremal acceptable trajectories



Computational complexity

- $h = 1000$ and $n = 3$ to 200



- CPU time: $O(h.n^2)$

- space memory: $(h \wedge \text{transient}).n^2$

Conclusion

- Algebraic modelling of p-time event graphs
  - Two formulations
- Extremal acceptable trajectories in P-time event graphs
  - Liveness analysis: series of matrices
  - Calculation of the greatest and lowest state trajectory
  - Initial conditions
- Perspective: fast (min, max, +) algorithm $x \leq f(x)$