Classification of mapping from $\mathbb{R}^2$ to $\mathbb{R}^2$

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IPA 2012 - Intervals Pavings and Applications - Uppsala
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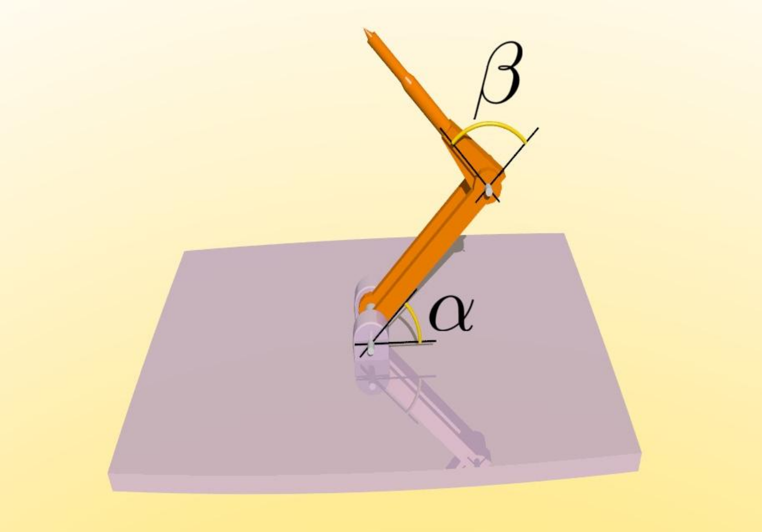
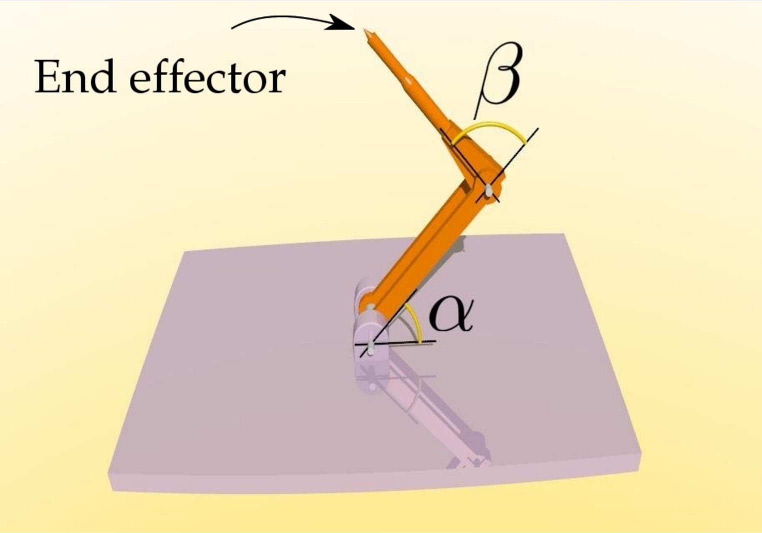
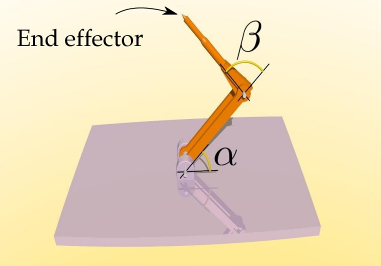
Outline


β
α


End effector
β
α



Position of the end effector depends on α and β

$$f : X \rightarrow \mathbb{R}^2$$
$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mapsto \begin{pmatrix} 2\cos(\alpha) + cos(\alpha+\beta) \\ 2\sin(\beta) + sin(\alpha+\beta) \end{pmatrix} \quad (1)$$

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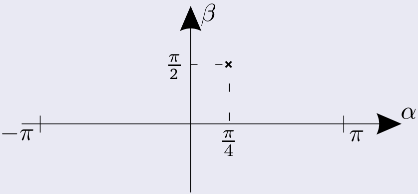
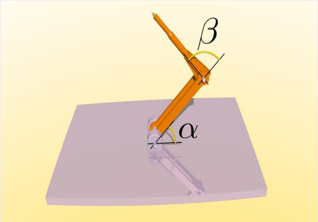


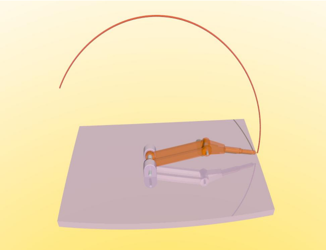
Configuration space



Working space

Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

$$f \circ \gamma = \delta$$



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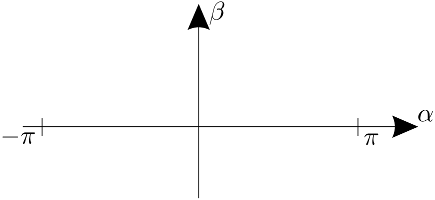

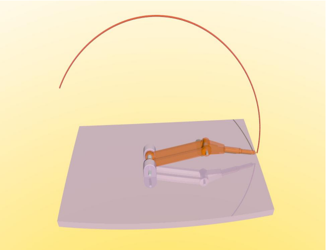

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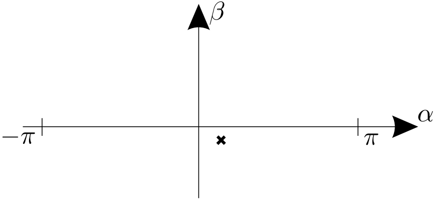

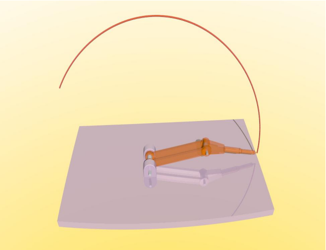

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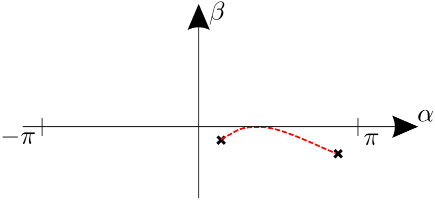

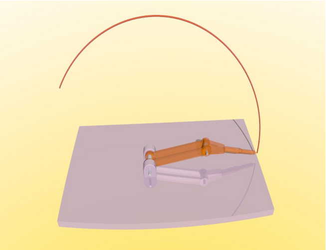

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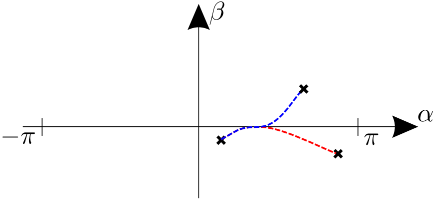
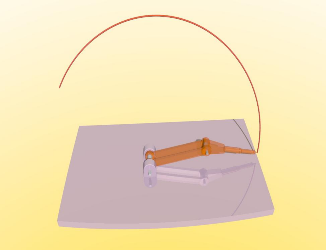
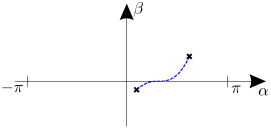
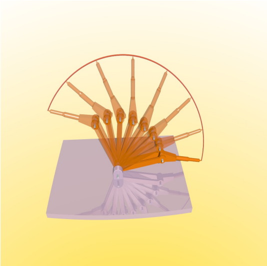
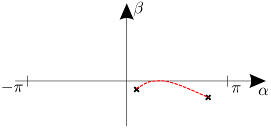





β
$-\pi$
π
α

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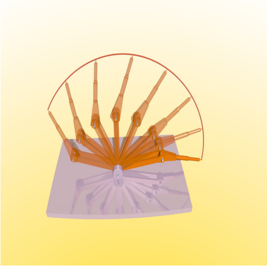
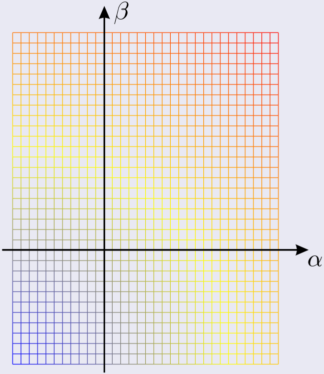
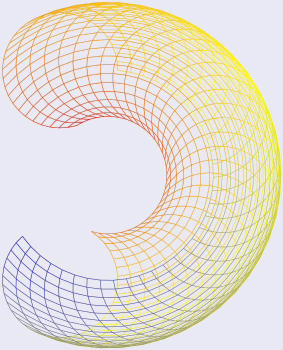


"Graph" of f





"Graph" of f

One wants a global "picture" of the map which does not depend on a choice of system of coordinates neither on the configuration space nor on the working space.

Equivalence

Let f and f' be two smooth maps. Then $f \sim f'$ if there exists diffeomorphisms $g : X \to X$ and $h : Y \to Y$ such that the diagram

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \downarrow g & & \uparrow h \\ X & \xrightarrow{f'} & Y \end{array}$$

commutes.

Examples

1. $f_1(x) = 2x + 1,\ f_2(x) = -x + 2,$
$$f_1 \sim f_2$$
2. $f_1(x) = x^2,\ f_2(x) = ax^2 + bx + c,$
$$f_1 \sim f_2$$
3. $f_1(x) = x + 1,\ f_2(x) = x^2 + 1,$
$$f_1 \not\sim f_2$$

Proposition

Let f and f' be smooth maps. Suppose that $f_1 \sim f_2$ with

$$\begin{array}{ccc} x_1 & \xrightarrow{f} & y_1 \\ \downarrow g & & \uparrow h \\ x_2 & \xrightarrow{f'} & y_2 \end{array}$$

then rank $df_{x_1} = df'_{x_2}$

Let us define the map *fold* by

$$
\begin{array}{rcccc}
folds & : & X \times X & \rightarrow & \mathbb{R}^4 \\
 & & \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} & \mapsto & \begin{pmatrix} \det df(x_1, y_1) \\ \det df(x_2, y_2) \\ f_1(x_1, y_1) - f_1(x_2, y_2) \\ f_2(x_1, y_1) - f_2(x_2, y_2) \end{pmatrix}
\end{array} \quad (2)
$$

The set $S^{\Delta 2}$ is directly connected to the map folds by the following relation

$$S^{\Delta 2} = folds^{-1}(\{0\}) - \Delta S / \sim .$$

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