Observer Synthesis for Max-plus Linear Systems : Application to Manufacturing Systems

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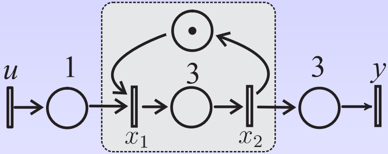
Problem Statement



(max,+) Linear Systems and Timed Event Graphs

Timed Event Graphs behavior is perfectly described by (max,+) linear systems.
The signals considered are the firing dates of each transition. Internal transitions x_i are called the state, (resp. u_i inputs, $y_i$outputs)

Questions ?

- Is it possible to compute a state estimation $\hat{x}$ by considering inputs u and output measurements y ?
- An Optimal Sub Observer is computed (Hardouin et al. 2010, IEEE TAC).
- Computation of bounds for the error between x and $\hat{x}$.

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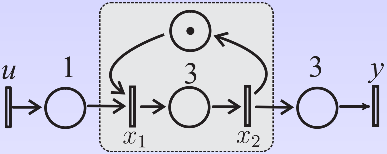

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- (max,+) algebra in few words
- Model of TEG in Idempotent Semiring
- Periodic series and Second order theory
- Sub Observer synthesis
- Performance Analysis
- Illustration
- Conclusion

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(max,+) algebra in few words

Idempotent Semiring $\mathcal{T}$

- Sum $\oplus$, associative,commutative, zero element denoted ε,
- Product $\otimes$, associative, identity element denoted e,
- Product $\otimes$ distributes with respect of sum, $(a \oplus b) \otimes c = a \otimes c \oplus b \otimes c$,
- Zero element ε is absorbing, $a \otimes \varepsilon = \varepsilon$
- The sum is idempotent, $a \oplus a = a$.
- $a \oplus b = a \vee b = a \Leftrightarrow b \preceq a \Leftrightarrow a \wedge b = b$ hence an idempotent semiring has a complete lattice structure, with (ε) as bottom element and ($T = \bigoplus_{x \in \mathcal{S}} x$) as top element.

Example :(max,+) algebra, $\overline{\mathbb{Z}}_{max}$

▸ More

Sum $\oplus$ is the operator *max*, product $\otimes$ is classical sum $+$, $\varepsilon = -\infty$ and $e = 0$, then :

$$1 \oplus 1 = 1 = max(1,1),$$
$$2 \otimes 1 = 3 = 2 + 1.$$

(max,+) algebra in few words

Fixed point equations

For order preserving (isotone) mapping, it is possible to compute fixed points $f(x) = x$.

Application : $x = ax \oplus b$

Theorem : Over a complete idempotent semiring $\mathcal{T}$, the least solution to $x = ax \oplus b$ is $x = a^*b$ with $a^* = \bigoplus_{i \in \mathbb{N}_0} a^i = e \oplus a \oplus a^2 \oplus \dots$

$*$ is called Kleene star operator.

We will denote $\mathcal{M}$ the mapping defined over $\mathcal{T}$ s.t. $\mathcal{M} : x \mapsto x^*$

(max,+) algebra in few words

Residuation Theory (Croisot 56, Blyth 05)

It allows to define a kind of inverse for order preserving mapping, defined over ordered sets.

Inequality $a \otimes x \preceq b$

Over a complete idempotent semiring $\mathcal{T}$, inequality $a \otimes x \preceq b$ admits a greatest solution , denoted, $x = a \diamond\!\!\!\backslash b$.

Example : (max,+) algebra $\overline{\mathbb{Z}}_{max}$

Inequality $3 \otimes x \preceq 5$ admits a greatest solution $x = 3 \diamond\!\!\!\backslash 5 = 5 - 3 = 2$. It achieves equality in the scalar case.

(max,+) algebra in few words

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(max,+) algebra in few words

Residuation of a closure mapping (Blyth 05)

Let $f : \mathcal{T} \to \mathcal{T}$ be a closure mapping, i.e., a mapping s.t. $f \succeq Id$ and $f \circ f = f$. Restriction of f to its image $_{\mathrm{Im}f|}f$ is residuated

Application : $x^* \preceq a$

Mapping $\mathcal{M}$ is a closure mapping. Hence $_{\mathrm{Im}\mathcal{M}|}\mathcal{M}$ is residuated.
Practically : If $a \in \mathrm{Im}\mathcal{M}$ then inequality $x^* \preceq a$ admits a greatest solution, it is given by $x = a^* = a$

Application : $x^+ \preceq a$

Mapping $\mathcal{P} : x \mapsto x^+ = \bigoplus_{i \in \mathbb{N}} x^i = x \oplus x^2 \oplus \ldots$ is a closure mapping. Hence $_{\mathrm{Im}\mathcal{P}|}\mathcal{P}$ is residuated.
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(max,+) algebra in few words

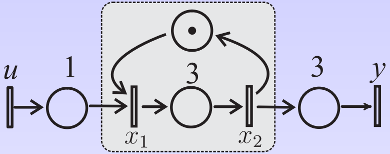
Matrix

Let A, B, C three matrices in $\overline{\mathbb{Z}}_{\max}^{n\times n}$

- $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$
- $(A \otimes B)_{ik} = \bigoplus_{j=1\ldots n} (A_{ij} \otimes B_{jk})$
- $(A \backslash\!\!\circ B)_{ik} = \bigwedge_{j=1\ldots n} (A_{ji} \backslash\!\!\circ B_{jk})$, where $A \backslash\!\!\circ B$ is the greatest matrix s.t. $AX \preceq B$
- $(B \phi A)_{ik} = \bigwedge_{j=1\ldots n} (A_{ij} \phi B_{kj})$, where $A \phi B$ is the greatest such $XA \preceq B$
- $(X)_{ij} = A^*_{ij}$ is the greatest matrix s.t. $X \preceq A^*$

More

TEG Model in $\overline{\mathbb{Z}}_{max}$



Firing Date [Cohen et al., IEEE TAC 85]

$x_i(k)$: date of the firing numbered k for the transition labelled x_i.

For each transition :

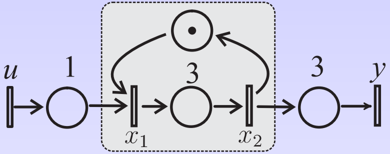
$$\begin{aligned} x_1(k) &= \max(1 + u(k), x_2(k-1)) \\ x_2(k) &= 3 + x_1(k) \\ y(k) &= 3 + x_2(k) \end{aligned}$$

In $\overline{\mathbb{Z}}_{max}$

▸ $\overline{\mathbb{Z}}_{max}$:

$$\begin{aligned} x_1(k) &= 1 \otimes u(k) \oplus x_2(k-1) \\ x_2(k) &= 3 \otimes x_1(k) \\ y(k) &= 3 \otimes x_2(k) \end{aligned}$$

TEG Model in $\overline{\mathbb{Z}}_{max}$



Firing Date [Cohen et al., IEEE TAC 85]

$x_i(k)$: date of the firing numbered k for the transition labelled i.

Dynamic Model

$$
\begin{aligned}
x(k) &= Ax(k-1) \oplus Bu(k) \\
y(k) &= Cx(k)
\end{aligned}
$$

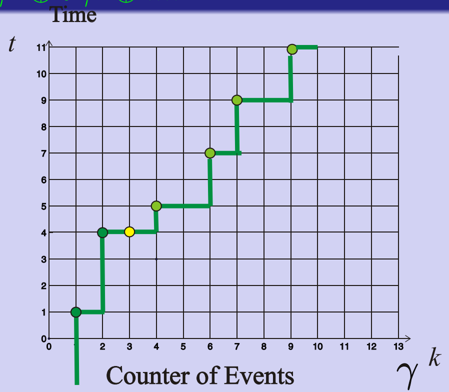
TEG Model in $\overline{\mathbb{Z}}_{max}[\![\gamma]\!]$

γ transform [Cohen, Quadrat et al. IEEE TAC 89]

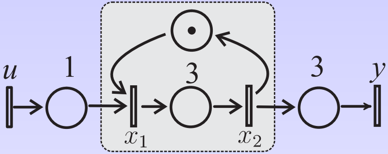
More

γ transform of $s(k)$ is a formal series $s(\gamma) = \bigoplus_{k \in \mathbb{N}} \gamma^k s(k)$. The set of series is a semiring denoted $\overline{\mathbb{Z}}_{max}[\![\gamma]\!]$. A series with a finite support is called a polynomial, and a monomial if there is only one element.

$s = 1\gamma \oplus 4\gamma^2 \oplus 5\gamma^4 \oplus 7\gamma^6 \oplus \ldots$



TEG Model in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$



The previous system in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$:

$$
\begin{aligned}
x(\gamma) &= Ax(\gamma) \oplus Bu(\gamma) &= \begin{pmatrix} \varepsilon & \gamma \\ 3 & \varepsilon \end{pmatrix} x(\gamma) \oplus \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix} u(\gamma) \\
y(\gamma) &= Cx(\gamma) &= \begin{pmatrix} \varepsilon & 3 \end{pmatrix} x(\gamma)
\end{aligned}
$$

The previous system in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$: ▸ More

$$
\begin{aligned}
x(\gamma) &= A^* Bu(\gamma) &= \begin{pmatrix} (3\gamma)^* & \gamma(3\gamma)^* \\ 3(3\gamma)^* & (3\gamma)^* \end{pmatrix} \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix} u(\gamma) \\
y(\gamma) &= CA^* Bu(\gamma) &= \begin{pmatrix} 7(3\gamma)^* \end{pmatrix} u(\gamma)
\end{aligned}
$$

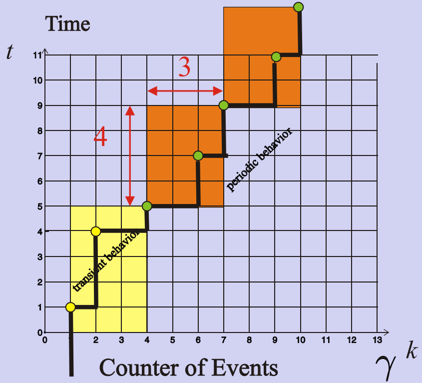
Semiring of periodic series (Cohen, Quadrat et al. IEEE TAC 89)

A periodic series in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$

Back

$s = p \oplus q(\tau\gamma^{\nu})^*$ where $p = \bigoplus_i t_i \gamma^{n_i}$ and $q = \bigoplus_j t_j \gamma^{n_j}$ are polynomials and $\sigma_\infty(s) = \nu/\tau$ is the asymptotic slope (the throughput).

$s = (1\gamma \oplus 4\gamma^2) \oplus (5\gamma^4 \oplus 7\gamma^6)(4\gamma^3)^*$ and $\sigma_\infty(s) = 3/4$



Semiring of periodic series (Cohen, Quadrat et al. IEEE TAC 89)

Operations over semiring of periodic series over $\overline{\mathbb{Z}}_{max}[\![\gamma]\!]$

- $s = s_1 \oplus s_2$ is a periodic series, asymptotic slope $\sigma_\infty(s) = min(\sigma_\infty(s_1), \sigma_\infty(s_2))$
- $s = s_1 \otimes s_2$ is a periodic series, asymptotic slope $\sigma_\infty(s) = min(\sigma_\infty(s_1), \sigma_\infty(s_2))$
- $s = s_1 \wedge s_2$ is a periodic series, asymptotic slope $\sigma_\infty(s) = max(\sigma_\infty(s_1), \sigma_\infty(s_2))$
- $s = s_1 \circ\!\!\!\backslash s_2$ is a periodic series, $\sigma_\infty(s) = \sigma_\infty(s_2)$ if $\sigma_\infty(s_2) \leq \sigma_\infty(s_1)$ else $s = \varepsilon$.

Software Tools

Software allowing to handle periodic series is available on :
http://www.istia.univ-angers.fr/~hardouin/outils.html

Second order theory (MAXPLUS, IEEE CDC 91)

Counter associated to a series

Let $s = \bigoplus_{k\in\mathbb{Z}} s(k)\gamma^k$ be a series and $\mathcal{C}_s$ the counter function associated to s defined by $s = \bigoplus_{t\in\mathbb{Z}} t\gamma^{\mathcal{C}_s(t)}$.

Distance in the event domain between 2 series (Santos Mendes et al. ETFA 05)

Let s_1 and s_2 be two series, the distance in the event domain is denoted $\Delta_{s_1 s_2}$ and defined by

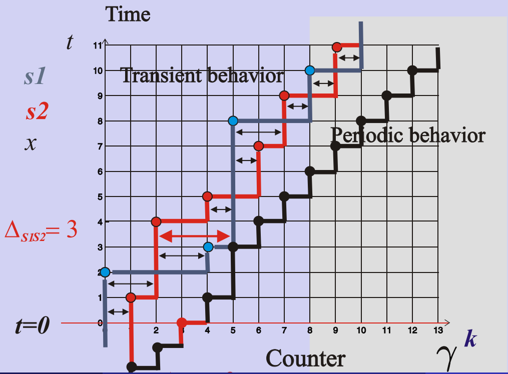
$$\Delta_{s_1 s_2} = max\{|\mathcal{C}_{s_1}(t) - \mathcal{C}_{s_2}(t)| \text{ s.t. } t \in \mathbb{Z}\}$$

it can be evaluated by considering
$\Delta_{s_1 s_2} = \mathcal{C}_{d_{12}}(0)$ where $d_{12} = (s_1 \wedge s_2) \not\circ (s_1 \oplus s_2)$

Distance in the event domain (Santos Mendes et al. ETFA 05)

Illustration : practical computation of the distance

Let $s_1 = 2 \oplus 3\gamma^4 \oplus 8\gamma^5 \oplus 10\gamma^8(2\gamma^2)^*$ and $s_2 = 1\gamma \oplus 4\gamma^2 \oplus 5\gamma^4 \oplus 7\gamma^6 \oplus 9\gamma^7(2\gamma^2)^*$ be two series. Series $d_{12} = (s_1 \wedge s_2) \phi (s_1 \oplus s_2) = -2\gamma \oplus -1\gamma \oplus 0\gamma^3 \oplus 1\gamma^4 \oplus 3\gamma^5(1\gamma)^*$ and the associated counter $\Delta_{s_1 s_2} = \mathcal{C}_{d_{12}}(0) = 3$.



Distance in the event domain (Santos Mendes et al. ETFA 05)

Application : bound computation for the difference between firing of two transitions subject to the same inputs

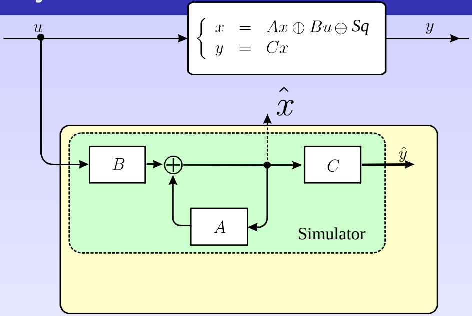
Let $x_1 = s_1 u$ and $x_2 = s_2 u$ two series describing the behavior of two states. The distance between these trajectories can be computed by considering $du_{12} = (s_1 u \wedge s_2 u) \phi (s_1 u \oplus s_2 u) = (x_1 \wedge x_2) \phi (x_1 \oplus x_2)$. We can prove that $du_{12} \succeq (s_1 \wedge s_2) \phi (s_1 \oplus s_2) = d_{12}$, hence $\mathcal{C}_{du_{12}}(0) \leq \mathcal{C}_{d_{12}}(0)\ \forall u$ and consequently

$$\Delta_{x_1 x_2} \leq \mathcal{C}_{d_{12}}(0)\ \forall u.$$

Corollary

Extension in the matrix case is straight forward, the bound for the event difference between two transitions will be the maximum for each entry.

Sub Observer Synthesis :

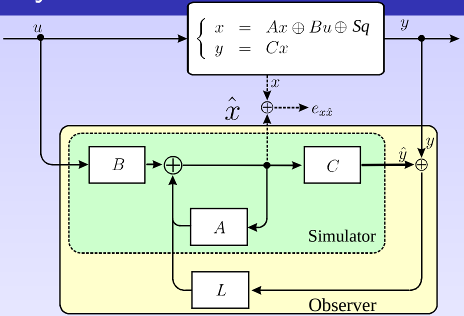


Objective :

Compute the greatest $\hat{x}$ such that

$$\hat{x} \preceq x.$$

Sub Observer Synthesis :

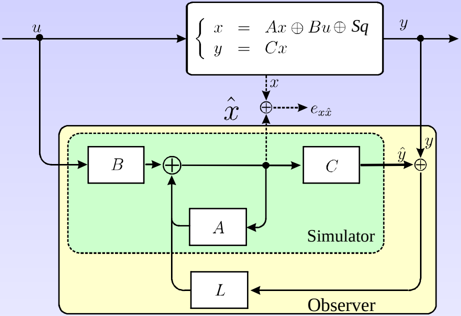


Objective :

Compute the greatest observer matrix L such that

$$\hat{x} \preceq x.$$

Sub Observer Synthesis :



System Equations :

▸ Matrix S

$$\begin{aligned} x &= Ax \oplus Bu \oplus Sq = A^* Bu \oplus A^* Sq \\ y &= Cx = CA^* Bu \oplus CA^* Sq. \end{aligned}$$

Estimated State Equations :

$$\begin{aligned} \hat{x} &= A\hat{x} \oplus Bu \oplus L(\hat{y} \oplus y) \\ \hat{y} &= C\hat{x}. \end{aligned}$$

Sub Observer Synthesis :

Constraints Satisfaction :

Compute the greatest observer matrix L such that

$$\begin{array}{llll} (A \oplus LC)^* Bu & \preceq & A^* Bu & \forall u \\ (A \oplus LC)^* LCA^* Sq & \preceq & A^* Sq & \forall q, \end{array}$$

Constraints Satisfaction :

Compute the greatest matrix L such that

$$\begin{array}{lll} (A \oplus LC)^* B & \preceq & A^* B \Leftrightarrow L \preceq (A^* B)\phi\!\!/(CA^* B) \\ (A \oplus LC)^* LCA^* S & \preceq & A^* S \Leftrightarrow L \preceq (A^* S)\phi\!\!/(CA^* S). \end{array}$$

Sub Observer Synthesis :

Optimal Matrix : (Hardouin et al. IEEE TAC 2010)

$$L_{opt} = ((A^*B) \phi (CA^*B)) \wedge ((A^*S) \phi (CA^*S))$$

is the greatest such that

$$\hat{x} \preceq x.$$

Performance Analysis :

Equality of the asymptotic slope (Hardouin et al. IEEE TAC 2010)

If matrix C linking state vector to the output is connected to all connected components of the graph then

$$\sigma_\infty(\hat{x}_i) = \sigma_\infty(x_i)\ \forall i$$

Distance between x_i and $\hat{x}_i$

$$d_{x\hat{x}_i} = (x_i \wedge \hat{x}_i) \phi\!\!/ (x_i \oplus \hat{x}_i)$$

From this vector it is possible to establish distance $\Delta_{\hat{x}_i x_i}$, and results given previously allow to get a bound

Bounded Error, sufficient condition

If state x_i belongs to a connected component whose at least one transition is measured then $\Delta_{\hat{x}_i x_i} \in \mathbb{N}_0$.

Conclusion :

Next works :

- may be used in a state feedback control strategy
- may be used in a fault detection structure for manufacturing systems
- extension in semiring of intervals (DiLoreto et al. 2009)
- in a future work it will be compared with a particular filter used to state estimation of non linear systems.

Scilab Toolboxes and animation

- http://www.istia.univ-angers.fr/~hardouin
- http://cermics.enpc.fr/~cohen-g//SED/book-online.html

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References 1 :

(Cohen et al. IEEE TAC 85)

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title=A linear system theoretic view of discrete event processes and its use for performance evaluation in manufacturing,
journal=IEEE Trans. on Automatic Control,
volume=AC–30,
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year =2010,
month=February,
volume=55-2,
title =Observer Design for (max,plus) Linear Systems,
journal =IEEE Transactions on Automatic Control,
note=istia.univ-angers.fr/~hardouin/Observer.html

DiLoreto et al. 2009

author=M. DiLoreto and S. Gaubert and R. Katz and J-J Loiseau,
title=Duality between invariant spaces for max-plus linear discrete event systems,
note=http://fr.arXiv.org/abs/0901.2915,
year= 2009,

(max,+) algebra in few words

Sum of matrices $A \oplus B = C$

Back

$$\begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix} \oplus \begin{pmatrix} e & 8 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ 3 & 7 \end{pmatrix}$$

Product of matrices $A \otimes B = C$

Back

$$\begin{pmatrix} 2 & 5 \\ \varepsilon & 3 \\ 1 & 8 \end{pmatrix} \otimes \begin{pmatrix} e \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \otimes e \oplus 5 \otimes 1 \\ \varepsilon \otimes e \oplus 3 \otimes 1 \\ 1 \otimes e \oplus 8 \otimes 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 9 \end{pmatrix}$$

Residuation of matrices $A \backslash\!\!\!\circ B$ is the greatest solution of $A \otimes X \preceq B$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \backslash\!\!\!\circ \begin{pmatrix} 8 \\ 9 \\ 10 \end{pmatrix} = \begin{pmatrix} (1 \backslash\!\!\!\circ 8) \wedge (3 \backslash\!\!\!\circ 9) \wedge (5 \backslash\!\!\!\circ 10) \\ (2 \backslash\!\!\!\circ 8) \wedge (4 \backslash\!\!\!\circ 9) \wedge (6 \backslash\!\!\!\circ 10) \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

TEG Model in $\overline{\mathbb{Z}}_{\max}$

$\overline{\mathbb{Z}}_{\max}$

$\overline{\mathbb{Z}}_{\max} = (\mathbb{Z} \cup \{-\infty, +\infty\}, \max, +)$ is an idempotent semiring.

$$(\forall a \in \overline{\mathbb{Z}}_{\max}, a \oplus a = a)$$

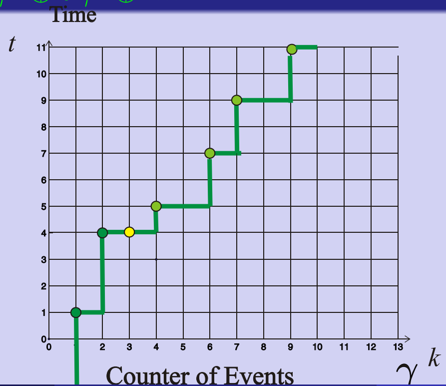
Back

Series in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$

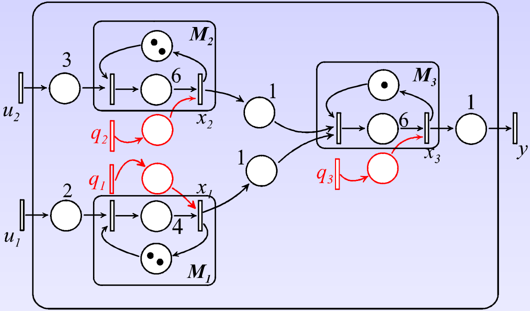
◂ Back

A series : $s = \bigoplus_{k \in \mathbb{Z}} s(k)\gamma^k$ codes a non decreasing trajectory. The set of series is a semiring denoted $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$. A series with a finite support is called a polynomial, and a monomial if there is only one element.

$s = 1\gamma \oplus 4\gamma^2 \oplus 5\gamma^4 \oplus 7\gamma^6 \oplus \ldots$



Sub Observer Synthesis :



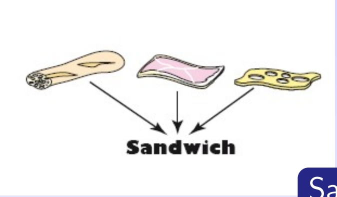
Matrix S and input q :

Back

- vector q represents a vector of exogenous uncontrollable inputs (disturbance) which act on the system through matrix S.
- These disturbances lead to disable the transition firing, that is they decrease system performances and delay tokens output.

(max,+) algebra in few words

Back



Sandwiches Algebra [Cohen et al.]

1 piece of Bread + 1 slice of ham + 1 slice of cheese is equal to 1 sandwich. Another way of counting !

A periodic series in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$

Back

$s = p \oplus q(\tau\gamma^{\nu})^*$ where $p = \bigoplus_i t_i\gamma^{n_i}$ and $q = \bigoplus_j t_j\gamma^{n_j}$ are polynomials and $\sigma_\infty(s) = \nu/\tau$ is the throughput.

$s = (1\gamma \oplus 4\gamma^2) \oplus (5\gamma^4 \oplus 7\gamma^6)(4\gamma^3)^*$ and $\sigma_\infty(s) = 3/4$