Guaranteeing the homotopy type of a set defined by non-linear inequalities.

Nicolas Delanoue

Topological Methods For The Study Of Discrete Structures

February 2010

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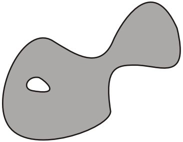

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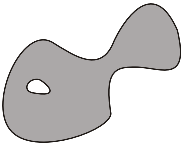

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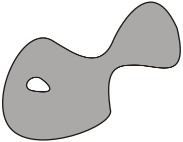

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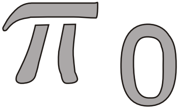

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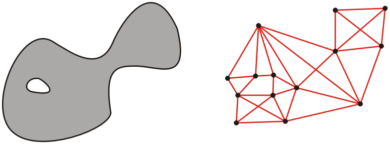

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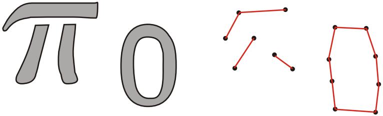
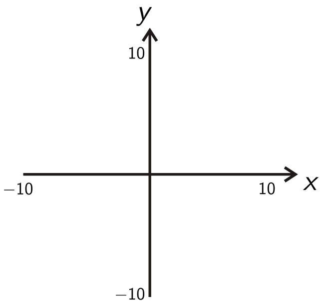


Outline

1. Introduction to interval analysis
2. Motivation - topological recall
3. Computing the number of connected components
   - A sufficient condition for proving that a set is star-shaped
   - The idea
4. More topological information
5. What I am doing now

Example 1

$$\mathbb{S} = \{(x,y) \in [-10;10]^2 \mid f(x,y) = x^2 + y^2 + xy - 30 \leq 0\}$$



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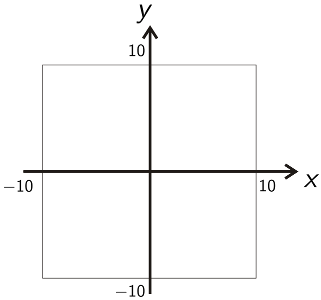

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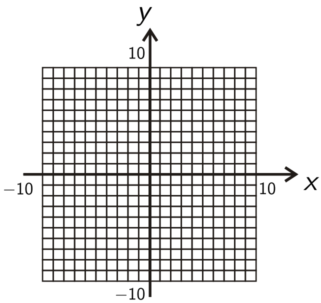

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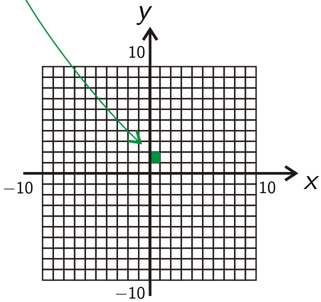
$$\begin{array}{lllllll} x & \in & [& 0 & ; & 1 &] \\ y & \in & [& 1 & ; & 2 &] \end{array}$$



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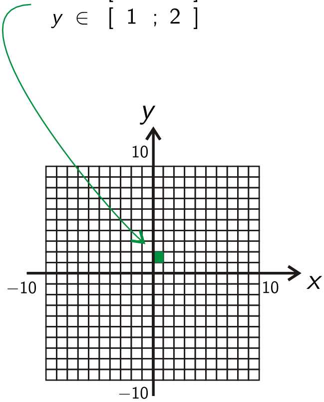

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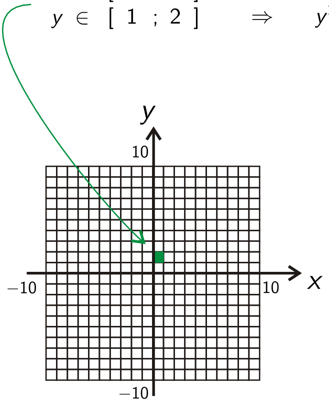

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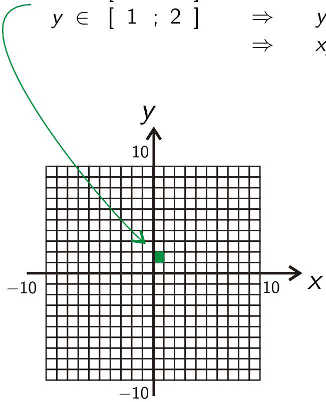

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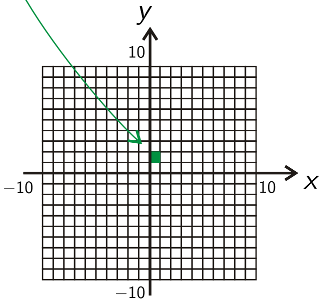

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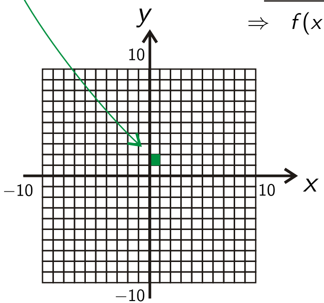

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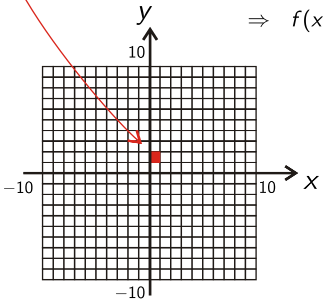

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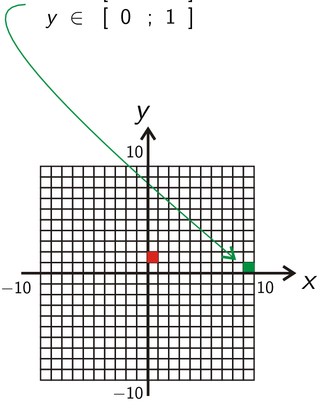
$$\mathbb{S} = \{(x,y) \in [-10;10]^2 \mid f(x,y) = x^2 + y^2 + xy - 30 \leq 0\}$$

$$\begin{array}{lcl} x & \in & [\ 9\ ;\ 10\ ] \\ y & \in & [\ 0\ ;\ 1\ ] \end{array}$$



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$$\begin{array}{lllcl}
x \in [\,9\,;10\,] & \Rightarrow & x^2 & \in & [\,81\,;\,100\,] \\
y \in [\,0\,;\,1\,] & \Rightarrow & y^2 & \in & [\,0\,;\,1\,] \\
 & \Rightarrow & xy & \in & [\,0\,;\,10\,] \\
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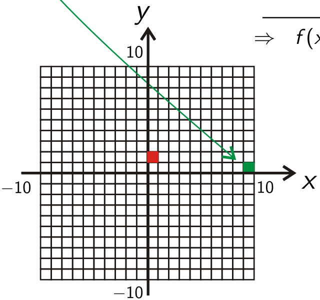
\hline
 & \Rightarrow & f(x,y) & \in & [\,51\,;\,81\,]
\end{array}$$



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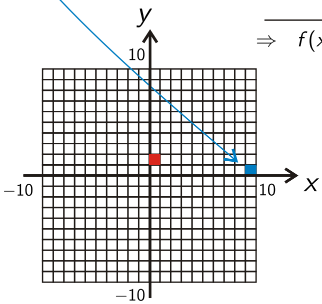

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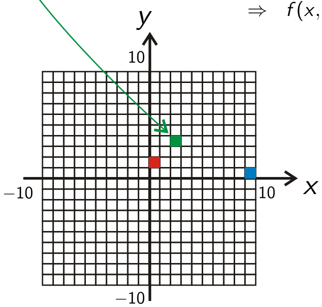
$$\begin{array}{lll} x & \in & [\ 2\ ;\ 3\ ] \\ y & \in & [\ 3\ ;\ 4\ ] \end{array}
\qquad
\begin{array}{llll} \Rightarrow & x^2 & \in & [\ 4\ ;\ 9\ ] \\ \Rightarrow & y^2 & \in & [\ 9\ ;\ 16\ ] \\ \Rightarrow & xy & \in & [\ 6\ ;\ 12\ ] \\ \Rightarrow & -30 & \in & [\ -30\ ;\ -30\ ] \\ \hline \Rightarrow & f(x,y) & \in & [\ -11\ ;\ 7\ ] \end{array}$$



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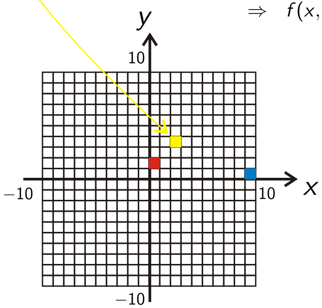

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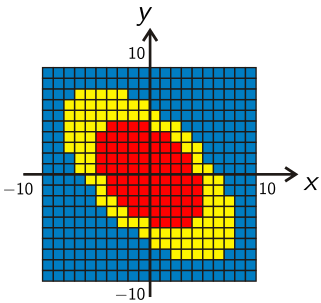

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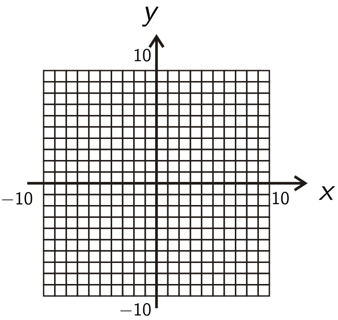
Proving that an equation has no solution

Example 2

$$\mathbb{S}' = \{(x, y) \in [-10; 10]^2 \mid f'(x, y) = x^2 + y^2 + xy + 10 \leq 0\}$$



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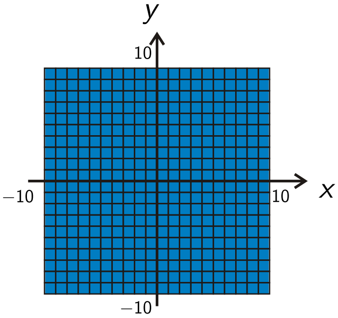

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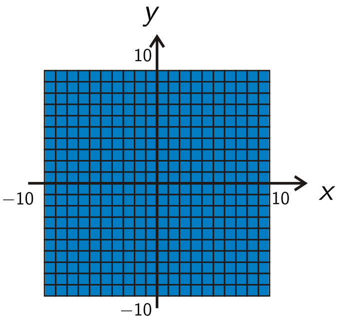
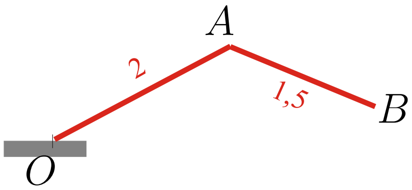
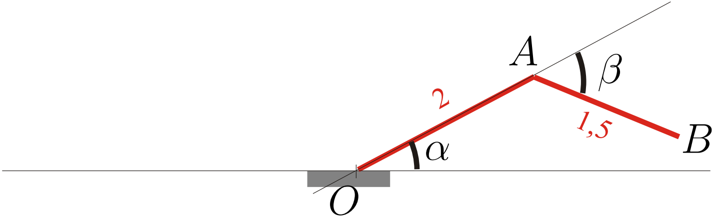


$$\Rightarrow \quad \mathbb{S}' = \emptyset$$


A
2
1,5
B
O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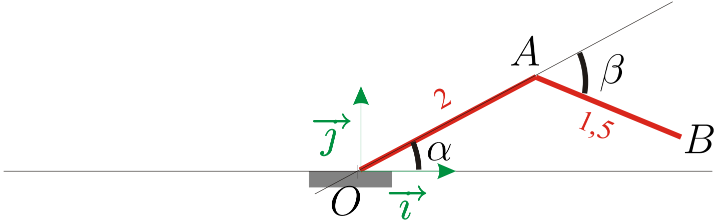
A
β
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$\vec{j}$
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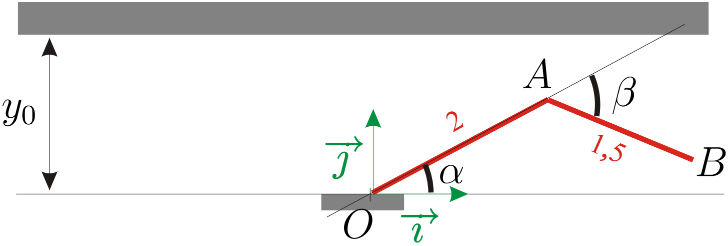
y_0
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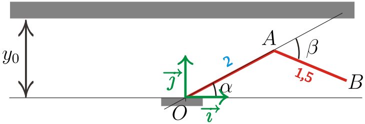
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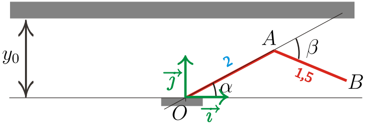
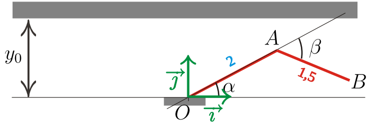


Coordinates of A

$$\begin{cases} x_A = 2\cos(\alpha) \\ y_A = 2\sin(\alpha) \end{cases}$$

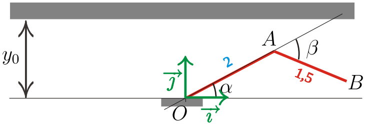


Coordinates of A

$$\begin{cases} x_A = 2\cos(\alpha) \\ y_A = 2\sin(\alpha) \end{cases}$$

Coordinates of B

$$\begin{cases} x_B = 2\cos(\alpha) + 1.5\cos(\alpha+\beta) \\ y_B = 2\sin(\alpha) + 1.5\sin(\alpha+\beta) \end{cases}$$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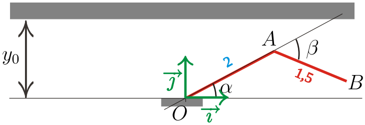
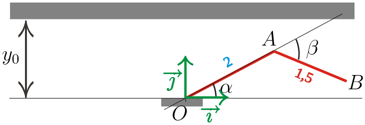


Constraints

$$y_A \in [0, y_0]$$



Constraints

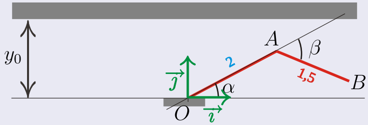
$$y_A \in [0, y_0]$$

Constraints

$$y_B \in]-\infty, y_0]$$

Constraints about α and β

$$\alpha \in [-\pi, \pi], \beta \in [-\pi, \pi]$$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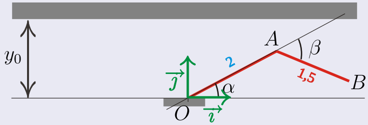


Feasible configuration set

$(\alpha, \beta) \in [-\pi, \pi]^2$ /

$$\begin{cases} f_1(\alpha, \beta) = 2\sin(\alpha) - y_0 & \leq \quad 0 \\ f_2(\alpha, \beta) = -2\sin(\alpha) & \leq \quad 0 \\ f_3(\alpha, \beta) = 2\sin(\alpha) + 1.5\sin(\alpha + \beta) - y_0 & \leq \quad 0 \end{cases}$$

Definition (*path-connected set*)

A topological space $\mathbb{S}$ is *path-connected* if and only if for every two points $x, y \in \mathbb{S}$, there is a continuous function f from $[0, 1]$ to $\mathbb{S}$ such that $f(0) = x$ and $f(1) = y$.

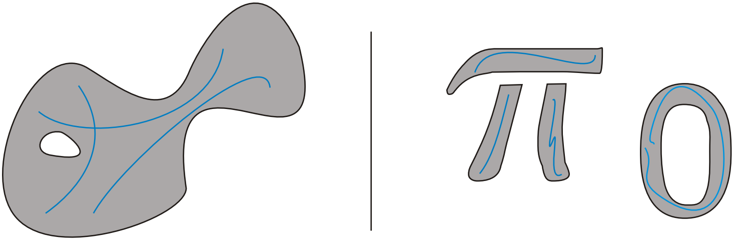
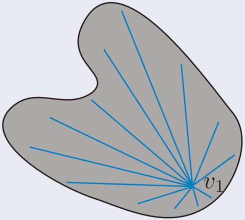


FIG.: Examples.

Definition (*star*)

The point v^* is a *star* for a subset X of an Euclidean space if $\forall x \in X$, the segment $[x, v^*]$ is include in X.



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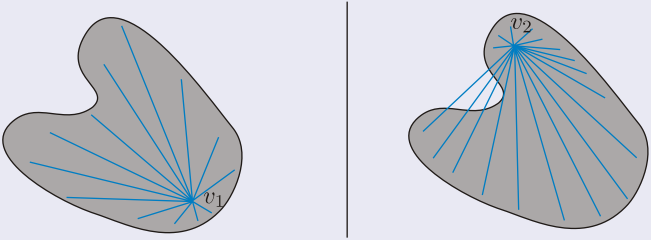

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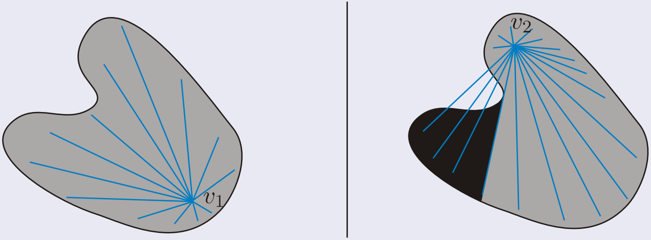
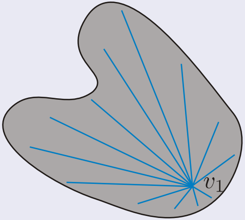


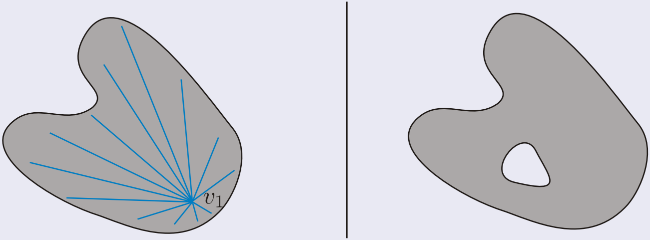
Definition (*star-shaped set*)

If there exists $v^* \in X$ such that v^* is a star for X, then we say that X is *star-shaped* or v^**-star-shaped*.



Definition (*star-shaped set*)

If there exists $v^* \in X$ such that v^* is a star for X, then we say that X is *star-shaped* or v^*-*star-shaped*.



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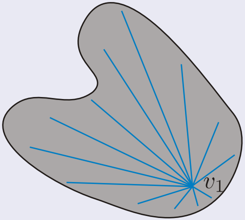
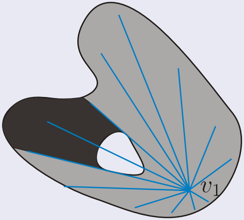
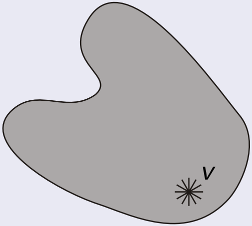




Proposition 1

A star-shaped set is a path-connected set.



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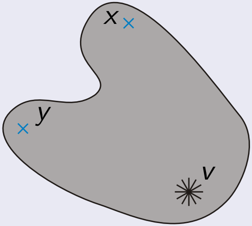

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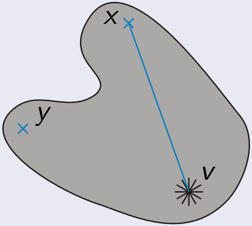

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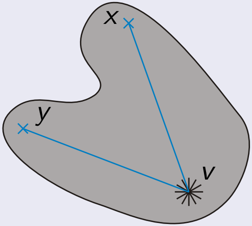
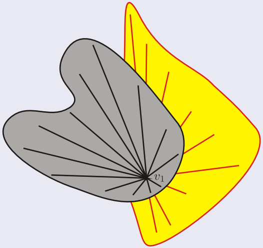


Proposition 2

Let X and Y two v^*-star-shaped set, then $X \cap Y$ and $X \cup Y$ are also v^*-star-shaped.



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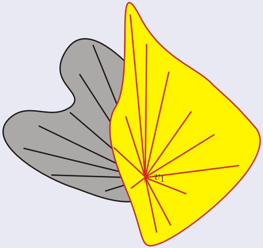

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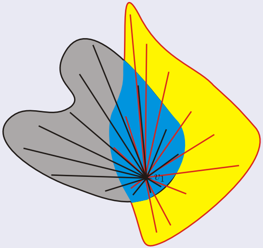
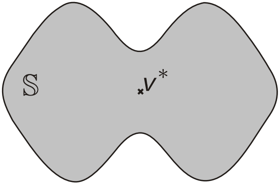


Theorem

Let $\mathbb{S} = \{x \in D \subset \mathbb{R}^n \ / \ f(x) \leq 0\}$ *where* f *is a* C^1 *function from* D *to* $\mathbb{R}$*,* D *a convex set,* v^* *be in* $\mathbb{S}$*. If*

$$f(x) = 0, \ Df(x).(x - v^*) \leq 0, \ x \in D$$

is inconsistent then v^* *is star a for* $\mathbb{S}$*.*



Geometric interpretation

Let us prove that $v^* = (0.6, -0.5)$ is a star for the set defined by

$$\mathbb{S} = \{(x, y) \in \mathbb{R}^2, \text{ such that } f(x, y) = x^2 + y^2 + xy - 2 \leq 0\}$$

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$$\Leftrightarrow \left\{ \begin{array}{l} x^2 + y^2 + xy - 2 = 0 \\ \partial_x f(x,y).(x-0.6) + \partial_y f(x,y).(y+0.5) \leq 0 \end{array} \right. \quad \text{is inconsistent}$$

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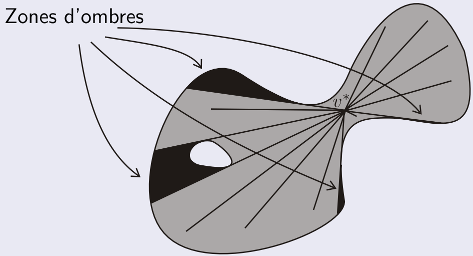
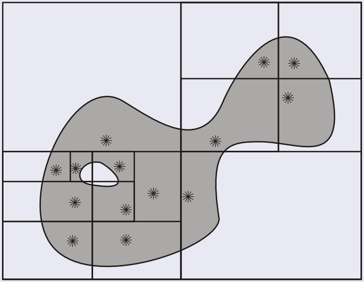

Zones d'ombres
v^*

The idea

To divide it with a paving $\mathcal{P}$ such that, on each part p, $\mathbb{S} \cap p$ is star-shaped.



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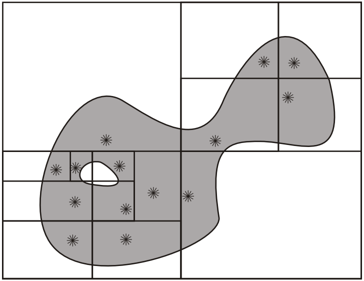
Let us define the notion of *star-spangled* graph with the relation : $\mathbb{S} \cap p \cap q \neq \emptyset$.



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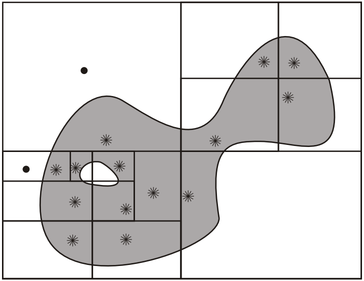

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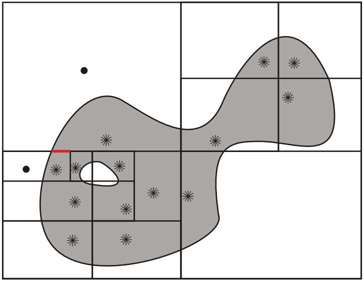

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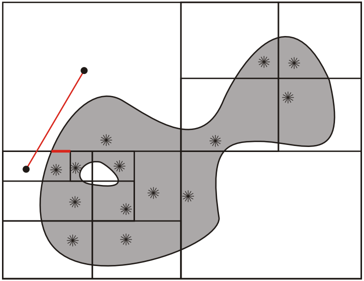

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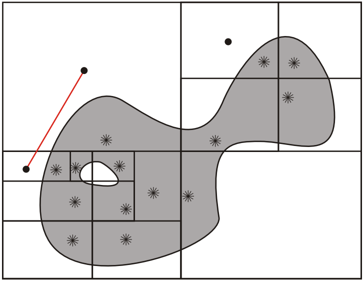

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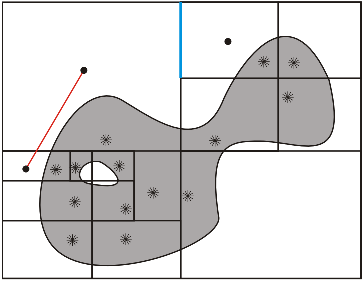

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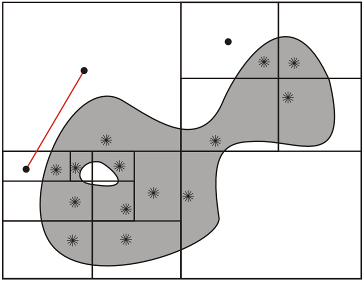

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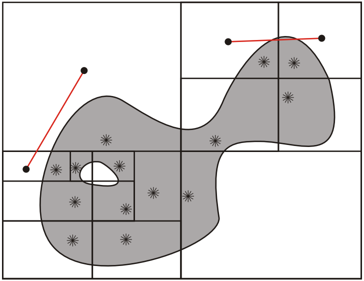

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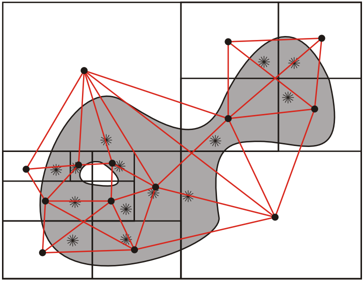
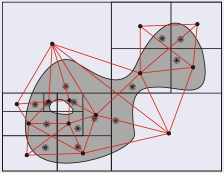


Definition

A *star-spangled graph* of a set $\mathbb{S}$, noted by $\mathcal{G}_{\mathbb{S}}$, is a relation $\mathcal{R}$ on a paving $\mathcal{P}$ where

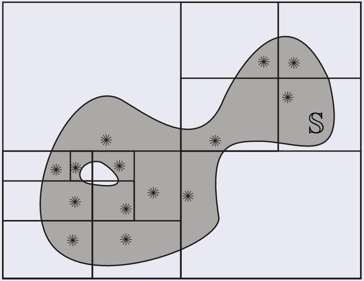
- $\mathcal{P} = (p_i)_{i\in I}$, for all p of $\mathcal{P}$, $\mathbb{S} \cap p$ is star-shaped. And $\mathbb{S} \subset \bigcup_{i\in I} p_i$
- $\mathcal{R}$ is the reflexive and symmetric relation on $\mathcal{P}$ defined by $p \,\mathcal{R}\, q \Leftrightarrow \mathbb{S} \cap p \cap q \neq \emptyset$.

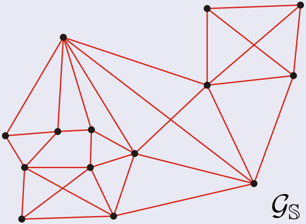


Theorem

Let $\mathcal{G}_{\mathbb{S}}$ be a star-spangled graph of a set $\mathbb{S}$.

$\mathbb{S}$ is path-connected $\Leftrightarrow \mathcal{G}_{\mathbb{S}}$ is connected .

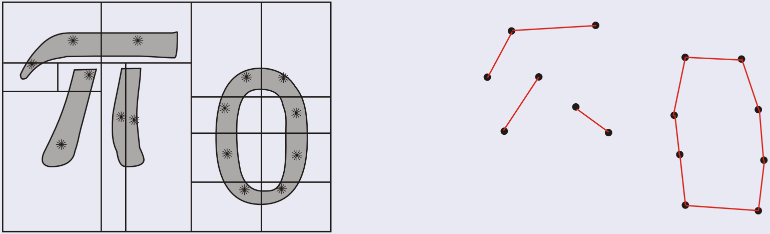




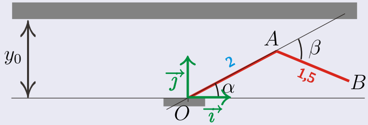
proof

Corollary

Let $\mathcal{G}_{\mathbb{S}}$ be a star-spangled graph of a set $\mathbb{S}$.
$\mathcal{G}_{\mathbb{S}}$ has the same number of connected components than $\mathbb{S}$. i.e. $\pi_0(\mathbb{S}) = \pi_0(\mathcal{G}_{\mathbb{S}})$.



Feasible configuration set, $y_0 = 2.3$



$$\mathbb{S} = \left\{ (\alpha,\beta) \in]-\pi,\pi[^2 / \left\{ \begin{array}{lcl} 2\sin(\alpha) & \in & [0, y_0] \\ 2\sin(\alpha) + 1.5\sin(\alpha+\beta) & \in &]-\infty, y_0] \end{array} \right. \right\}$$

The solver CIA (path-Connected via Interval Analysis)

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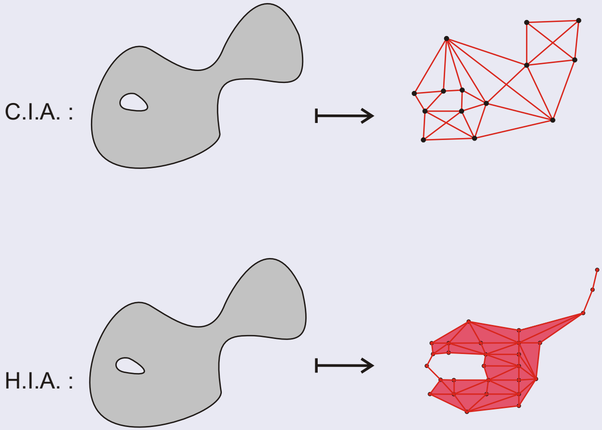
Feasible configuration set, $y_0 = 2.3$

$(\alpha,\beta) \in]-\pi,\pi[^2$ /

$$\left\{ \begin{array}{lcl} f_1(\alpha,\beta) = 2\sin(\alpha) - y_0 & \leq & 0 \\ f_2(\alpha,\beta) = -2\sin(\alpha) & \leq & 0 \\ f_3(\alpha,\beta) = 2\sin(\alpha) + 1.5\sin(\alpha+\beta) - y_0 & \leq & 0 \end{array} \right.$$


C.I.A. :
H.I.A. :

topological invariant

number of connected components

Let π_0 be :

$$\begin{array}{rcl} \pi_0 : \quad \{ \text{"Nice" sets} \} & \rightarrow & \mathbb{N} \\ X & \mapsto & \text{number of connected components of } X \end{array}$$

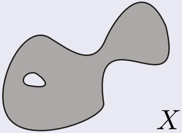
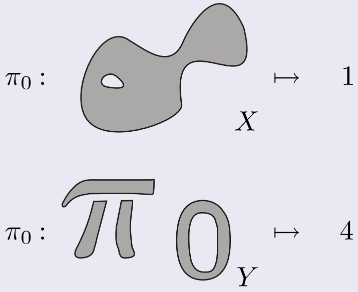
π_0 :  $\mapsto$ 1

FIG.: $\pi_0(X)$ and $\pi_0(Y)$.



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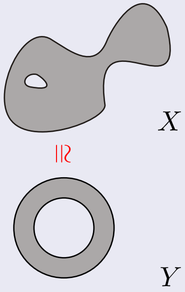
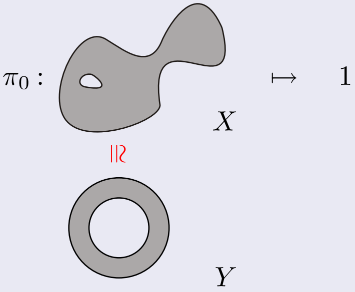


FIG.: $\pi_0(.)$ is a topological invariant.



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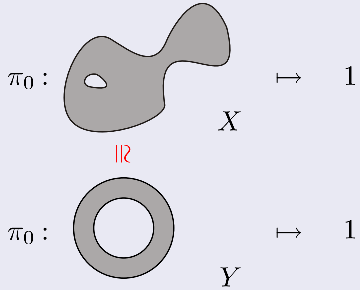

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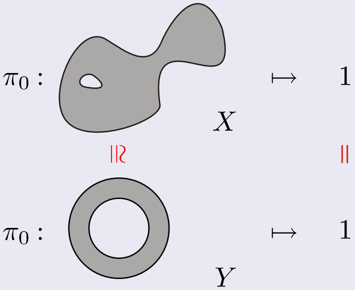

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If X and Y are such that $\pi_0(X) \neq \pi_0(Y)$
then $X \ncong Y$.

There exists sets such that

$$X \ncong Y \text{ and } \pi_0(X) = \pi_0(Y)$$

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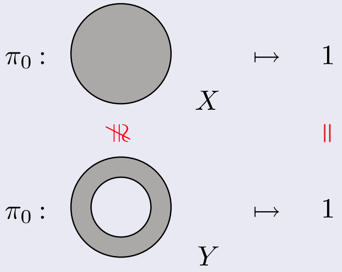

FIG.: $\pi_0(.)$ is not enough strong.

Definition - Homeomorphism

A function f between two topological spaces X and Y is called a homeomorphism if it has the following properties :

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Definition - homeomorphic spaces

Two spaces X and Y with a homeomorphism $f : X \to Y$ between them are called homeomorphic. From a topological viewpoint they are the same.

Examples

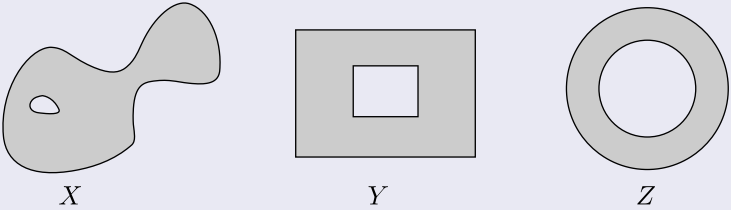


FIG.: Example.

Examples

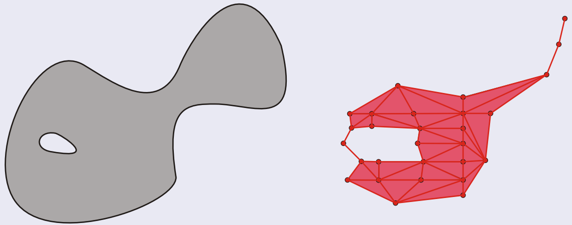
Example.

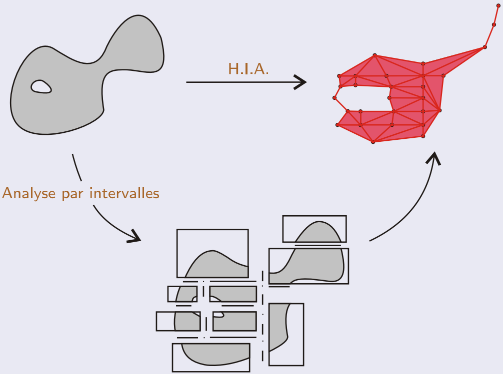
$X \cong Y \cong Z.$

Examples

Example.

- Create a triangulation to obtain more topological properties.
  - homotopy type, fundamental group ($\pi_1(\mathbb{S})$).
  - homology groups ($H_1(\mathbb{S}), H_2(\mathbb{S}), \ldots$).
  - Betti numbers.




H.I.A.
Analyse par intervalles

Definition - homotopy between the two functions

Two continuous functions $f, g : X \to Y$ are *homotopic*, $f \sim g$ if the exists a continuous function $F : X \times [0,1] \to Y$, such that :

$$F(x,0) = f(x) \text{ and } F(x,1) = g(x),\ \forall x \in X.$$

$$f \sim g$$

$$f \sim f^0$$

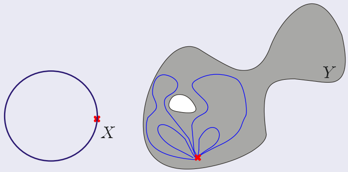


Fig.:

Definition : π_1 of a set

Let Y be a path connected space and $y_0 \in Y$,

$$\pi_1(Y, y_0) = \{f : \mathbb{S}_1 \to Y,\ f \text{ continuous },f(x_0) = y_0\}/\sim$$

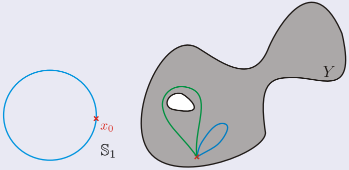


FIG.: f^1 and f^0.

Poincaré, H. 'Analysis situs', J. Ecole polytech. (2)1, 1-121 (1895).

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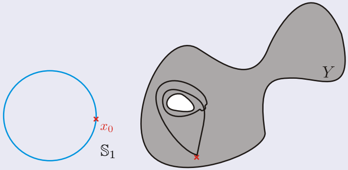


FIG.: f^2

Poincaré, H. 'Analysis situs', J. Ecole polytech. (2)1, 1-121 (1895).

Property

$\pi_1(Y, y_0)$ is a group.

$$f^0 \times f^1 \sim f^1.$$

Property

$\pi_1(Y, y_0)$ is a group.

$$f^1 \times f^{-1} \sim f^0$$

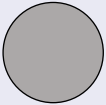
$\pi_1 :$  $X \mapsto \{0\}$

FIG.: $\pi_1(.)$ is a topological invariant.

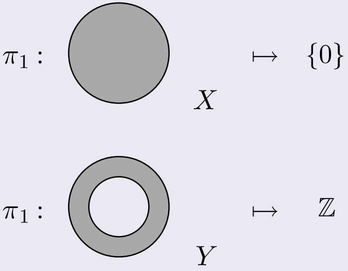
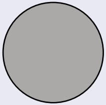


FIG.: $\pi_1(.)$ is a topological invariant.

$\pi_1 :$  $X \quad \mapsto \quad \{0\}$

$\nparallel$

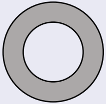
$\pi_1 :$  $Y \quad \mapsto \quad \mathbb{Z}$

FIG.: $\pi_1(.)$ is a topological invariant.

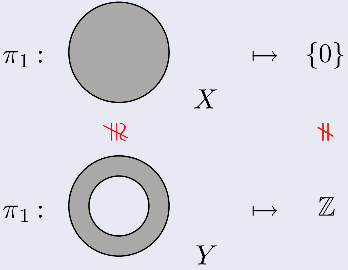


FIG.: $\pi_1(.)$ is a topological invariant.

Definition

Two spaces X and Y are homotopy equivalent or of the same homotopy type if there exist continuous maps $f : X \to Y$ and $g : Y \to X$ such that $g \circ f \sim 1_X$ et $f \circ g \sim 1_Y$. $X \simeq Y$.
One writes $X \simeq Y$.

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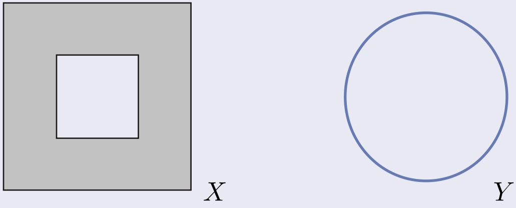

FIG.: $X \simeq Y$

Spaces that are homotopy equivalent.

Definition

Spaces that are homotopy equivalent to a point are called contractible.

$$X \simeq Y.$$

Proposition

Two homeomophic spaces are of the same homotopy type.

$$X \cong Y \Rightarrow X \simeq Y$$

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Remark

Most of the topological invariant can not distinguished two spaces that are homotopy equivalent.

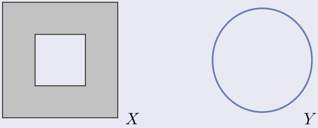

FIG.: $X \simeq Y \Rightarrow \pi_1(X) = \pi_1(Y)$

A triangulation

Definition - Abstract triangulation

Let $\mathcal{N}$ be a finite set of symbols $\{(a^0), (a^1), \dots, (a^n)\}$
An abstract triangulation $\mathcal{K}$ is a subset of the powerset of $\mathcal{N}$ satisfying : $\sigma \in \mathcal{K} \Rightarrow \forall \sigma_0 \subset \sigma, \sigma_0 \in \mathcal{K}$

$$\begin{aligned}\mathcal{K} = \{&(a^0),(a^1),(a^2),(a^3),(a^4),\\ &(a^0,a^1),(a^1,a^2),(a^0,a^2),(a^3,a^4),\\ &(a^0,a^1,a^2)\}\end{aligned}$$

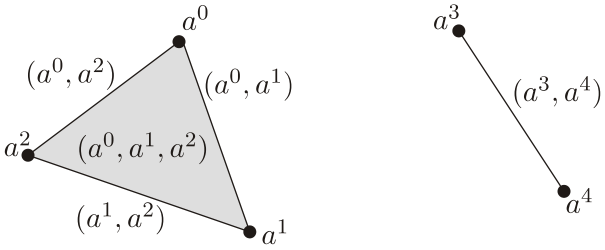

FIG.: A realisation of $\mathcal{K}$.

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This will be denoted by $a^0a^1a^2 + a^3a^4$

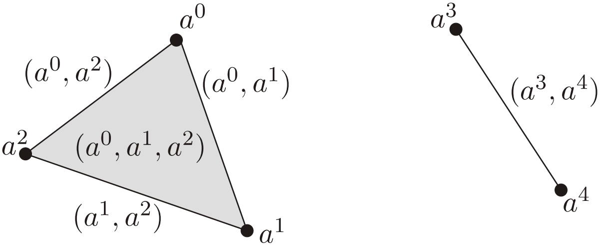
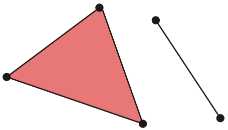


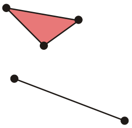
FIG.: A realisation of $\mathcal{K}$.

Theorem

If $|K_1|$ and $|K_2|$ are two realisations of an abstract triangulation $\mathcal{K}$, then $|K_1|$ and $|K_2|$ homeomorphic.



$|K_1|$



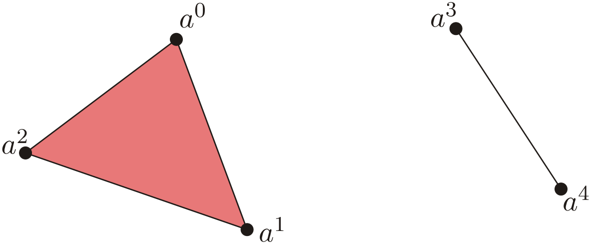
$|K_2|$

Definition

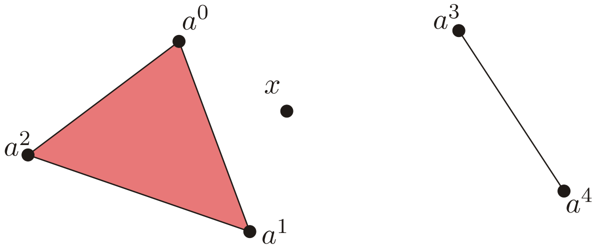
Let $\mathcal{K}$ be an abstract triangulation, an (x) a symbol. One notes by $\mathcal{C}(x, \mathcal{K})$ the set :

$$\mathcal{C}(x, \mathcal{K}) = \{(x)\} \cup \mathcal{K} \cup \bigcup_{\sigma \in \mathcal{K}} (x, \sigma).$$

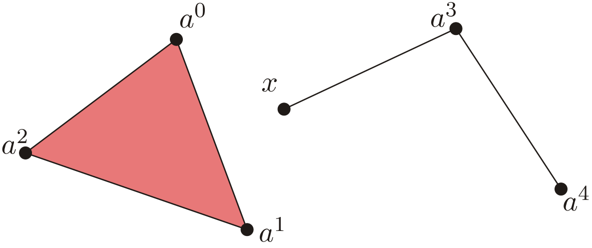
where $(x, \sigma) := (x, a^1, \ldots, a^n)$ with $\sigma = (a^1, \ldots, a^n) \in \mathcal{K}$.
$\mathcal{C}(x, \mathcal{K})$ is the *cone* with x as apex and $\mathcal{K}$ as base.



$$\begin{aligned}\mathcal{C}(x,\mathcal{K}) \quad = \quad & \{(x)\} \cup \mathcal{K} \cup \{(x,a^0),(x,a^1),(x,a^2),(x,a^3),(x,a^4),\\ & (x,a^0,a^1),(x,a^1,a^2),(x,a^0,a^2),(x,a^3,a^4),(x,a^0,a^1,a^2)\}\,.\end{aligned}$$



$$\begin{aligned}\mathcal{C}(x,\mathcal{K}) &= \{(x)\}\cup\mathcal{K}\cup\big\{(x,a^0),(x,a^1),(x,a^2),(x,a^3),(x,a^4),\\ &\quad (x,a^0,a^1),(x,a^1,a^2),(x,a^0,a^2),(x,a^3,a^4),(x,a^0,a^1,a^2)\big\}\,.\end{aligned}$$



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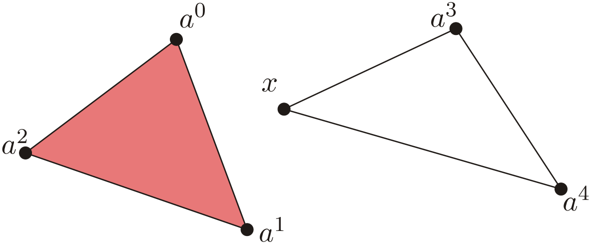
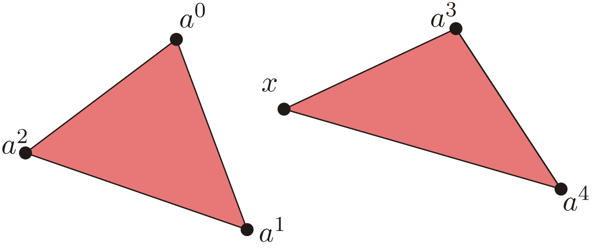


$$\begin{aligned}\mathcal{C}(x,\mathcal{K}) \quad = \quad & \{(x)\} \cup \mathcal{K} \cup \big\{(x,a^0),(x,a^1),(x,a^2),(x,a^3),(x,a^4),\\ & (x,a^0,a^1),(x,a^1,a^2),(x,a^0,a^2),(x,a^3,a^4),(x,a^0,a^1,a^2)\big\}\,.\end{aligned}$$



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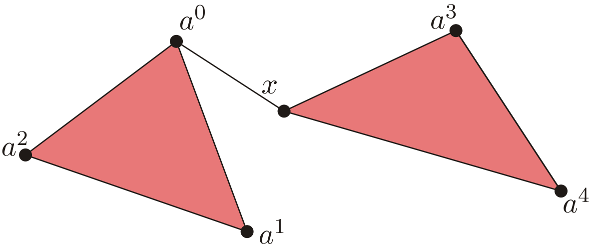

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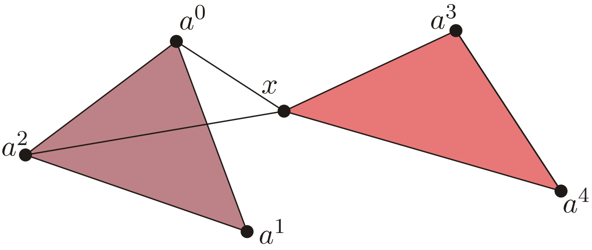

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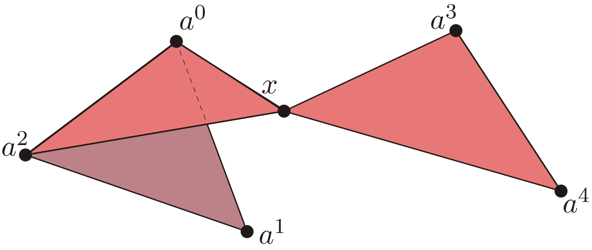

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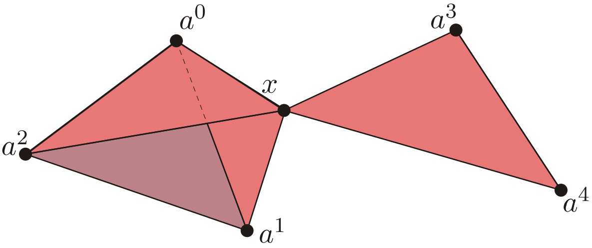

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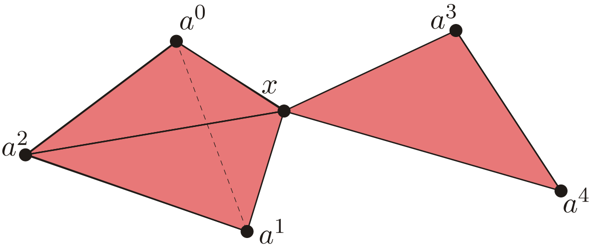

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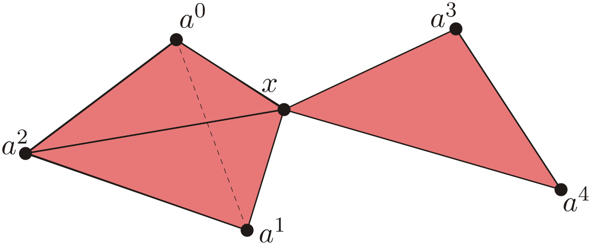
$$\begin{aligned}\mathcal{C}(x,\mathcal{K}) &= x(a^0a^1a^2 + a^3a^4)\\ &= xa^0a^1a^2 + xa^3a^4\end{aligned}$$

Property of a cone

A cone is contractile.



Main goal :

Create a triangulation homotopy equivalent to :

$$\mathbb{S} = \bigcup_{i=1}^{s} \bigcap_{j=1}^{r_i} \{x \in \mathbb{R}^n ; f_{i,j}(x) \leq 0\} \text{ where } f_{i,j} \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R})$$

$$\mathbb{S} = \left\{ (x,y) \in \mathbb{R}^2 \middle| \left\{ \begin{array}{rcl} x^2 + y^2 + xy - 2 & \leq & 0 \\ -x^2 - y^2 - xy + 1 & \leq & 0 \end{array} \right. \right\}$$

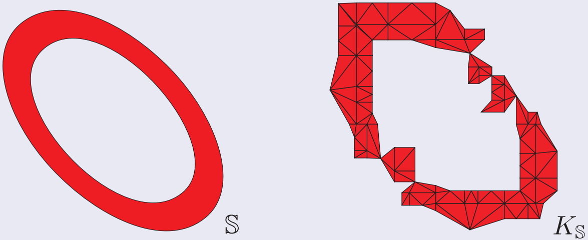


FIG.: Example of a set $\mathbb{S}$ and a triangulation generated by the algorithm `Homotopy via Interval Analysis`.

1. Create a covering $\{\mathbb{S}_i\}_{i \in I}$ de $\mathbb{S}$ such that

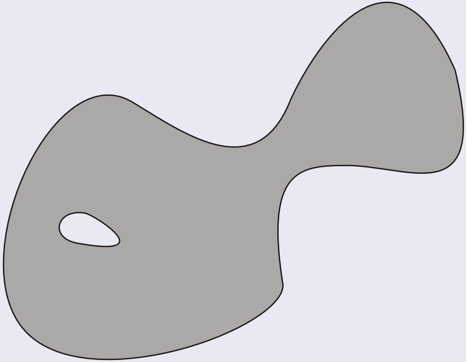
$$\forall J \subset I, \bigcap_{j \in J} \mathbb{S}_j \text{ is contractile or empty}$$

1. Create a covering $\{\mathbb{S}_i\}_{i\in I}$ de $\mathbb{S}$ such that

$$\forall J \subset I, \bigcap_{j\in J} \mathbb{S}_j \text{ is contractile or empty}$$

2. Create a triangulation homotopy equivalent to $\mathbb{S}$.

Divide $\mathbb{S}$ with a paving $\{p_i\}_{i \in I}$, $(\mathbb{S}_i := \mathbb{S} \cap p_i)$ such that $\forall J \subset I, \bigcap_{j \in J} \mathbb{S}_j$ is contractible of empty



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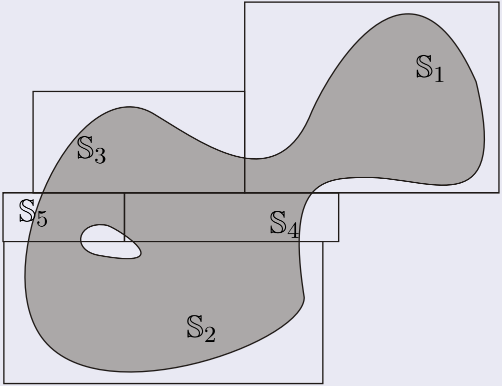

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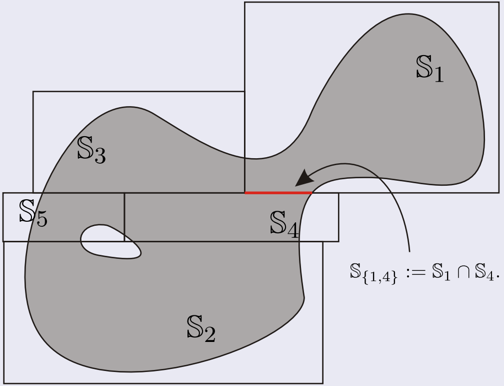

Divide $\mathbb{S}$ with a paving $\{p_i\}_{i\in I}$, ($\mathbb{S}_i := \mathbb{S} \cap p_i$) such that

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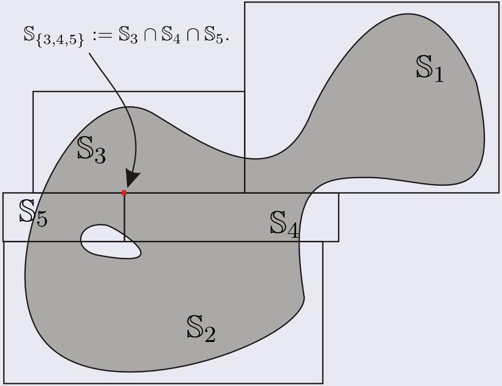
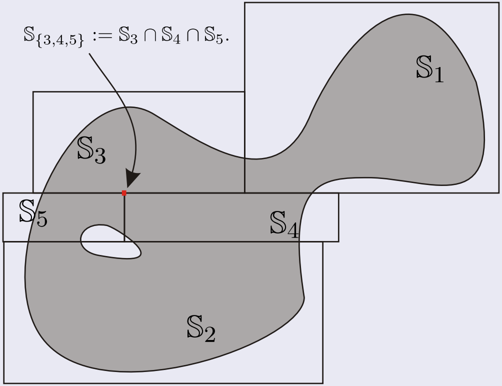
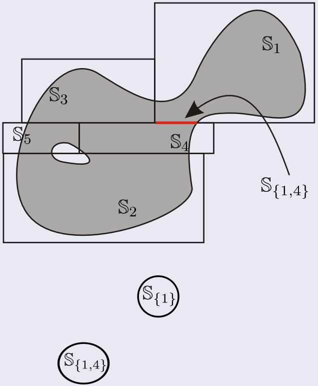


Let $\mathcal{F} = \{\mathbb{S}_J, J \subset I,$ such that $\mathbb{S}_J$ is contractible $\}$.



Order $\mathcal{F}$ with inclusion :



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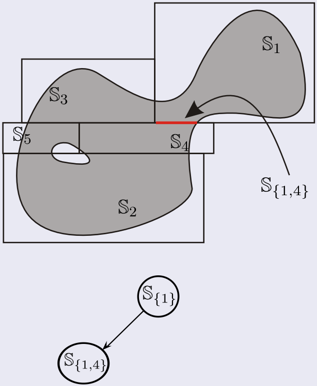

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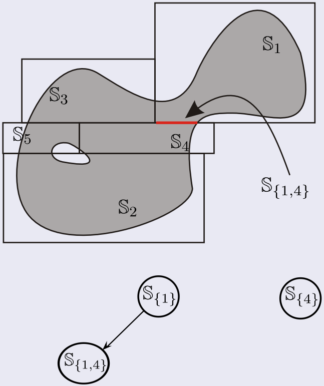

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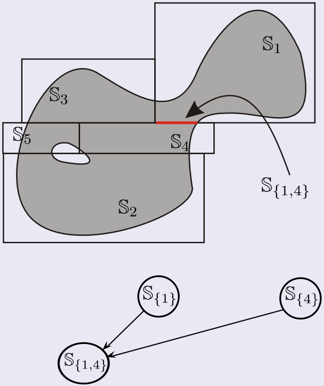
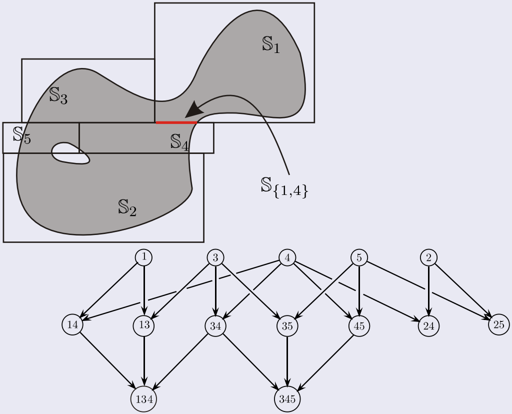
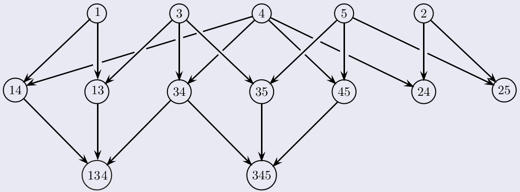


Order $\mathcal{F}$ with inclusion :

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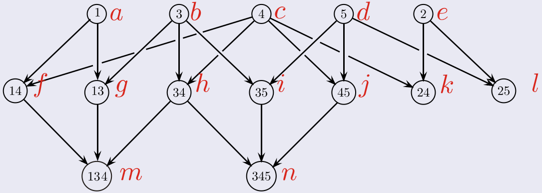
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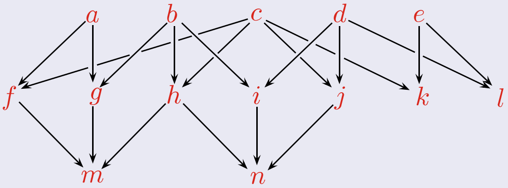
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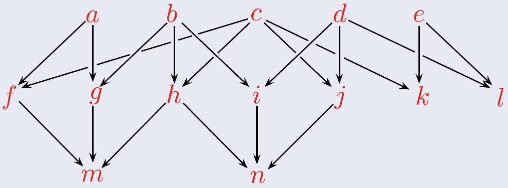
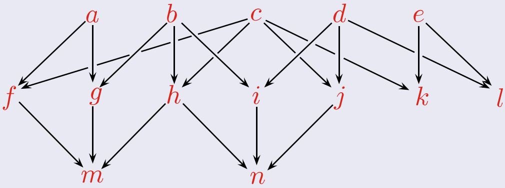


$$a(fm+gm)+b(gm+h(m+n)+in)+$$

$$+c(fm+h(m+n)+jn+k)+d(in+jn+l)+e(k+l)$$



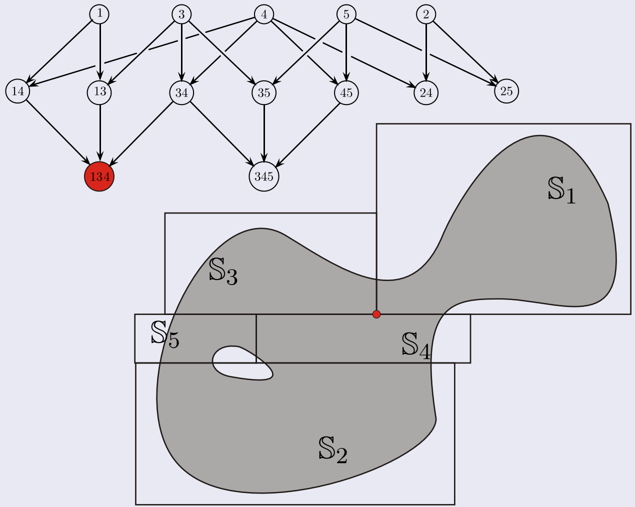
$$a(fm+gm)+b(gm+h(m+n)+in)+$$

$$+c(fm+h(m+n)+jn+k)+d(in+jn+l)+e(k+l)$$

$$=$$

$$afm+agm+bgm+bhm+bhn+bin+$$

$$+cfm+chm+chn+cjn+ck+din+djn+dl+ek+el$$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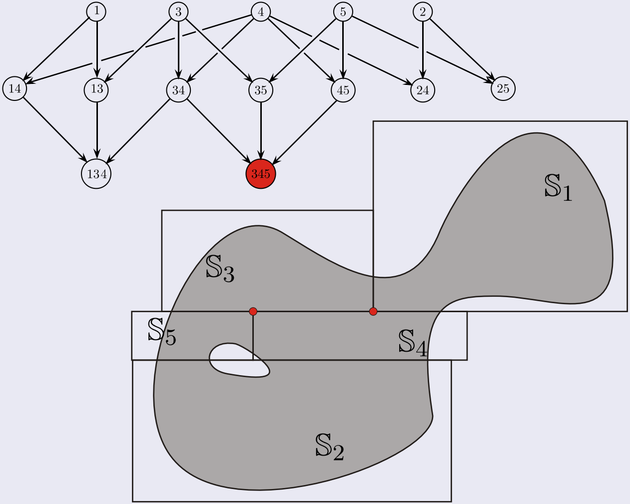
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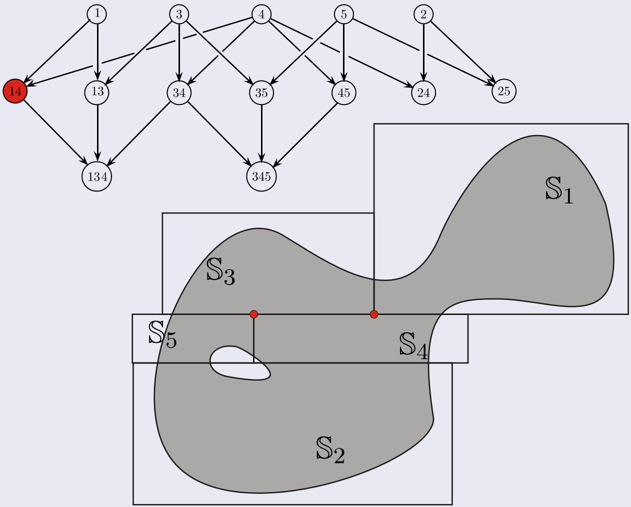
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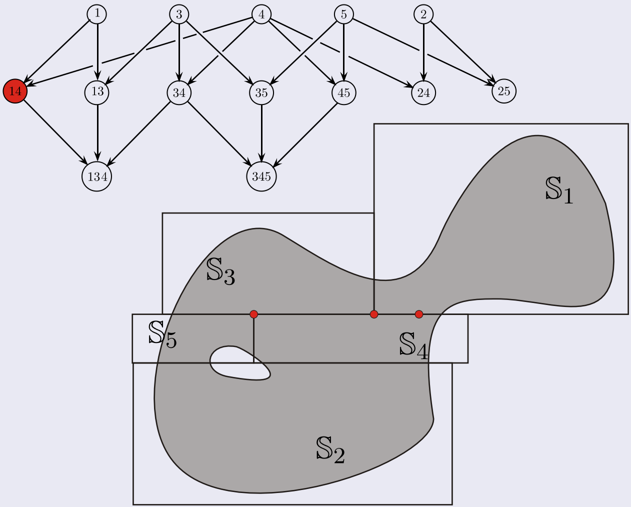
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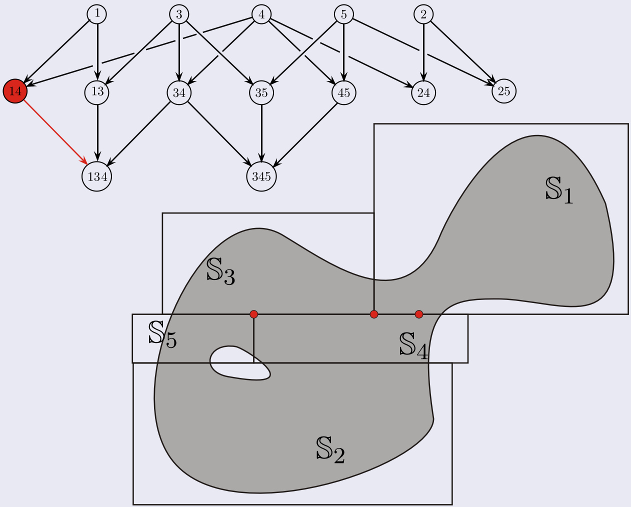
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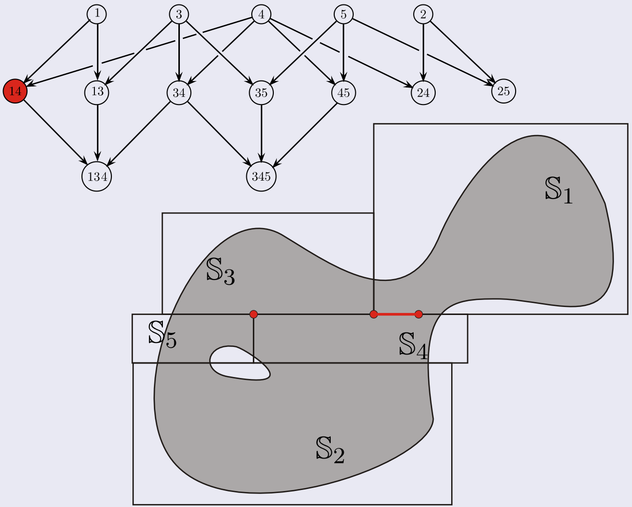
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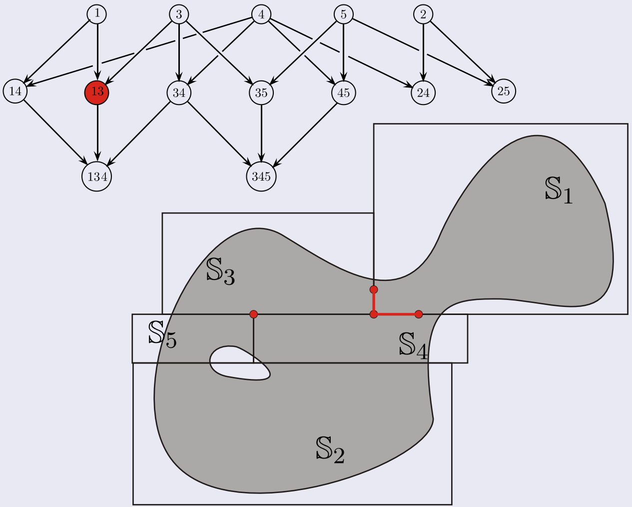
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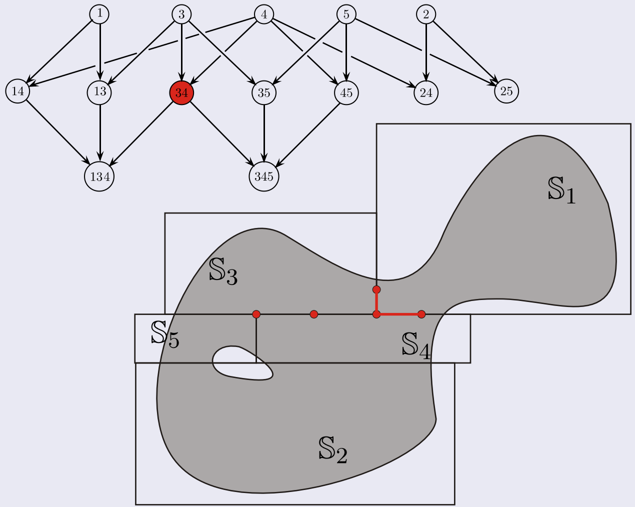
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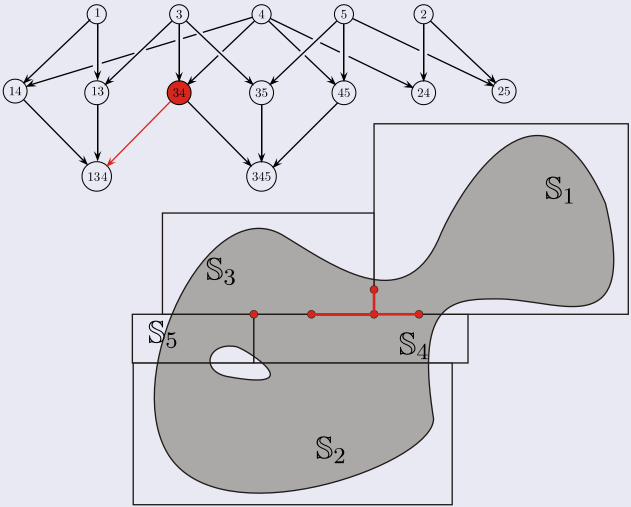
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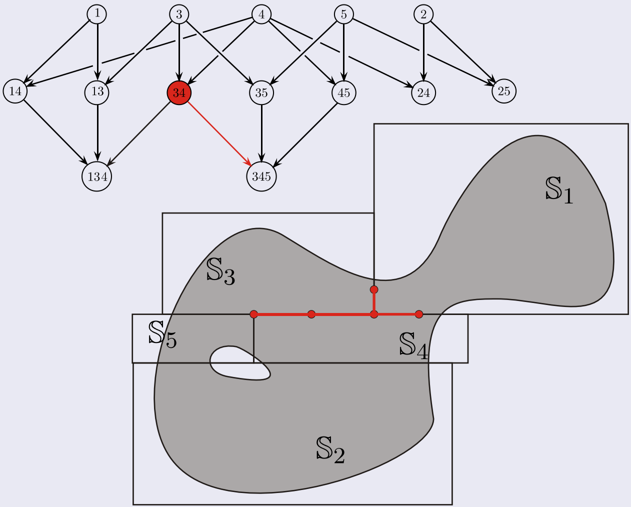
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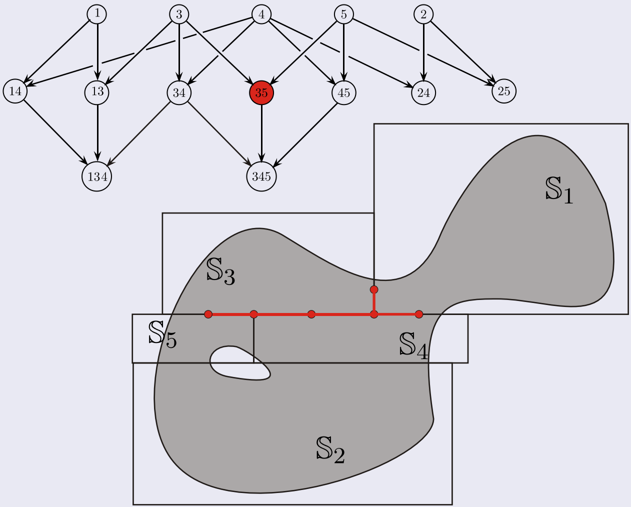
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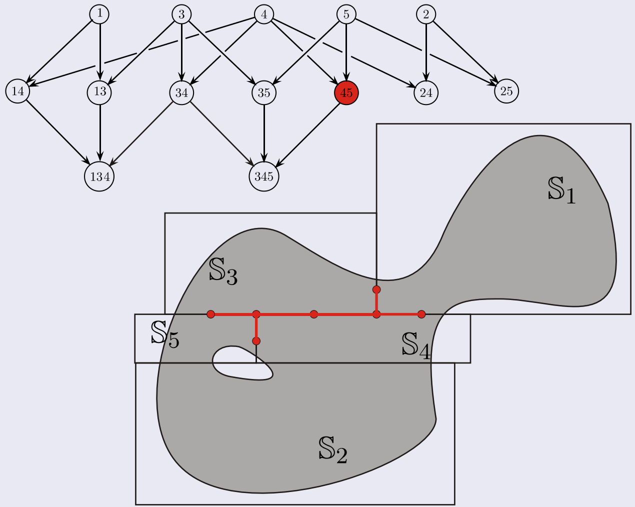
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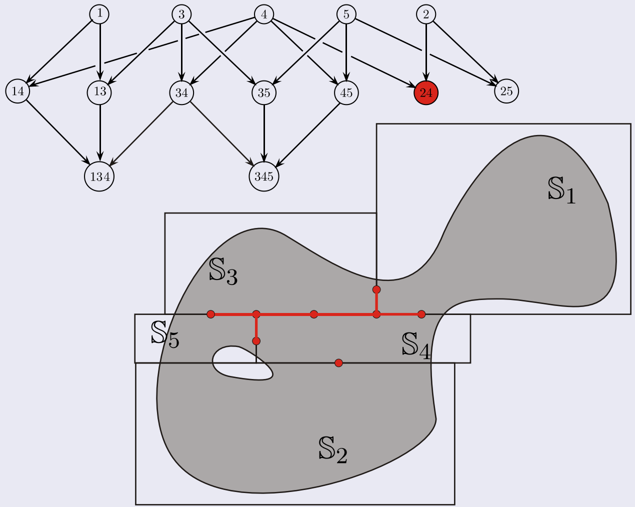
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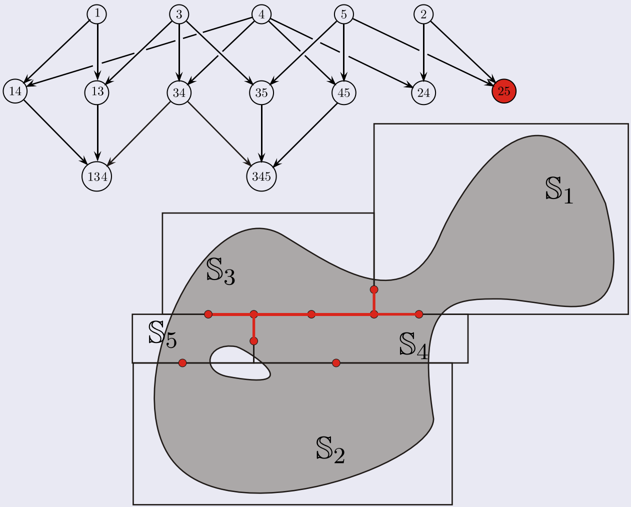
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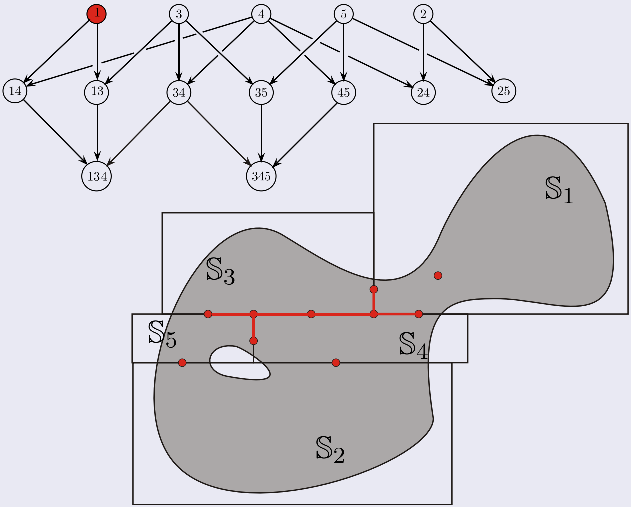
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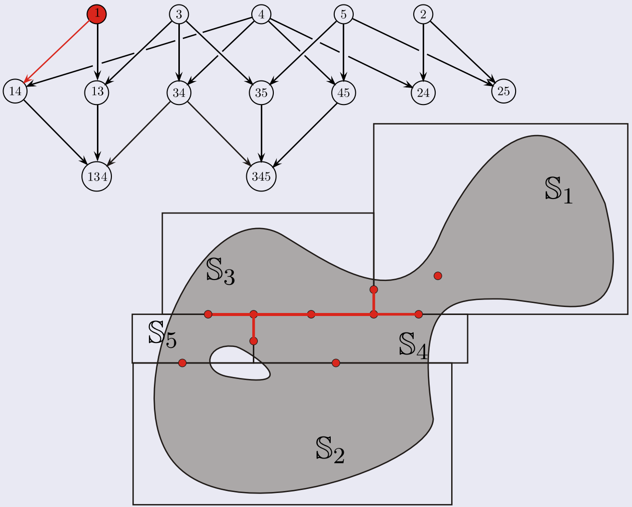
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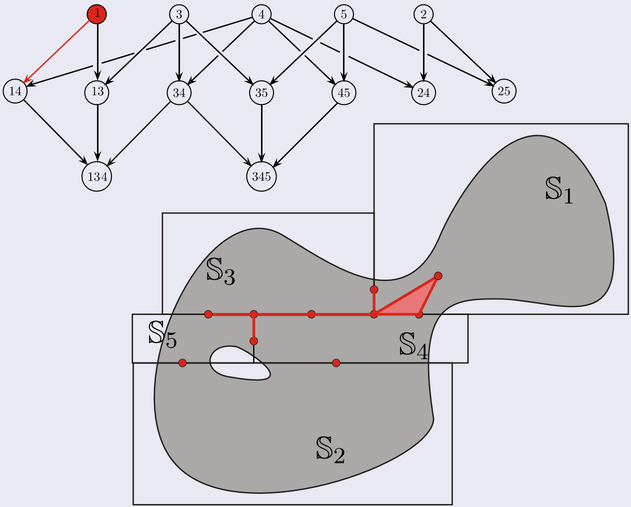
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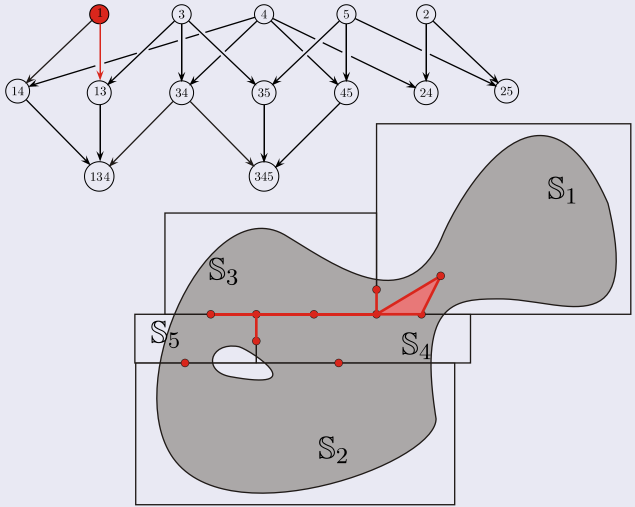
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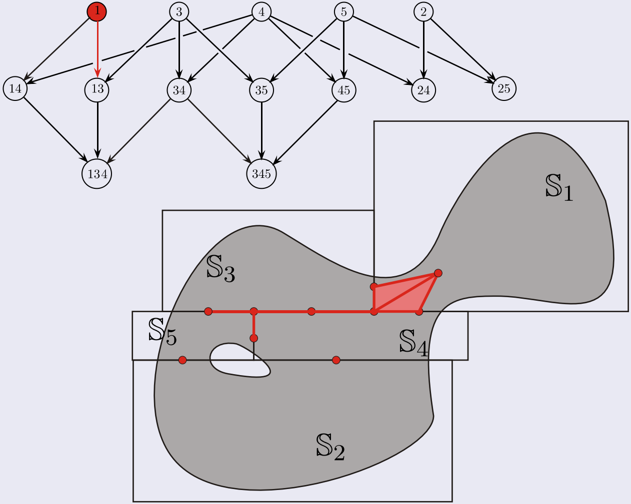
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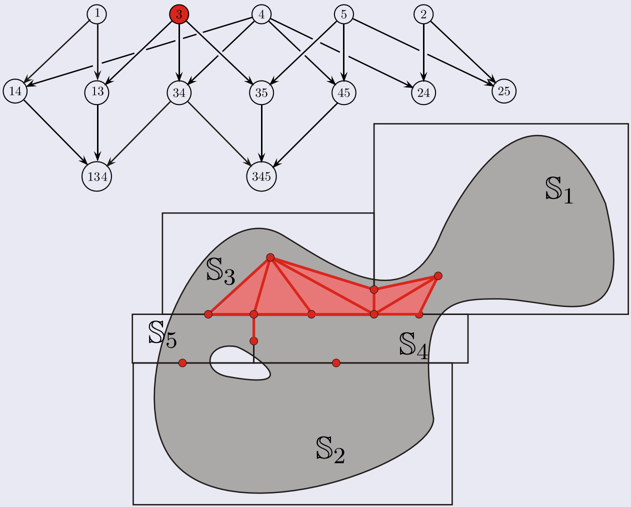
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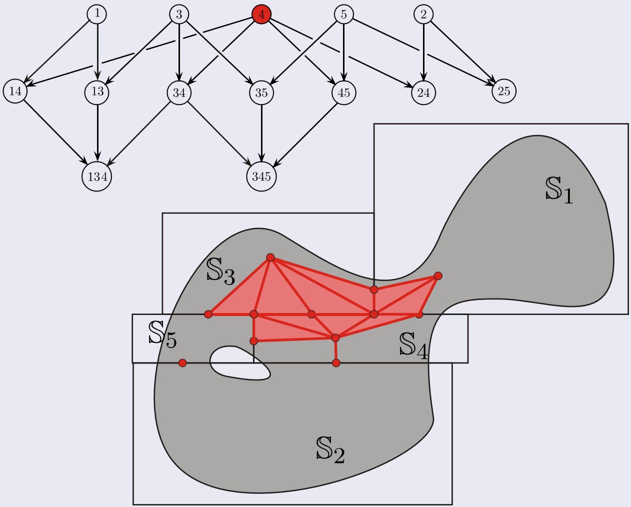
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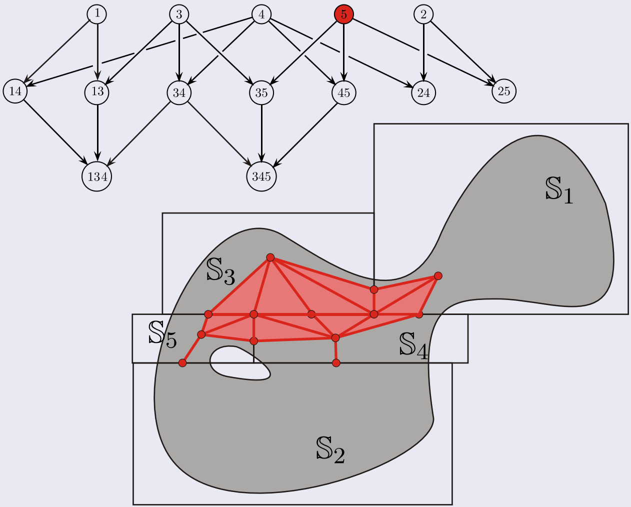
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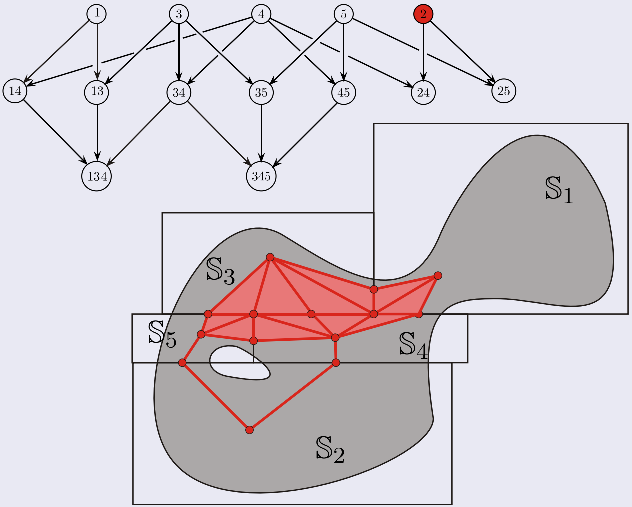
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- CIA : Connected components via Interval Analysis
  http://www.istia.univ-angers.fr/~delanoue/

- CIA : Connected components via Interval Analysis
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- HIA : Homotopy type via Interval Analysis
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What I am doing now

Stability and dynamical system with

- interval analysis.

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Stability and dynamical system with

- interval analysis.
- graph theory.

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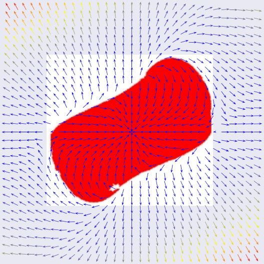


- Merci pour votre attention !

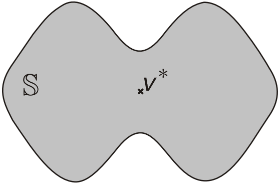
Theorem

If $\mathbb{S} = \{x \in D \subset \mathbb{R}^n \ / \ f(x) \leq 0\}$ where f is a C^1 function from D to $\mathbb{R}$, D a convex set, v^* be in $\mathbb{S}$ and if

$$f(x) = 0, \ Df(x).(x - v^*) \leq 0, \ x \in D$$

is inconsistent then v^* is star a for $\mathbb{S}$.

(1) est inconsistent $\Leftrightarrow \forall x \in D, f(x) = 0 \Rightarrow \partial f(x).(x - v^*) > 0$



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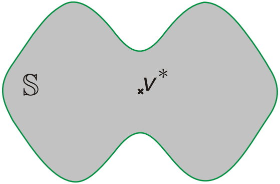

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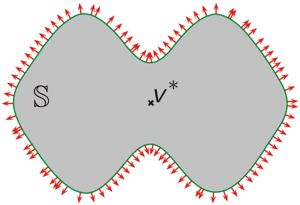
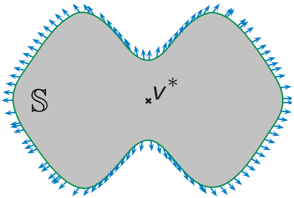


$\partial f(x)$



$x - v^*$

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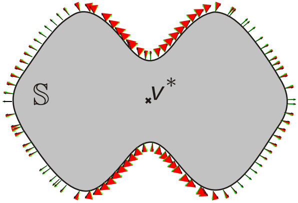

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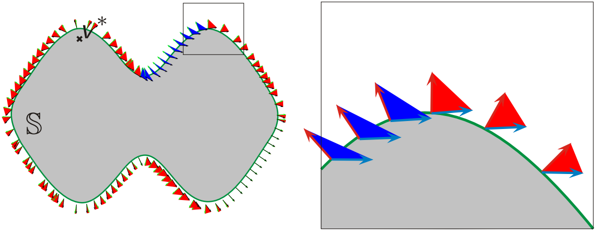
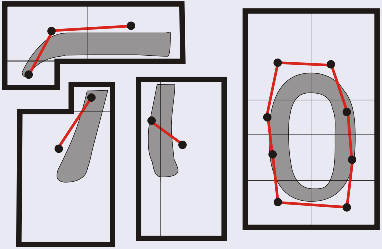


Back

Corollaire

Let $\mathcal{G}_{\mathbb{S}}$ be a star-spangled graph of a set $\mathbb{S}$.
$\mathcal{G}_{\mathbb{S}}$ has the same number of connected components than $\mathbb{S}$. i.e. $\pi_0(\mathbb{S}) = \pi_0(\mathcal{G}_{\mathbb{S}})$.



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