

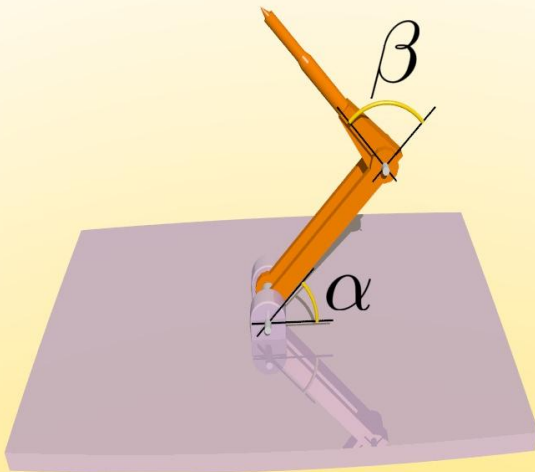
Classification of mapping from \mathbb{R}^2 to \mathbb{R}^2

Nicolas Delanoue - Sébastien Lagrange

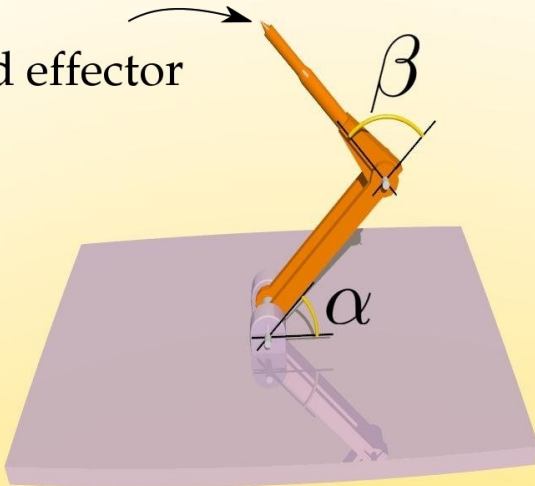
IPA 2012 - Intervals Pavings and Applications - Uppsala
<http://www.math.uu.se/ipa2012/>

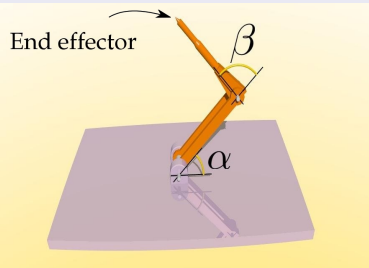
Outline

- 1 Robotics
 - Motion planning
- 2 Stable mappings and their singularities
 - Equivalence between smooth mappings
 - Genericity and Thom transversality theorem
 - Stability
 - Discretization - Portrait of a map
- 3 Interval analysis and mappings from \mathbb{R}^2 to \mathbb{R}^2 .
 - Boundary - Boundary
 - Fold - Fold
 - Cusp
 - Boundary - Fold
- 4 Algorithm computing an invariant
- 5 Conjecture and conclusion



End effector





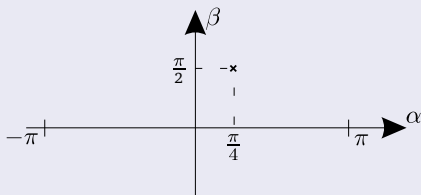
Position of the end effector depends on α and β

$$\begin{aligned}
 f : \quad X &\rightarrow \mathbb{R}^2 \\
 \begin{pmatrix} \alpha \\ \beta \end{pmatrix} &\mapsto \begin{pmatrix} 2 \cos(\alpha) + \cos(\alpha + \beta) \\ 2 \sin(\beta) + \sin(\alpha + \beta) \end{pmatrix} \quad (1)
 \end{aligned}$$

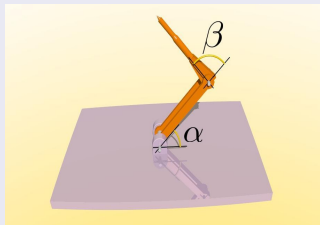
Position of the end effector depends on α and β

$$f : X \rightarrow \mathbb{R}^2$$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mapsto \begin{pmatrix} 2 \cos(\alpha) + \cos(\alpha + \beta) \\ 2 \sin(\beta) + \sin(\alpha + \beta) \end{pmatrix}$$



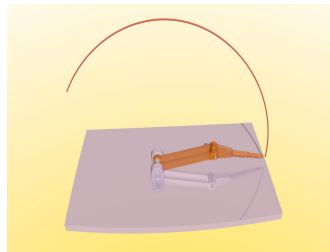
Configuration space



Working space

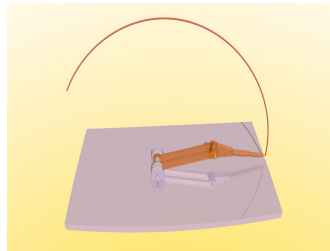
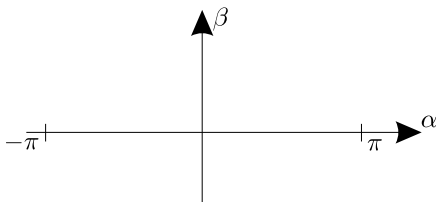
Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

$$f \circ \gamma = \delta$$



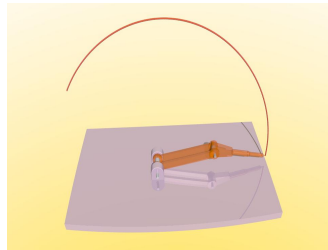
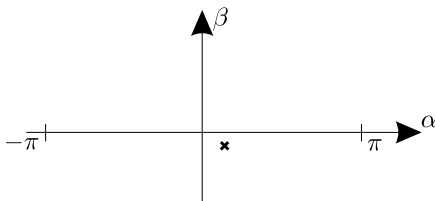
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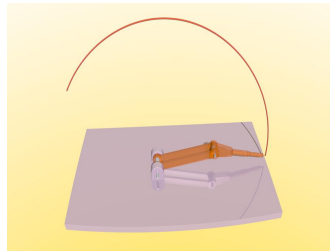
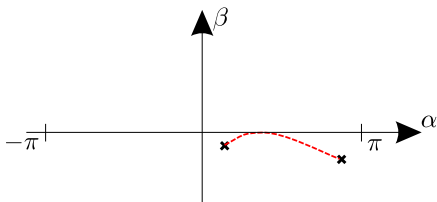
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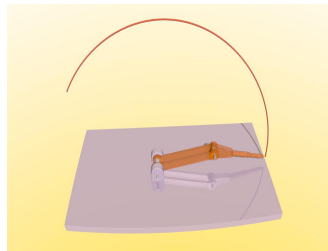
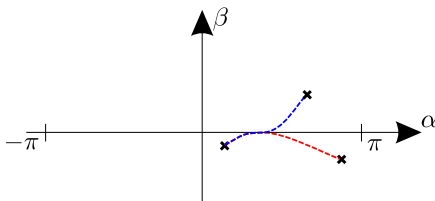
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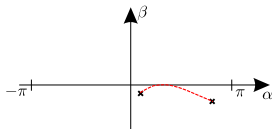
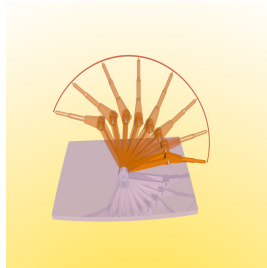
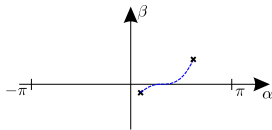
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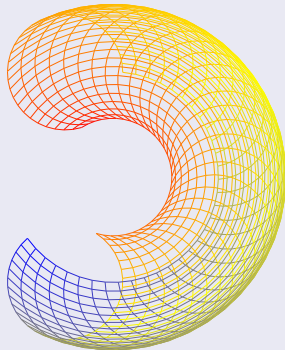
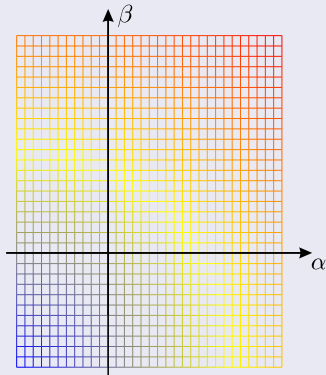


Given a path δ for the end-effector in the working space, find a curve γ in the configuration space such that

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"Graph" of f 

"Graph" of f

One wants a global "picture" of the map which does not depend on a choice of system of coordinates neither on the configuration space nor on the working space.

Equivalence

Let f and f' be two smooth maps. Then $f \sim f'$ if there exists diffeomorphisms $g : X \rightarrow X$ and $h : Y \rightarrow Y$ such that the diagram

$$\begin{array}{ccc}
 X & \xrightarrow{f} & Y \\
 \downarrow g & & \uparrow h \\
 X & \xrightarrow{f'} & Y
 \end{array}$$

commutes.

Examples

$$\textcircled{1} \quad f_1(x) = 2x + 1, \quad f_2(x) = -x + 2,$$

$$f_1 \sim f_2$$

$$\textcircled{2} \quad f_1(x) = x^2, \quad f_2(x) = ax^2 + bx + c,$$

$$f_1 \sim f_2$$

$$\textcircled{3} \quad f_1(x) = x + 1, \quad f_2(x) = x^2 + 1,$$

$$f_1 \not\sim f_2$$

Proposition

Let f and f' be smooth maps. Suppose that $f_1 \sim f_2$ with

$$\begin{array}{ccc}
 x_1 & \xrightarrow{f} & y_1 \\
 \downarrow g & & \uparrow h \\
 x_2 & \xrightarrow{f'} & y_2
 \end{array}$$

then $\text{rank } df_{x_1} = \text{rank } df'_{x_2}$

Let us define the map *fold* by

$$\begin{aligned} \text{folds} : \quad X \times X &\rightarrow \mathbb{R}^4 \\ \left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right) &\mapsto \begin{pmatrix} \det df(x_1, y_1) \\ \det df(x_2, y_2) \\ f_1(x_1, y_1) - f_1(x_2, y_2) \\ f_2(x_1, y_1) - f_2(x_2, y_2) \end{pmatrix} \end{aligned} \quad (2)$$

The set $S^{\Delta 2}$ is directly connected to the map *fold*s by the following relation

$$S^{\Delta 2} = \text{folds}^{-1}(\{0\}) - \Delta S / \sim .$$

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