A guaranteed numerical method to classify smooth mappings from $\mathbb{R}^2$ to $\mathbb{R}^2$

Nicolas Delanoue - Sébastien Lagrange
LARIS - Université d'Angers - France

SMART 2014
1st Small Symposium on Set-Membership: Applications, Reliability and Theory
The University of Manchester, Aerospace Research Institute, Manchester, UK
http://www.umari.manchester.ac.uk/aboutus/events/

Outline

Objects

The set of square matrices of order n (denoted by $\mathcal{M}_n$)

Objects

The set of square matrices of order n (denoted by $\mathcal{M}_n$)

Equivalence

We can defined an equivalence relation $\sim$ between elements of $\mathcal{M}_n$ with

$$A \sim B \Leftrightarrow \exists P \in GL, A = PBP^{-1}$$

Objects

The set of square matrices of order n (denoted by $\mathcal{M}_n$)

Equivalence

We can defined an equivalence relation $\sim$ between elements of $\mathcal{M}_n$ with

$$A \sim B \Leftrightarrow \exists P \in GL, A = PBP^{-1}$$

Invariant

The set of eigenvalues is an invariant because

$$A \sim B \Rightarrow sp(A) = sp(B)$$

Invariant

The set of eigenvalues is *not a strong enough* invariant since

$$\exists A, B \in \mathcal{M}_n, A \not\sim B \text{ and } sp(A) = sp(B)$$

Invariant

The set of eigenvalues is *not a strong enough* invariant since

$$\exists A, B \in \mathcal{M}_n, A \not\sim B \text{ and } sp(A) = sp(B)$$

Example

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Invariant

The set of eigenvalues is *not a strong enough* invariant since

$$\exists A, B \in \mathcal{M}_n, A \not\sim B \text{ and } sp(A) = sp(B)$$

Example

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

A really strong invariant

Let us call by J the Jordan method, we have

$$A \sim B \Leftrightarrow J(A) = J(B)$$

Objects	*Equivalence*	*Invariants*
Square matrices	$A \sim B \Leftrightarrow \exists P \in GL, A = PBP^{-1}$	Eigenvalues,

Objects	*Equivalence*	*Invariants*
Square matrices	$A \sim B \Leftrightarrow \exists P \in GL, A = PBP^{-1}$	Eigenvalues, Jordanisation

Objects	*Equivalence*	*Invariants*
Square matrices	$A \sim B \Leftrightarrow \exists P \in GL, A = PBP^{-1}$	Eigenvalues, Jordanisation
Real bilinear forms	$A \sim B \Leftrightarrow \exists U, V \in SO, A = UBV$	Singularvalues

Objects	*Equivalence*	*Invariants*
Square matrices	$A \sim B \Leftrightarrow \exists P \in GL, A = PBP^{-1}$	Eigenvalues, Jordanisation
Real bilinear forms	$A \sim B \Leftrightarrow \exists U, V \in SO, A = UBV$	Singularvalues
Smooth maps	$f \sim f'$ if there exist diffeomorphic changes of variables (g, h) on X and Y such that $f = g \circ f' \circ h$	?

Global picture

One wants a global "picture" of a given map which does not depend on a choice of system of coordinates neither on the source set nor on the target set.

Global picture

One wants a global "picture" of a given map which does not depend on a choice of system of coordinates neither on the source set nor on the target set.

Definition - Equivalence

Let f and f' be two smooth maps. Then $f \sim f'$ if there exists diffeomorphisms $g : X \to X'$ and $h : Y' \to Y$ such that the diagram

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \downarrow g & & \uparrow h \\ X' & \xrightarrow{f'} & Y' \end{array}$$

commutes.

Example

$$\begin{array}{ccc} \mathbb{R} & \xrightarrow{2x+6} & \mathbb{R} \\ \downarrow g & & \uparrow h \\ \mathbb{R} & \xrightarrow{x+1} & \mathbb{R} \end{array}$$

Example

$$\begin{array}{ccc} \mathbb{R} & \xrightarrow{2x+6} & \mathbb{R} \\ \downarrow{\scriptstyle x+2} & & \uparrow{\scriptstyle 2y} \\ \mathbb{R} & \xrightarrow{x+1} & \mathbb{R} \end{array}$$

Examples

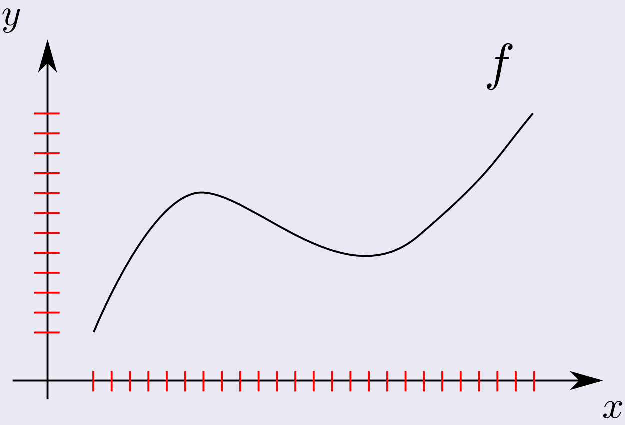
1. $f_1(x) = x^2,\ f_2(x) = ax^2 + bx + c,\ a \neq 0$

$$f_1 \sim f_2$$

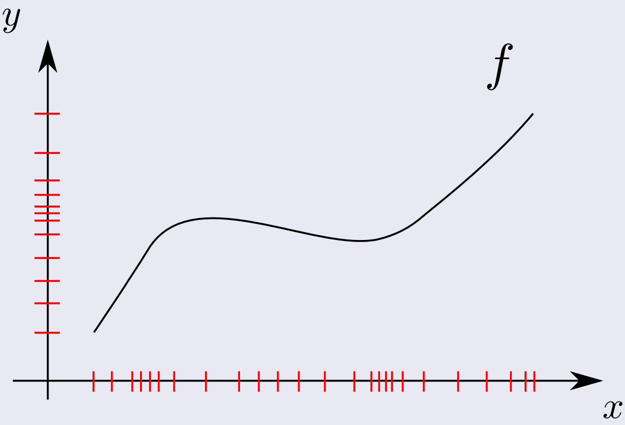
2. $f_1(x) = x^2 + 1,\ f_2(x) = x + 1,$

$$f_1 \not\sim f_2$$

Examples



Examples



Proposition

Suppose that $f \sim f'$ with

$$\begin{array}{ccc} X_1 & \xrightarrow{f} & Y_1 \\ \Big\downarrow g & & \Big\uparrow h \\ X_2 & \xrightarrow{f'} & Y_2 \end{array}$$

then $f^{-1}(\{y_1\})$ is homeomorphic to $f'^{-1}(\{y_2\})$.

Proposition

Suppose that $f \sim f'$ with

$$\begin{array}{ccc} x_1 & \xrightarrow{f} & y_1 \\ \downarrow g & & \uparrow h \\ x_2 & \xrightarrow{f'} & y_2 \end{array}$$

then $\operatorname{rank} df_{x_1} = \operatorname{rank} df'_{x_2}$.

Proof

Chain rule, $df = dh \cdot df' \cdot dg$

Definition

Let us defined by S_f the set of critical points of f :

$$S_f = \{x \in X \mid df(x) \text{ is singular } \}.$$

Corollary

$$f \sim f' \Rightarrow S_f \simeq S_{f'}$$

where $\simeq$ means homeomorphic.
i.e. the topology of the critical points set is an invariant.

This is not a strong enough invariant,
there exists smooth maps $f, f' : [0,1] \to [0,1]$ such that

$$S_f \simeq S_{f'} \text{ and } f \not\sim f'.$$

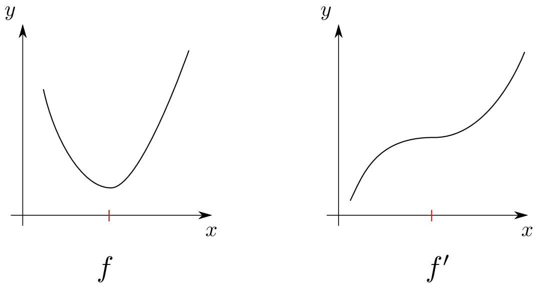


FIGURE : Singularity theory.

This is not a strong enough invariant,
there exists smooth maps $f, f' : [0,1] \to [0,1]$ such that

$$S_f \simeq S_{f'} \text{ and } f \not\sim f'.$$

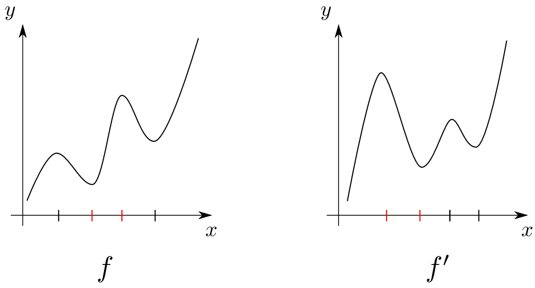
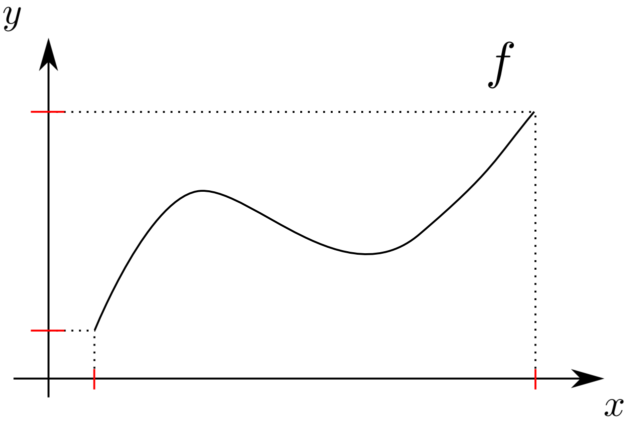
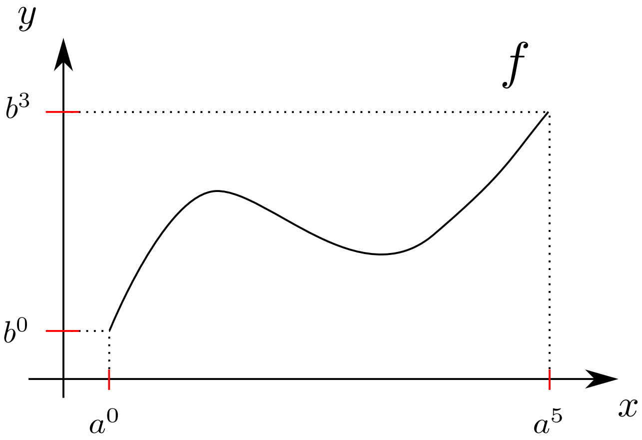
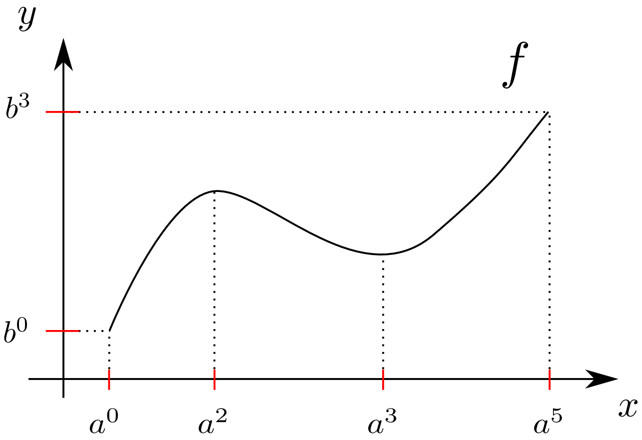
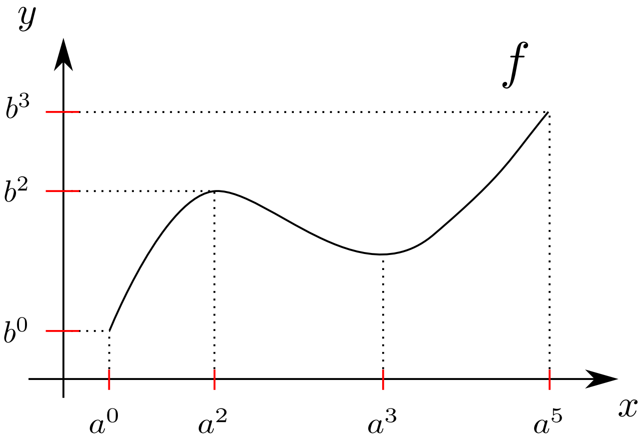


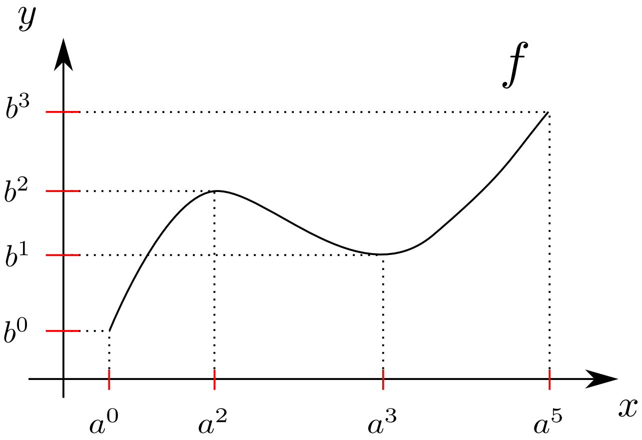
FIGURE : Topology of X.

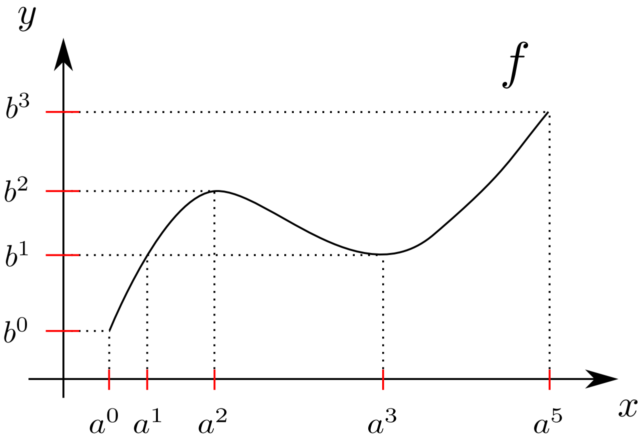

y
f
x

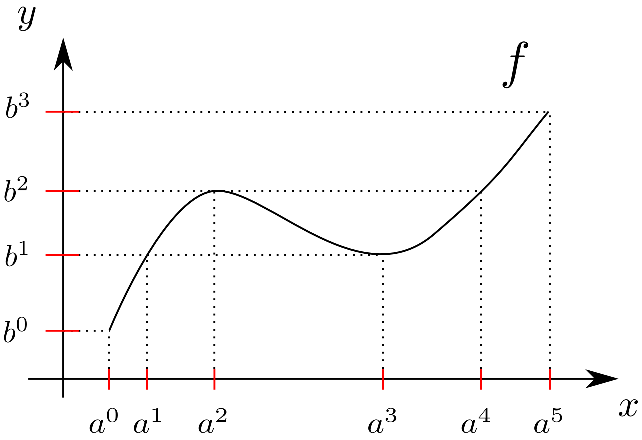

y
f
b^3
b^0
a^0
a^5
x

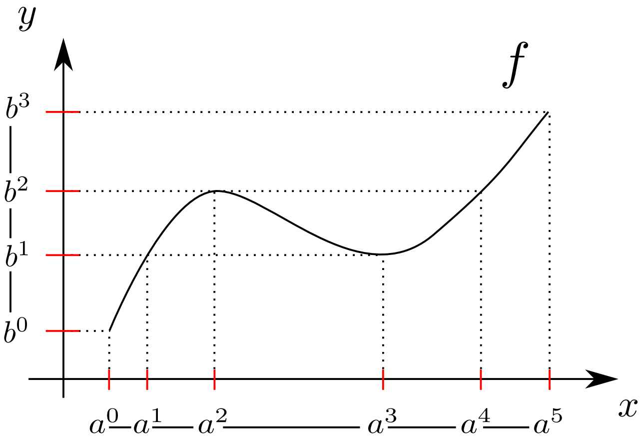

y
f
b^3
b^0
a^0
a^2
a^3
a^5
x


y
f
b^3
b^2
b^0
a^0
a^2
a^3
a^5
x


y
f
b^3
b^2
b^1
b^0
a^0
a^2
a^3
a^5
x


y
f
b^3
b^2
b^1
b^0
a^0
a^1
a^2
a^3
a^5
x


y
f
b^3
b^2
b^1
b^0
a^0
a^1
a^2
a^3
a^4
a^5
x

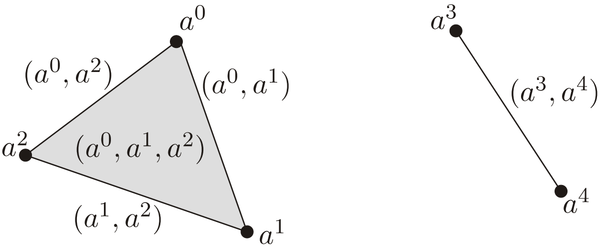

y
f
b^3
b^2
b^1
b^0
x
a^0 a^1 a^2 a^3 a^4 a^5

Definition - Abstract simplicial complex

Let $\mathcal{N}$ be a finite set of symbols $\{(a^0), (a^1), \ldots, (a^n)\}$
An abstract simplicial complex $\mathcal{K}$ is a subset of the powerset of $\mathcal{N}$ satisfying : $\sigma \in \mathcal{K} \Rightarrow \forall \sigma_0 \subset \sigma, \sigma_0 \in \mathcal{K}$

$$\mathcal{K} = \{(a^0),(a^1),(a^2),(a^3),(a^4),$$
$$(a^0,a^1),(a^1,a^2),(a^0,a^2),(a^3,a^4),$$
$$(a^0,a^1,a^2)\}$$

This will be denoted by $a^0a^1a^2 + a^3a^4$



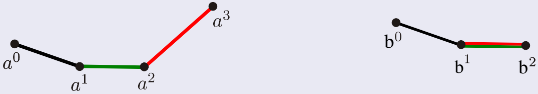
Definition

Given abstract simplicial complexes $\mathcal{K}$ and $\mathcal{L}$, a *simplicial map* $F : \mathcal{K}^0 \to \mathcal{L}^0$ is a map with the following property :

$$(a^0, a^1, \dots, a^n) \in \mathcal{K} \Rightarrow (F(a^0), F(a^1), \dots, F(a^n)) \in \mathcal{L}.$$

Example - Simplicial map

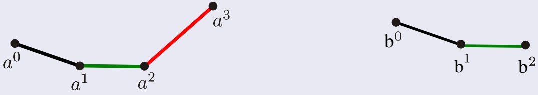
$$\mathcal{K} = a_0 a_1 + a_1 a_2 + a_2 a_3, \quad \mathcal{L} = b_0 b_1 + b_1 b_2$$



$$\begin{array}{lllll} F & : & a^0 & \mapsto & b^0 \\ & & a^1 & \mapsto & b^1 \\ & & a^2 & \mapsto & b^2 \\ & & a^3 & \mapsto & b^1 \end{array}$$

Example - NOT a Simplicial map

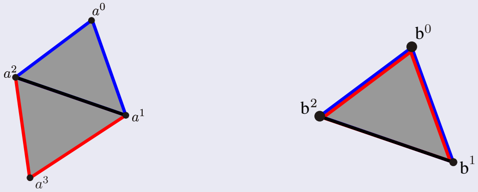
$$\mathcal{K} = a_0 a_1 + a_1 a_2 + a_2 a_3, \quad \mathcal{L} = b_0 b_1 + b_1 b_2$$



$$F \; : \; \begin{array}{ccc} a^0 & \mapsto & b^0 \\ a^1 & \mapsto & b^1 \\ a^2 & \mapsto & b^2 \\ a^3 & \mapsto & b^0 \end{array}$$

Example - Simplicial map

$$\mathcal{K} = a_0 a_1 a_2 + a_1 a_2 a_3, \quad \mathcal{L} = b_0 b_1 b_2$$



$$\begin{array}{cclcl} F & : & a^0 & \mapsto & b^0 \\ & & a^1 & \mapsto & b^1 \\ & & a^2 & \mapsto & b^2 \\ & & a^3 & \mapsto & b^0 \end{array}$$

Definition

Let f and f' be continous maps. Then f and f' are *topologically conjugate* if there exists homeomorphism $g : X \to X'$ and $h : Y \to Y'$ such that the diagram

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, "g"] & Y \\ X' \arrow[r, "f'"] & Y' \arrow[u, "h"] \end{tikzcd}$$

commutes.

Proposition

$$f \sim f' \Rightarrow f \sim_0 f'$$

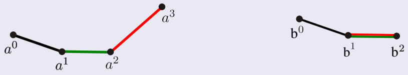
Definition

Let f be a smooth map and F a simplicial map, F is a *portrait* of f if

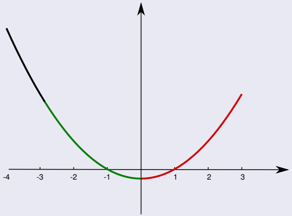
$$f \sim_0 F$$

Example - Simplicial map

The simplicial map



is a portrait of $[-4, 3] \ni x \mapsto x^2 - 1 \in \mathbb{R}$



Proposition

For every closed subset A of $\mathbb{R}^n$, there exists a smooth real valued function f such that

$$A = f^{-1}(\{0\})$$

Proposition

For every closed subset A of $\mathbb{R}^n$, there exists a smooth real valued function f such that

$$A = f^{-1}(\{0\})$$

We are not going to consider all cases ...

Definition

Let f be a smooth map, f is *stable* if their exists a nbrd N_f such that

$$\forall f' \in N_f, f' \sim f$$

Examples

1. $g : x \mapsto x^2$ is stable,
2. $f_0 : x \mapsto x^3$ is not stable, since with $f_\epsilon : x \mapsto x(x^2 - \epsilon)$,

$$\epsilon \neq 0 \Rightarrow f_\epsilon \not\sim f_0.$$

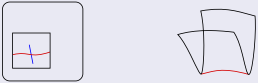
Whitney Theorem - 1955

Let X and Y be 2-dimentional manifolds and f be generic. The critical point set S_f is a regular curve. With $p \in S_f$, one has

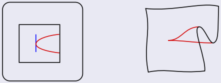
$$T_pS_f \oplus \ker df_p = T_pX \text{ or } T_pS_f = \ker df_p$$

Geometric representation

1. if $T_pS_f \oplus \ker df_p = T_pX$,

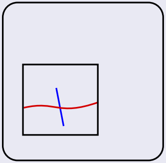


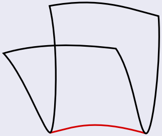
2. if $T_pS_f = \ker df_p$,



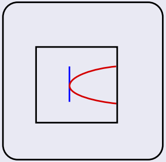
Geometric representation

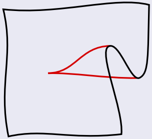
1. $T_pS_f \oplus \ker df_p = T_pX$: fold point





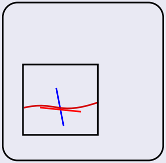
2. $T_pS_p = \ker df_p$: cusp point

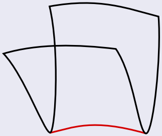




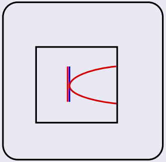
Geometric representation

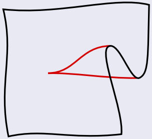
1. $T_pS_f \oplus \ker df_p = T_pX$: fold point





2. $T_pS_p = \ker df_p$: cusp point





Normal forms

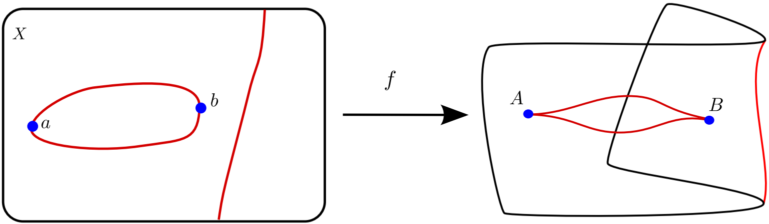
1. If $T_pS_f \oplus \ker df_p = T_pX$, then there exists a nbrd N_p such that
$$f|N_p \sim (x,y) \mapsto (x,y^2).$$
2. If $T_pS_f = \ker df_p$, then there exists a nbrd N_p such that
$$f|N_p \sim (x,y) \mapsto (x,xy+y^3).$$

Definition

Let f a smooth map from $X \to \mathbb{R}^2$ with X a simply connected compact subset of $\mathbb{R}^2$ with smooth boundary ∂X. The *apparent contour* of f is

$$f(S_f \cup \partial X)$$

The topology of the Apparent contour is an invariant.

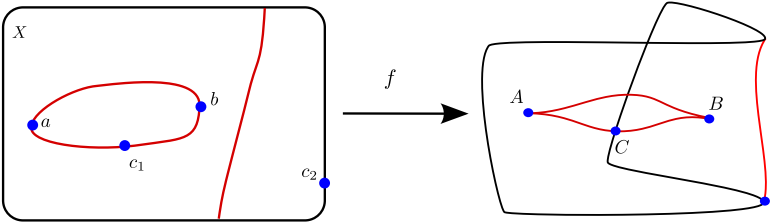


Definition

Let f a smooth map from $X \to \mathbb{R}^2$ with X a simply connected compact subset of $\mathbb{R}^2$ with smooth boundary ∂X. The *apparent contour* of f is

$$f(S_f \cup \partial X)$$

The topology of the Apparent contour is an invariant.

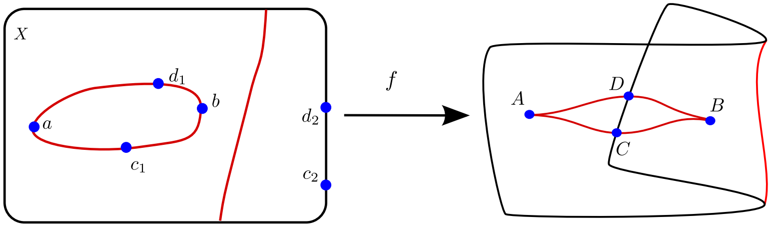


Definition

Let f a smooth map from $X \to \mathbb{R}^2$ with X a simply connected compact subset of $\mathbb{R}^2$ with smooth boundary ∂X. The *apparent contour* of f is

$$f(S_f \cup \partial X)$$

The topology of the Apparent contour is an invariant.

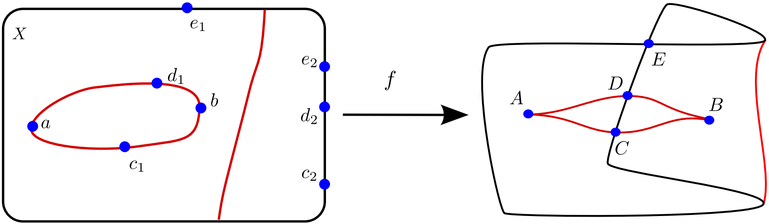


Definition

Let f a smooth map from $X \to \mathbb{R}^2$ with X a simply connected compact subset of $\mathbb{R}^2$ with smooth boundary ∂X. The *apparent contour* of f is

$$f(S_f \cup \partial X)$$

The topology of the Apparent contour is an invariant.

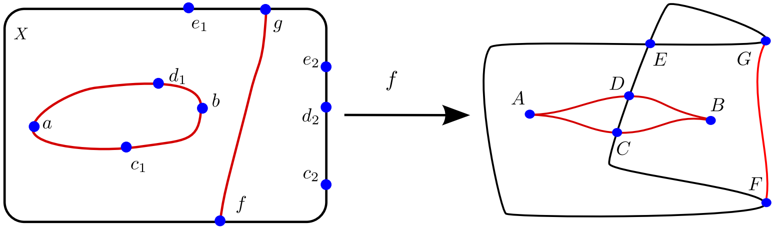


Definition

Let f a smooth map from $X \to \mathbb{R}^2$ with X a simply connected compact subset of $\mathbb{R}^2$ with smooth boundary ∂X. The *apparent contour* of f is

$$f(S_f \cup \partial X)$$

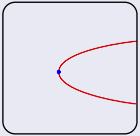
The topology of the Apparent contour is an invariant.

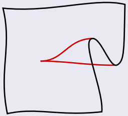


Theorem (Global properties of generic maps)

Let X be a compact simply connected domain of $\mathbb{R}^2$ with $\partial X = \Gamma^{-1}(\{0\})$. A generic smooth map f from X to $\mathbb{R}^2$ has the following properties :

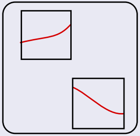
1. *S_f is regular curve. Moreover, elements of S are folds and cusp. The set of cusp is discrete.*

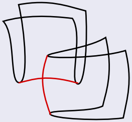




Theorem

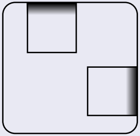
2. *3 singular points do not have the same image,*
3. *2 singular points having the same image are folds points and they have normal crossing.*

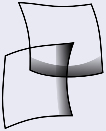




Theorem

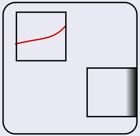
4. *3 boundary points do not have the same image,*
5. *2 boundary points having the same image cross normally.*

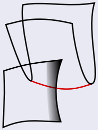




Theorem

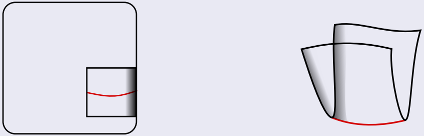
6. *3 different points belonging to $S_f \cup \partial X$ do not have the same image,*
7. *If a point on the singularity curve and a boundary have the same image, the singular point is a fold and they have normal crossing.*

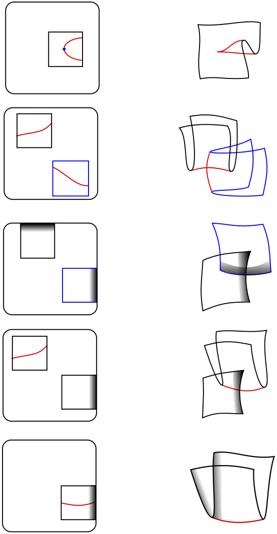


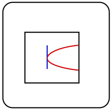


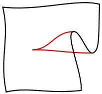
Theorem

8. *if the singularity curve intersects the boundary, then this point is a fold,*
9. *moreover tangents to the singularity curve and boundary curve are different.*









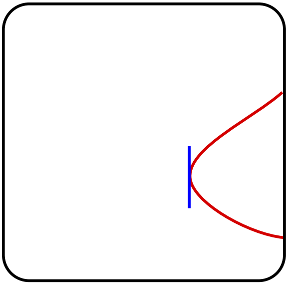
Proposition

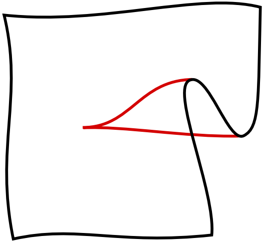
Let f be a smooth generic map from X to $\mathbb{R}^2$, let us denote by c the map defined by :

$$\begin{array}{cccc} c & : & X & \rightarrow & \mathbb{R}^2 \\ & & p & \mapsto & df_p \xi_p \end{array} \qquad (1)$$

where ξ is the vector field defined by $\xi_p = \begin{pmatrix} \partial_2 \det df_p \\ -\partial_1 \det df_p \end{pmatrix}$.

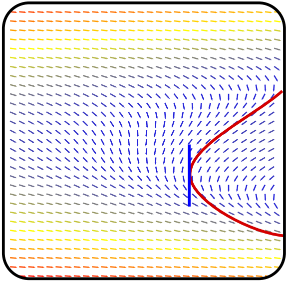
If $c(p) = 0$ and dc_p is invertible then p is a simple cusp. This sufficient condition is locally necessary.

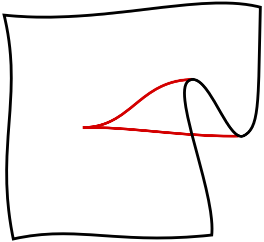




Interval Newton method

$$\begin{array}{cccccc} c & : & X & \to & \mathbb{R}^2 \\ & & p & \mapsto & df_p\xi_p \end{array} \tag{2}$$

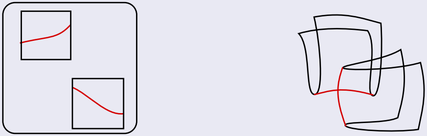




Interval Newton method

$$\begin{array}{rcccc} c & : & X & \rightarrow & \mathbb{R}^2 \\ & & p & \mapsto & df_p \xi_p \end{array} \quad (3)$$

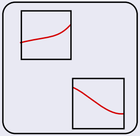
2 different folds

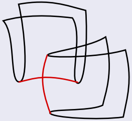


$$S^{\Delta 2} = \{(x_1, x_2) \in S \times S - \Delta(S) \mid f(x_1) = f(x_2)\}/\simeq$$

where $\simeq$ is the relation defined by
$(x_1, x_2) \simeq (x_1', x_2') \Leftrightarrow (x_1, x_2) = (x_2', x_1')$.

2 different folds





$$S^{\Delta 2} = \{(x_1, x_2) \in X \times X - \Delta(X) \mid \begin{cases} \det df_{x_1} = 0 \\ \det df_{x_2} = 0 \\ f_1(x_1) = f_1(x_2) \\ f_2(x_1) = f_2(x_2) \end{cases} \}/\simeq$$

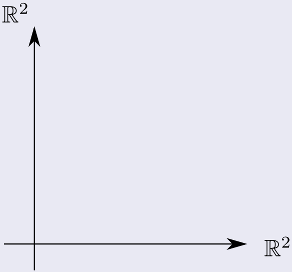
where $\simeq$ is the relation defined by
$(x_1, x_2) \simeq (x_1', x_2') \Leftrightarrow (x_1, x_2) = (x_2', x_1')$.

Method

Bisection scheme on $X \times X$.

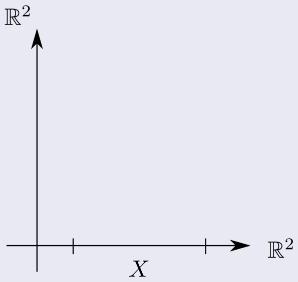
Method

Bisection scheme on $X \times X$.



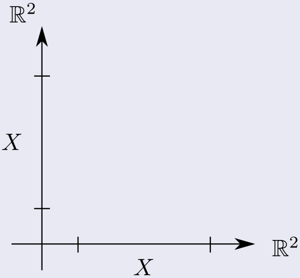
Method

Bisection scheme on $X \times X$.



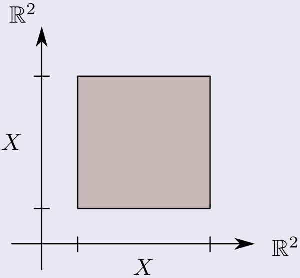
Method

Bisection scheme on $X \times X$.



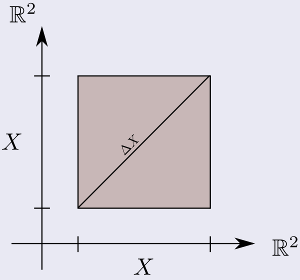
Method

Bisection scheme on $X \times X$.



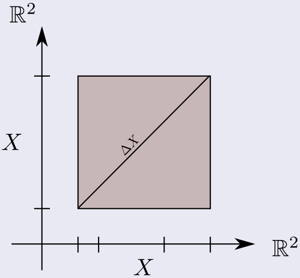
Method

Bisection scheme on $X \times X$.



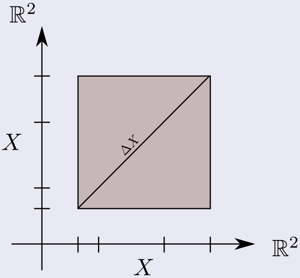
Method

Bisection scheme on $X \times X$.



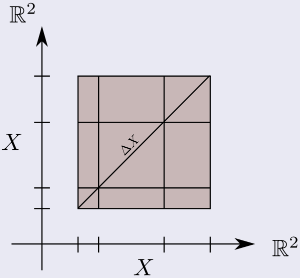
Method

Bisection scheme on $X \times X$.



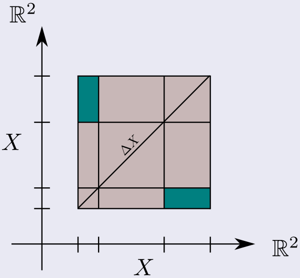
Method

Bisection scheme on $X \times X$.



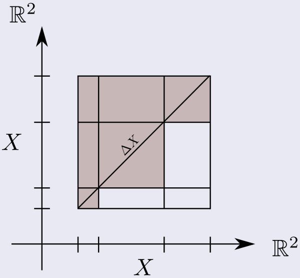
Method

Bisection scheme on $X \times X$.



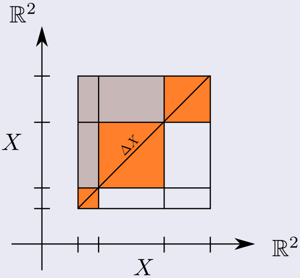
Method

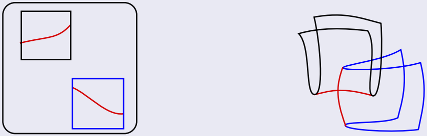
Bisection scheme on $X \times X$.



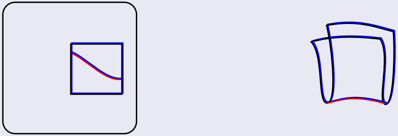
Method

Bisection scheme on $X \times X$.





$$[x_1] \neq [x_2]$$



$$[x_1] = [x_2]$$

Let us define the map *folds* by

$$
\begin{array}{cccc}
folds : & X \times X & \rightarrow & \mathbb{R}^4 \\
& \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} & \mapsto & \begin{pmatrix} \det df(x_1, y_1) \\ \det df(x_2, y_2) \\ f_1(x_1, y_1) - f_1(x_2, y_2) \\ f_2(x_1, y_1) - f_2(x_2, y_2) \end{pmatrix}
\end{array}
$$

One has

$$S^{\Delta 2} = folds^{-1}(\{0\}) - \Delta S / \simeq .$$

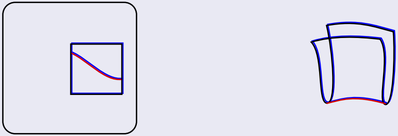
For any (α, α) in ΔS, the d *folds* is conjugate to

$$\begin{pmatrix} a & b & 0 & 0 \\ 0 & 0 & a & b \\ a_{11} & a_{12} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{21} & a_{22} \end{pmatrix}$$

which is not invertible since $\det \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \det df(\alpha) = 0$. In other words, as any box of the form $[x_1] \times [x_1]$ contains ΔS, the interval Newton method will fail.

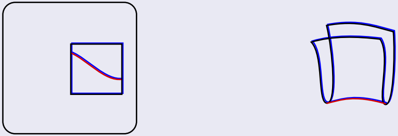
One needs a method to prove that $f|S \cap [x_1]$ is an embedding.

One needs a method to prove that $f|S \cap [x_1]$ is an embedding.



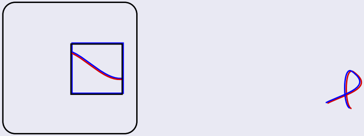
$$[x_1] = [x_2]$$

One needs a method to prove that $f|S \cap [x_1]$ is an embedding.



$$[x_1] = [x_2]$$

Not in this case ...



Corollary

Let $f : X \to \mathbb{R}^2$ be a smooth map and X a compact subset of $\mathbb{R}^2$. Let $\Gamma : X \to \mathbb{R}$ be a submersion such that the curve $S = \{x \in X \mid \Gamma(x) = 0\}$ is contractible. If

$$\forall J \in \tilde{d}f(X) \cdot \begin{pmatrix} \partial_2 \Gamma(X) \\ -\partial_1 \Gamma(X) \end{pmatrix}, \text{rank } J = 1$$

then $f|S$ is an embedding.

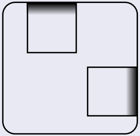
The last condition is not satisfiable if $[x_1]$ contains a cusp . . .

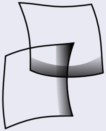
Proposition

Suppose that there exists a unique simple cusp p_0 in the interior of X. Let $\alpha \in \mathbb{R}^{2*}$, s.t. $\alpha \cdot \mathrm{Im}\, df_{p_0} = 0$, and ξ a non vanashing vector field such that $\forall p \in S, \xi_p \in T_pS$ (S contractible).
If $g = \sum \alpha_i \xi^3 f_i : X \to \mathbb{R}$ is a nonvanishing function then $f|S$ is injective. This condition is locally necessary.

Here the vector field ξ is seen as the derivation of $\mathcal{C}^\infty(X)$ defined by

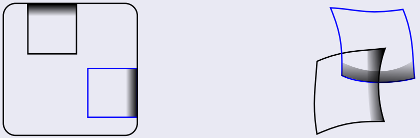
$$\xi = \sum \xi_i \frac{\partial}{\partial x_i}.$$



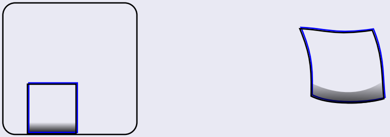


$$\partial X^{\Delta 2} = \{(x_1, x_2) \in \partial X \times \partial X - \Delta(\partial X) \mid f(x_1) = f(x_2)\} / \simeq$$

$[x_1] \neq [x_2]$



$[x_1] = [x_2]$



Let us define the map *boundaries* by

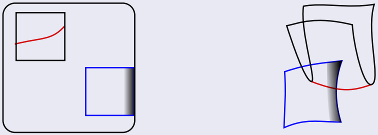
$$\begin{array}{rccc} boundaries : & X \times X & \to & \mathbb{R}^4 \\ & \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} & \mapsto & \begin{pmatrix} \Gamma(x_1, y_1) \\ \Gamma(x_2, y_2) \\ f_1(x_1, y_1) - f_1(x_2, y_2) \\ f_2(x_1, y_1) - f_2(x_2, y_2) \end{pmatrix} \end{array}$$

One has

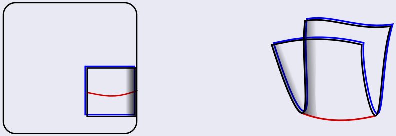
$$\partial X^{\Delta 2} = boundaries^{-1}(\{0\}) - \Delta \partial X / \simeq .$$

$$BF = \{(x_1, x_2) \in \partial X \times S \mid f(x_1) = f(x_2)\}$$

$[x_1] \neq [x_2]$



$[x_1] = [x_2]$



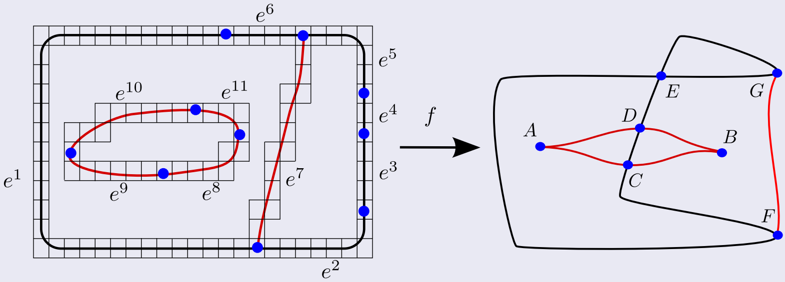
$[x_1] \neq [x_2]$

$$
\begin{array}{ccc}
X \times X & \to & \mathbb{R}^4 \\
\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} & \mapsto & \begin{pmatrix} \det df(x_1, y_1) \\ \Gamma(x_2, y_2) \\ f_1(x_1, y_1) - f_1(x_2, y_2) \\ f_2(x_1, y_1) - f_2(x_2, y_2) \end{pmatrix}
\end{array}
$$

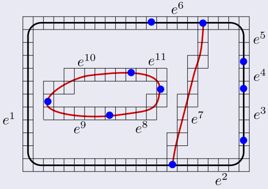
$[x_1] = [x_2]$

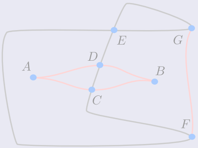
$$
\begin{array}{ccc}
X & \to & \mathbb{R}^2 \\
\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} & \mapsto & \begin{pmatrix} \det df(x_1, y_1) \\ \Gamma(x_1, y_1) \end{pmatrix}
\end{array}
$$

Analysis



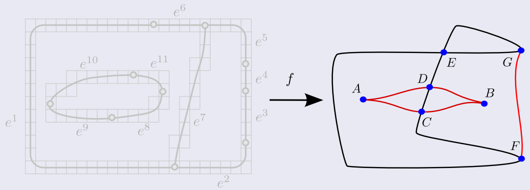
Synthesis





$$\mathcal{X} = \begin{array}{c} \\ a \\ b \\ c_1 \\ c_2 \\ d_1 \\ d_2 \\ e_1 \\ e_2 \\ f \\ g \end{array} \begin{array}{c} \begin{array}{ccccccccccc} e^1 & e^2 & e^3 & e^4 & e^5 & e^6 & e^7 & e^8 & e^9 & e^{10} & e^{11} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Synthesis



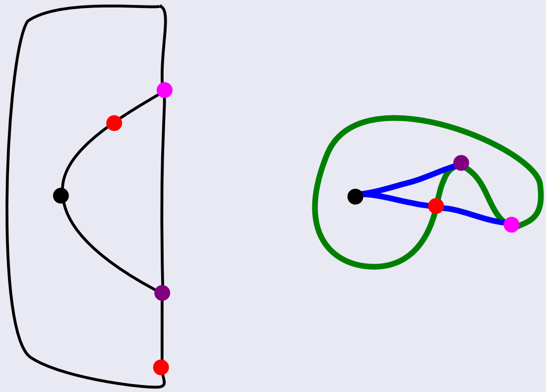
$$\mathcal{X}/f = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccccccc} e^1 & e^2 & e^3 & e^4 & e^5 & e^6 & e^7 & e^8 & e^9 & e^{10} & e^{11} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Theorem

For every portrait F of f, the 1-skeleton of ImF contains a subgraph that is an expansion of $\mathcal{X}/f$.

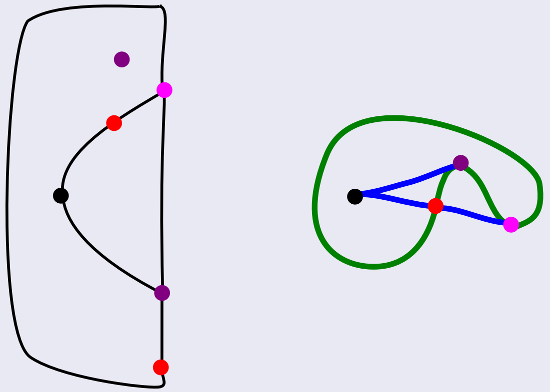
Conjecture

From $\mathcal{X}/f$ and its right embedding in $\mathbb{R}^2$ it is possible to create a portrait for f.



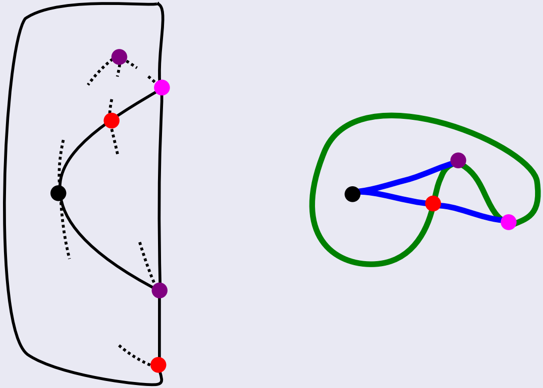
Conjecture

From $\mathcal{X}/f$ and its right embedding in $\mathbb{R}^2$ it is possible to create a portrait for f.



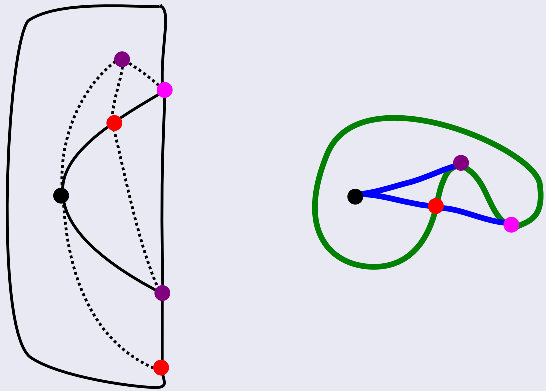
Conjecture

From $\mathcal{X}/f$ and its right embedding in $\mathbb{R}^2$ it is possible to create a portrait for f.



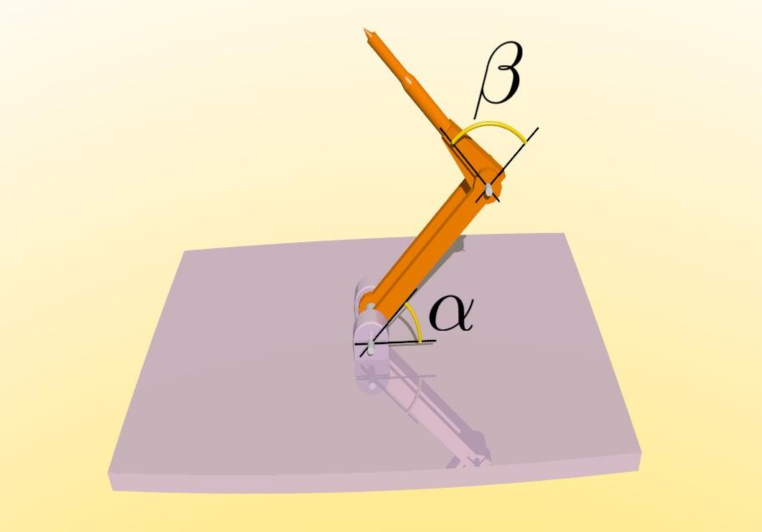
Conjecture

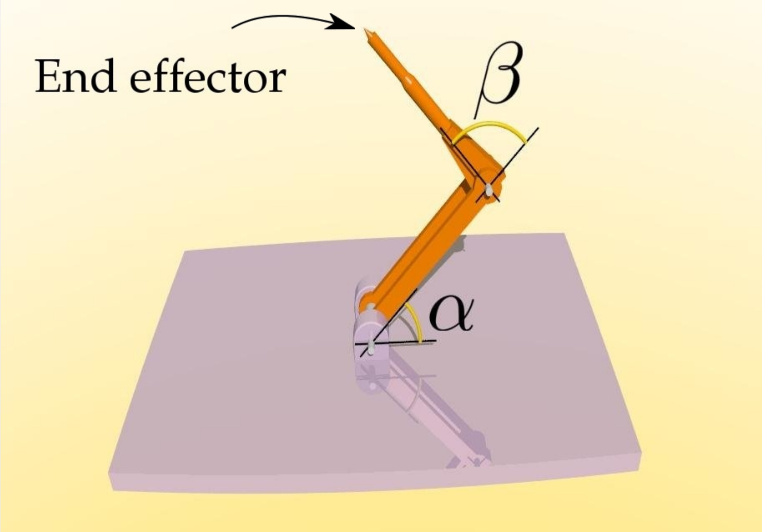
From $\mathcal{X}/f$ and its right embedding in $\mathbb{R}^2$ it is possible to create a portrait for f.



References

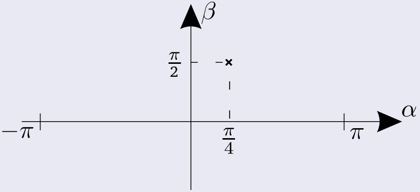
- *A numerical approach to compute the topology of the Apparent Contour of a smooth mapping from $\mathbb{R}^2$ to $\mathbb{R}^2$*,
  Nicolas Delanoue, Sébastien Lagrange,
  Journal of Computational and Applied Mathematics
- *Stable Mappings and Their Singularities*,
  M. Golubitsky, V. Guillemin,
  Springer


β
α

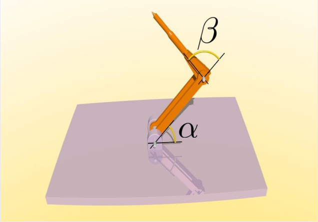

End effector
β
α

Position of the end effector depends on α and β

$$f : X \to \mathbb{R}^2$$
$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mapsto \begin{pmatrix} 2\cos(\alpha) + cos(\alpha+\beta) \\ 2\sin(\beta) + sin(\alpha+\beta) \end{pmatrix}$$



Configuration space

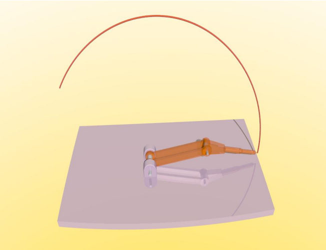


Working space

Motion planning

Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

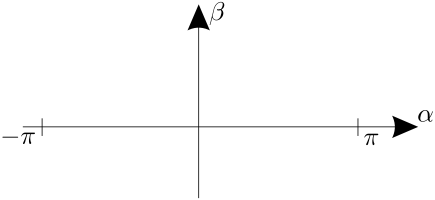
$$f \circ \gamma = \delta$$

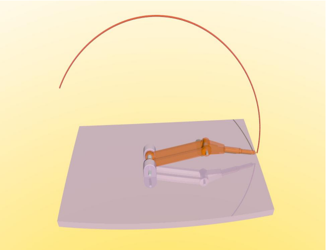


Motion planning

Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

$$f \circ \gamma = \delta$$

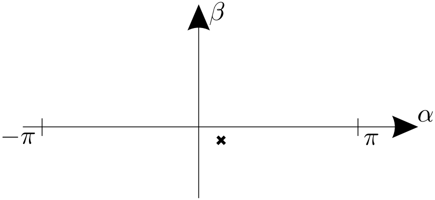


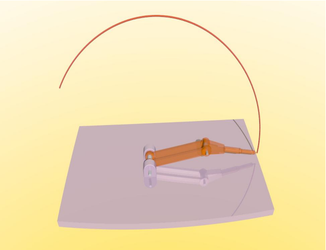


Motion planning

Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

$$f \circ \gamma = \delta$$

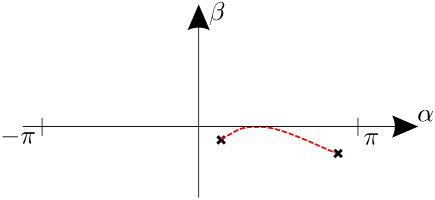


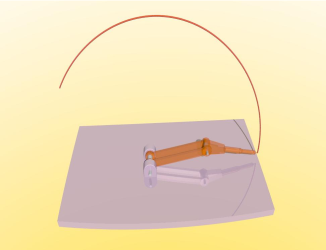


Motion planning

Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

$$f \circ \gamma = \delta$$

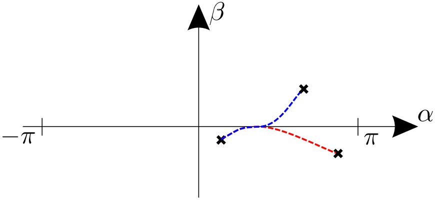


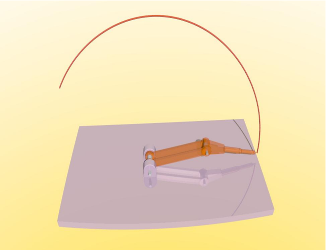


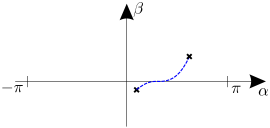
Motion planning

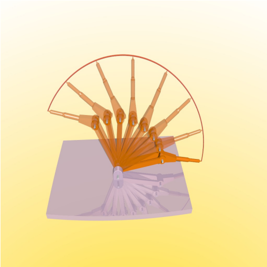
Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

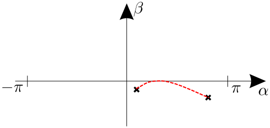
$$f \circ \gamma = \delta$$

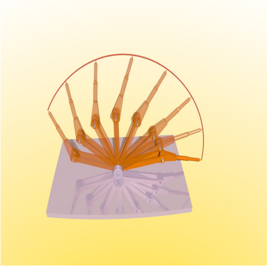





β
$-\pi$
π
α



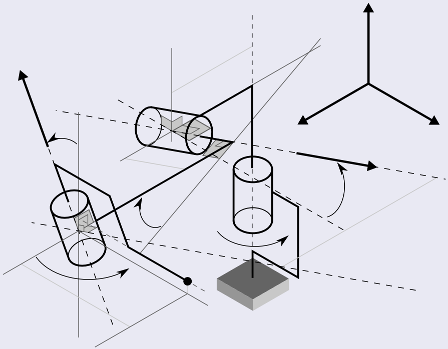

β
$-\pi$
π
α



Application

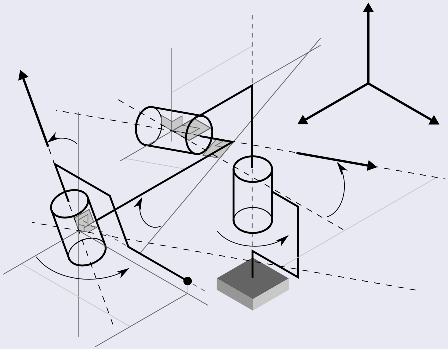
Computing intrisic properties of a robot to understand its behaviour.

$$x = f(\alpha)$$



Application

Guaranteed detection of the singularities of 3R robotic manipulators by Romain BENOIT in EUCOMES 2014.



Source code is available on my webpage.

Source code is available on my webpage.

Merci pour votre attention.