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An Active Anechoic Termination for Low Frequencies with Mean Flow

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Summary
This paper deals with the behavior of an active anechoic termination for low frequencies (<150 Hz) in the presence of mean flow. The auxiliary source is an oscillating valve positioned within the flow which can be considered as an acoustic source in series with a primary source. This system was previously used to eliminate flow periodic fluctuations in exhaust from reciprocating engines. The modeling (briefly recalled here) is based on the propagation of plane waves in flow, the principle of the superposition of linear acoustics, and electroacoustic analogies. It is first shown that the reflection coefficient can be minimized for a uniform duct in the case of a fixed valve positioned in incidence. It is then demonstrated that the reflection coefficient can be controlled by oscillating the valve to provide an active anechoic termination. This active method allows the passive influence of the valve to be reduced in an acoustic circuit through limitation of head-loss. The theoretical study was validated by an experimental application confirming the viability of this low-frequency termination. A 5% lower reflection coefficient was obtained from 40 to 110 Hz, with a Mach number of around 0.05.

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1. Introduction

The use of anechoic termination is essential in various acoustic experiments, e.g. normalized measurements of the transmission loss (TL) of mufflers. This type of information can be useful for the design and optimization of these exhaust systems. However, it is very difficult to achieve efficient anechoic termination at low frequencies (<200 Hz) in the presence of flow. Classically, an exponential open-end duct covered with absorbant material is used. This system is efficient for wavelengths shorter than the thickness of the material, so considerable space would be required for the absorption of low frequencies.

The few studies performed in this field have concerned ducts with low-frequency traveling waves, notably the work (Dalmont *et al.*, 1989) in adapting impedance within a cylindrical duct by means of a microchannel plate and a coupling adapter. The principle is to use the acoustic resistance provided by a bundle of capillary tubes. This method allows a reflection coefficient of less than 5% to be obtained between 0 and 2000 Hz. Other studies have employed active methods. Thenail and Galland (Thenail and Galland, 1992) used a loudspeaker positioned at the end of the duct to eliminate reflected waves. The principle in this case was to control the loudspeaker in order to obtain local absorption of reflected waves, i.e. acoustic pressure p and velocity u near the loudspeaker were related by the equation $p = \rho c u$, where ρc is the specific impedance of the air. Acoustic velocity was measured on the loudspeaker membrane by means of an accelerometer. A similar method developed by Haw et al.

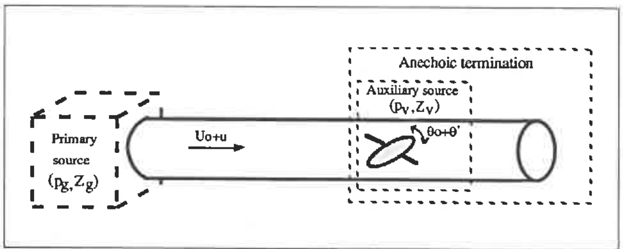

Figure 1. A schematic diagram of the active anechoidal termination.

(Haw *et al.*, 1992) for the characterization of materials used in underwater acoustics consists in direct minimization of the reflection coefficient determined by the two-microphone method.

In fact, we found no description in the literature of a device limiting the reflection of low-frequency waves propagating in a flow which could be easily integrated into an acoustic circuit without perturbation, particularly within exhaust systems which constitute a corrosive environment.

The present study proposes a solution consisting in the development of an active anechoic termination involving an auxiliary source generating high acoustic pressure levels at low frequencies in the presence of flow. This secondary source is particularly robust since it consists of an oscillating valve positioned within the flow (Figure 1). This system was previously used to eliminate flow periodic fluctuations in exhaust from reciprocating engines (Hardouin *et al.*, 1993) which cause considerable noise pollution. In these conditions, it was shown that the oscillating valve behaves like an active termination characterized by a perfect reflection of the incident wave generated by the primary source. The purpose of the present study is to use this same auxiliary source to

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abolish low-frequency waves reflected by an open-end duct in the presence of flow.

The modeling of this system, as proposed in (Hardouin, 1993), is based on one-dimensional propagation of plane waves in the presence of flow and the principle of the superposition of linear acoustics. Classically, the study of acoustic circuits and mufflers (Munjal, 1987) involves the use of electroacoustic analogies consisting in the replacement of voltage and current by respectively acoustic pressure p and mass velocity q. In this formulation, an acoustic source (primary source) is associated with a Thévenin generator characterized by a source term p_g and an internal impedance Z_g. Each passive element of the circuit is represented by a transfer matrix. The boundary conditions are modeled by a radiation impedance Z_0.

The auxiliary source is modeled as a serial Thévenin generator within the circuit. Its internal impedance Z_v represents the passive influence of the valve as a function of the induced mean head-loss coefficient Λ_0, which is itself a nondimensional function of mean incidence θ_0. This gives a passive element with variable impedance. The source term p_v is a function of the fluctuations λ of the head-loss coefficient due to the harmonic oscillations of the valve.

The first part of this article provides a brief description of the modeling of the system using an oscillating valve in pulsed flow as previously developed by the Laboratoire d'Etudes Aérodynamiques. It is then demonstrated that the insertion of a valve at a fixed incidence within a duct modifies the reflection coefficient of the open-end, and that adaptation of passive impedance can be achieved in certain conditions. However, the major drawback with this approach is that the load is too large, making the method unusable for exhaust systems. Thus, we subsequently propose an active anechoic termination and provide an analysis of its passive influence within the circuit. Finally, an experimental study is presented to validate the theoretical results obtained by modeling and to illustrate the efficacy of the termination proposed.

2. System modeling

This first part of the paper provides a brief reminder of the modeling of the active system; this being composed of a primary source connected to a cylindrical duct of uniform cross-section including an oscillating valve (Figure 1). We consider this active system as a series of passive and active elements. So, first, we propose the electroacoustic representation of each composing the system in particular the oscillating valve considered as an auxiliary source. Then the active system is modeled using the principle of the superposition of linear acoustics.

2.1. Transfer matrix of a cylindrical duct in the presence of flow

The harmonic solutions for acoustic pressure p and mass velocity $q\,(=\rho S u)$, relative to the one-dimensional equation for the propagation of a plane wave with a pulsation ω in an isentropic medium in the presence of flow, can be expressed as:

$$p(x,t) = \left[p_f(x) + p_r(x)\right] \mathrm{e}^{\mathrm{j}\omega t} \tag{1}$$

$$= \left[p_f(0)\,\mathrm{e}^{-\mathrm{j}k^+x} - p_r(0)\,\mathrm{e}^{\mathrm{j}k^-x}\right] \mathrm{e}^{\mathrm{j}\omega t}$$

$$= \frac{1}{Z_c}\left[p_f(0)\,\mathrm{e}^{-\mathrm{j}k^+x} - p_r(0)\,\mathrm{e}^{\mathrm{j}k^-x}\right] \mathrm{e}^{\mathrm{j}\omega t} \tag{2}$$

where $p_f(x)$ and $p_r(x)$ represent respectively the incident and reflected waves, the direction of the axis x being that indicated by the direction of flow. k^+ (respectively k^-) is the complex wave number for waves propagating in the direction (respectively in the opposite direction) of flow:

$$k^+ = \frac{k_0 - \mathrm{j}\alpha(M)}{1+M} \tag{3}$$

$$k^- = \frac{k_0 - \mathrm{j}\alpha(M)}{1-M}. \tag{4}$$

$k_0 = \omega/c$ is the wave number for an inviscous stationary medium, c the local sound velocity and M the Mach number. The factors $1 \pm M$ indicate the convection due to mean flow. $\alpha(M)$ represents the total attenuation of the wave in a viscous medium in movement:

$$\alpha(M) = \alpha + \xi M \tag{5}$$

where α is a constant characterizing the dissipation of the wave in a viscous flow, and ξ a coefficient of wall friction.

Z_c is the characteristic impedance of a duct defined by:

$$Z_c = Z_{co}(1 - \frac{\alpha(M)}{k_0} + \mathrm{j}\frac{\alpha(M)}{k_0} \tag{6}$$

with $Z_{co} = c/S$ being the characteristic impedance for an inviscous stationary medium in a uniform cylindrical duct with a cross-section S.

The reflection coefficient R is also defined at the point of the abscissa x by:

$$R(x) = p_f(x)/p_r(x). \tag{7}$$

Thanks to equations (1) and (2), the matrix transfer linking state variables p and q is established between two cross-sections (1 and 2) of a uniform duct separated by a length L:

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \mathrm{e}^{-\mathrm{j}k_c ML} \begin{bmatrix} \cos k_c L & \mathrm{j}Z_c \sin k_c L \\ \mathrm{j}\frac{\sin k_c L}{Z_c} & \cos k_c L \end{bmatrix} \begin{bmatrix} p_2 \\ q_2 \end{bmatrix} \tag{8}$$

with

$$k_c = \frac{k_0 - \mathrm{j}\alpha(M)}{1-M^2}. \tag{9}$$

If the sound wave progresses in the direction opposed to flow, the matrix becomes:

$$\begin{bmatrix} p_1 \\ q_1 \end{bmatrix} = \mathrm{e}^{\mathrm{j}k_c ML} \begin{bmatrix} \cos k_c L & \mathrm{j}Z_c \sin k_c L \\ \mathrm{j}\frac{\sin k_c L}{Z_c} & \cos k_c L \end{bmatrix} \begin{bmatrix} p_2 \\ q_2 \end{bmatrix} \tag{10}$$

2.2. Modeling of the primary source

In a standing wave analysis, a source of acoustic energy, as a compressor or an engine, is classicaly (Boden and Abom, 1995) characterized by means of the electroacoustic analogies. This approach makes it possible to represent the acoustic source as a Thevenin generator characterized by a source pressure p_g and an internal impedance Z_g, connected to a load represented by its impedance Z_l (Figure 2a). The pressure at the level of this load is expressed as:

$$p_l = p_f + p_r = p_g \frac{Z_l}{Z_g + Z_l} \tag{11}$$

and the mass velocity as

$$q_l = \frac{p_f - p_r}{Z_c} = \frac{p_g}{Z_g + Z_l}. \tag{12}$$

p_f is the incident wave generated by the source, and p_r the wave reflected by the load and thus incident on the generator. The combination of equations (11) and (12) gives the expression of the incident wave p_f as a function of p_g and p_r:

$$p_f = p_g \frac{Z_c}{Z_g + Z_c} + p_r \frac{Z_g - Z_c}{Z_g + Z_c}. \tag{13}$$

The first term of the right-hand member expresses the pressure generated by the acoutic source connected to the characteristic impedance Z_c, i.e. to an infinite-length duct or one equiped with an anechoic termination. The second term characterizes the wave reflected by the impedance of source Z_g if p_r is incident on it. We may recall that the reflection coefficient associated with the load resistance Z_l is defined as follows:

$$R = \frac{Z_l - Z_c}{Z_l + Z_c}. \tag{14}$$

2.3. Auxiliary source

Like the primary source, the auxiliary one, consisting of an oscillating valve positioned in flow, is represented by an internal impedance Z_v and a pressure source p_v. The movement of the valve consists of a mean incidence θ_0 and a fluctuating incidence θ':

$$\theta(t) = \theta_0 + \theta'. \tag{15}$$

The oscillation of the valve in an initially steady flow causes an instantaneous variation in head-loss and resulting fluctuations of physical variables (pressure, velocity, etc.) directly correlated with the movement $q(t)$ of the valve. This variation in head-loss is represented by an nondimensional coefficient $\Lambda(t)$ relating the difference in instantaneous static pressure upstream and downstream from the valve with a dynamic reference pressure $\rho_0 U_0^2/2$ (ρ_0 being the density of the fluid and U_0 the mean velocity of flow through a straight section of the duct). This nondimensional coefficient $\Lambda(t)$ can also be expressed in the following form:

$$\Lambda(t) = \Lambda_0 + \lambda. \tag{16}$$

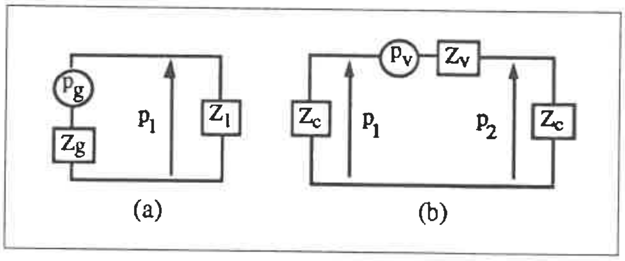

Figure 2. Modelling of the sources: a) The primary source (p_g, Z_g), b) the auxiliary source: an oscillating valve (p_v, Z_v) in an infinite length duct.

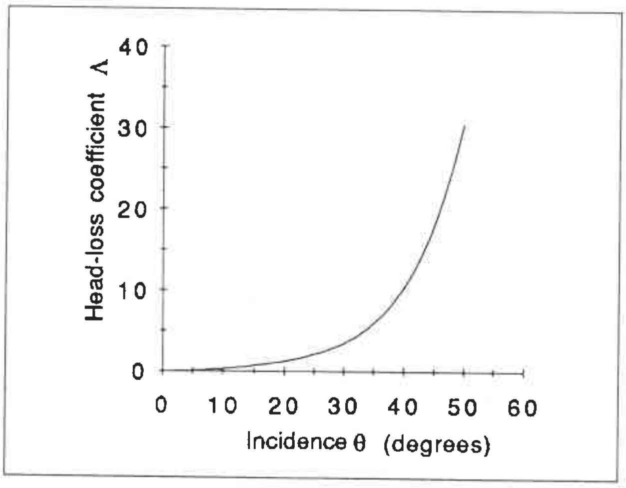

Figure 3. Changes in the head-loss coefficient Λ as a function of incidence θ for a circular valve positioned in a cylindrical duct with the same diameter.

For a circular valve positioned in a cylindrical duct of uniform diameter, the head-loss coefficient is a nonlinear universal function of incidence. This function $\Lambda = F(\theta)$ represented in Figure 3, can be approached by a polynomial relation. However, for low amplitudes of valve movement, the principle of linearization around the mean incidence θ_0 is used (Laumonier, 1990).

To model this source, Hardouin used one-dimensional continuity and momentum equations as well as the first principle of thermodynamics (Hardouin, 1992; Hardouin, 1993); these equations being applied to the fixed domain around the valve. Supposing also an isentropic evolution, a homogeneous medium with a low Mach number ($M^2 \ll 1$) and a relatively low generated head- loss, the oscillating valve is considered to be an acoustic source possessing the following characteristics:

$$\text{Pressure source:} \quad p_v = \lambda \rho_0 \frac{U_0^2}{2} \tag{17}$$

$$\text{Internal impedance:} \quad Z_v = \Lambda_0 M Z_c. \tag{18}$$

The internal impedance Z_v of this generator reflects the passive influence of the oscillating valve which is related to the mean head-loss Λ_0 which it induces. The use of this source in an acoustic circuit in the presence of flow thus creates an additional load which it is necessary to control.

The weight of the generator p_v corresponds to the product of the fluctuating head-loss coefficient λ and dynamic pressure $p_0 U_0^2/2$. The presence of the Mach number in these equations indicates that the oscillating valve behaves like an acoustic source in the presence of flow only. Assuming that this source is positioned in an infinite-length duct, the electrical representation shown in Figure 2b can be associated with it by analogy. p_1 and p_2 represent the fluctuating pressures generated upstream and downstream from the oscillating valve. These pressures can be expressed as follows:

$$p_1 = \frac{Z_c}{2Z_c + Z_v} p_v \tag{19}$$

$$p_2 = -\frac{Z_c}{2Z_c + Z_v} p_v . \tag{20}$$

Equations (17) and (18) clearly show that in the absence of movement ($\theta = \theta_0$, i.e. $\Lambda = \Lambda_0$ and $\lambda = 0$) the valve is characterized by an impedance $Z_v = \Lambda_0 M Z_c$ in series with the system. Thus, the transfer matrix characterizing the influence of the stationary value is as follows:

$$\begin{bmatrix} p \\ q \end{bmatrix}_1 = \begin{bmatrix} 1 & \Lambda_0 M Z_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}_2 . \tag{21}$$

2.4. System modeling in the presence of the two sources

The active system shown in Figure 4 is composed of a primary source and an oscillating valve positioned in a duct of uniform cross-section with an open-end characterized by the radiation impedance Z_0. This oscillating valve plays the role of an auxiliary source. It is governed by a feedforward filter which calculates the control by means of a pressure signal picked up at a point "i" upstream from the auxiliary source. This filter is represented by its transfer function H from which is derived the expression of the generative pressure of the valve: $p_v = H p_i$.

The equivalent electrical scheme for this system is given in Figure 5a. L_g, L_i and L_o are respectively the distances primary source/point "i", point "i"/auxiliary source and auxiliary source/duct end.

The principle of the superposition of linear acoustics allows us to consider the behavior of the primary source (p_g, Z_g) with the fixed valve $(p_v = 0, Z_v)$, then the influence of the oscillating valve (p_v, Z_v) in steady flow $(p_g = 0, Z_g)$ and finally the superposition of the two configurations.

2.4.1. Case of the fixed valve ($p_v = 0$)

This configuration is given by the electrical formulation in Figure 5b. The following is the transfer equation characterizing this system:

$$\begin{bmatrix} p_g \\ q_g \end{bmatrix} = \begin{bmatrix} 1 & Z_g \\ 0 & 1 \end{bmatrix} \begin{bmatrix} Mat \\ L_g \end{bmatrix}_i \begin{bmatrix} Mat \\ L_i \end{bmatrix}_1 \times \begin{bmatrix} 1 & Z_v \\ 0 & 1 \end{bmatrix}_2 \begin{bmatrix} Mat \\ L_o \end{bmatrix} \begin{bmatrix} 1 & Z_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ q_0 \end{bmatrix} \tag{22}$$

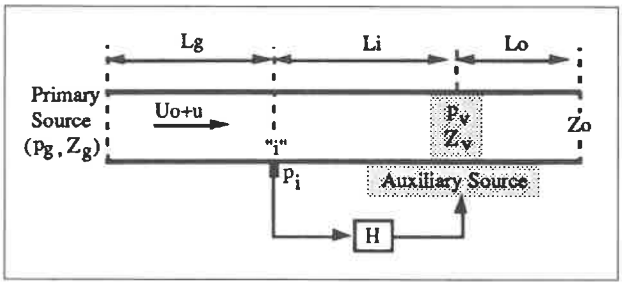

Figure 4. A schematic diagram of the active control device with a feedforward filter.

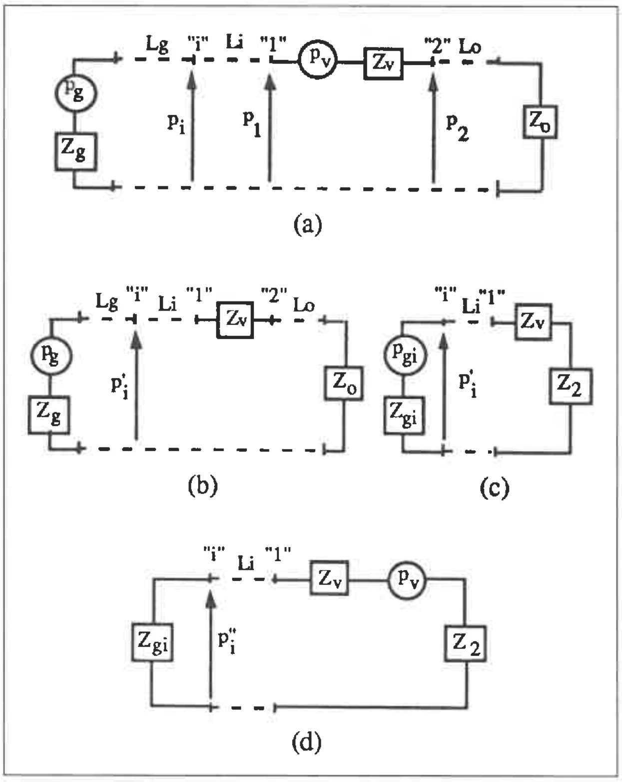

Figure 5. Analogic representation of the active system.
a) The two sources on; b), c) the auxiliary source off ($p_v = 0$);
d) the primary source off ($p_g = 0$).

where each matrix $\begin{bmatrix} Mat \\ L \end{bmatrix}$ is the transfer matrix of a uniform duct of length L when the wave moves in the direction of the flow. This matrix is defined by the relation (8).

By transferring the characteristics of the primary source in "i" (then referred to as p_{gi} and Z_{gi}) and by making the load resistance Z_i appear at the level of point "i", the simplified scheme shown in Figure 5c is obtained, and the matrix equation (22) becomes:

$$\begin{bmatrix} p_{gi} \\ q_{gi} \end{bmatrix} = \begin{bmatrix} 1 & Z_{gi} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_i' \\ p_i'/Z_i \end{bmatrix} \tag{23}$$

with

$$Z_i = \frac{(Z_2 + Z_v)\cos k_c L_i + \mathrm{j} Z_c \sin k_c L_i}{\cos k_c L_i + \mathrm{j} \frac{Z_2 + Z_v}{Z_c} \sin k_c L_i} . \tag{24}$$

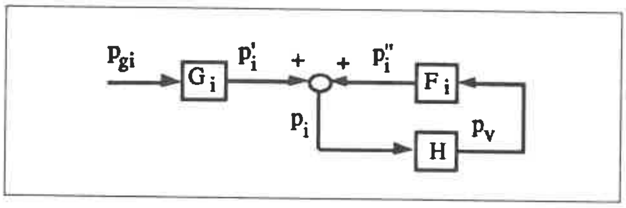

Figure 6. Block diagram representation of the active control device.

p'_i is the contribution to point "i" of the primary source, and Z_2 is the radiation impedance Z_0 transferred to point "2". It may be recalled that an impedance Z transferred from a distance L in a uniform cylindrical duct is expressed as follows:

$$Z(L) = \frac{Z \cos k_c L + \mathrm{j} Z_c \sin k_c L}{\cos k_c L + \mathrm{j}\frac{Z}{Z_c} \sin k_c L}. \tag{25}$$

By developing the preceding equation, Hardouin determined the contribution of the expression of the primary source upstream from the valve, represented by the transfer function G_i (Hardouin, 1993):

$$G_i = \frac{p'_i}{p_{gi}} = \frac{Z_i}{Z_i + Z_{gi}} \tag{26}$$

or analytically:

$$G_i = \frac{(Z_2 + Z_v) \cos k_c L_i + \mathrm{j} Z_c \sin k_c L_i}{A \cos k_c L_i + \mathrm{j} B \sin k_c L_i}, \tag{27}$$

where

$$A = Z_v + Z_2 + Z_{gi}, \quad B = Z_c + \frac{Z_{gi}}{Z_c}(Z_v + Z_2).$$

2.4.2. Case when the primary generator is stopped ($p_g = 0$)

An approach similar to the preceding one was adapted to characterize the behavior of the oscillating valve when the primary source was stopped. This configuration, represented by Figure 5d, can be easily written in matrix form as follows:

$$\begin{bmatrix} p_v \\ q_v \end{bmatrix} = \begin{bmatrix} 1 & Z_v + Z_2 \\ 0 & 1 \end{bmatrix}_1 \begin{bmatrix} Mat \\ L_i \end{bmatrix} \begin{bmatrix} p''_i \\ p''_i / Z_{gi} \end{bmatrix} \tag{28}$$

where p''_i characterizes the contribution, at point "i", of the auxillary source. The matrix $\begin{bmatrix} Mat \\ L_i \end{bmatrix}$ is in this case the transfer matrix of the uniform duct of length L_i for a acoustic wave propagating in the direction opposed to flow. This matrix is defined by the relation (10).

The contribution of the auxiliary source at point "i" upstream from the valve is then represented by the transfer function F_i which is written:

$$F_i = \frac{p''_i}{p_v} = \frac{Z_{gi} \mathrm{e}^{-\mathrm{j} k_c M L_i}}{A \cos k_c L_i + \mathrm{j} B \sin k_c L_i}. \tag{29}$$

2.4.3. Superposition of the two configurations

Figure 6 represents in block diagram form the active system given in Figure 4 in which the transfer functions F_i and G_i are those defined above. This representation leads immediately to an expression of the pressure at point "i" as a function of the pressures generated by the two sources:

$$p_i = p'_i + p''_i = G_i p_{gi} + F_i p_v. \tag{30}$$

By considering the expression of the control law ($p_v = H p_i$) of the auxiliary source, the preceding equation (30) allows the transfer relation to be determined between the primary source and the pressure at point "i":

$$p_i = \frac{G_i}{1 - H F_i} p_{gi}. \tag{31}$$

3. Minimization of the reflection coefficient

3.1. Passive minimization of the reflection coefficient

In this section, we shall study the conditions allowing the reduction of the reflection coefficient of an open-end duct by means of a valve at a fixed incidence θ_0 characterized by an impedance Z_v defined by equation (18). The valve is positioned at a distance L_o from the duct end. To determine the reflection coefficient R at point "i" located at a distance L_i upstream from the valve, equation (14) is applied to impedance Z_i defined by equation (24):

$$R = \frac{Z_2 + Z_v - Z_c}{Z_2 + Z_v + Z_c} \mathrm{e}^{-2\mathrm{j} k_c L_i} \tag{32}$$

i.e.

$$R = \frac{Z_2/Z_c + \Lambda_0 M - 1}{Z_2/Z_c + \Lambda_0 M + 1} \mathrm{e}^{-2\mathrm{j} k_c L_i}. \tag{33}$$

It is noteworthy that the reflection coefficient R is independent of the characteristics (p_{gi}, Z_{gi}) of the primary source. For a given configuration, the module of the reflection coefficient is minimal if there is a head-loss coefficient Λ_0, which is the solution of the following equation:

$$\frac{\partial |R|}{\partial \Lambda_0} = 0. \tag{34}$$

Inclusion of the ratio of complex impedances $Z_2/Z_c = X_2 + \mathrm{j} Y_2$ gives the solution:

$$\Lambda_0 = \frac{-X_2 \pm \sqrt{Y^2 + 1}}{M}. \tag{35}$$

If it is assumed that the open-end impedance differs little from zero ($Z_0 \simeq 0$), and if the effects of flow viscosity and wall friction are not taken into consideration, i.e.:

$$k_c \simeq k_{co} = \frac{k_0}{1 - M^2}, \tag{36}$$

then, after equation (25), we have $X_2 \simeq 0$ and $Y_2 \simeq \tan k_{co}L_o$. In these conditions, equation (35) becomes:

$$\Lambda_0 \simeq \frac{\sqrt{Y^2+1}}{M} \simeq \frac{\sqrt{(\tan k_{co}L_o)^2+1}}{M}. \quad (37)$$

which corresponds to the module of the following reflection coefficient:

$$|R| = \frac{\sqrt{\sqrt{1+(\tan k_{co}L_o)^2}-1}}{\sqrt{\sqrt{1+(\tan k_{co}L_o)^2}+1}}. \quad (38)$$

There is thus a head-loss coefficient Λ_0 which minimizes the reflection coefficient of an open-end. The value of this coefficient depends on the frequency, Mach number and distance L_o between the valve and the duct end. Equation (38) shows that, on first approximation ($Z_0 \simeq 0$ and $k_c \simeq k_{co}$), the reflection coefficient is cancelled out when $\tan k_{co}L_o = 0$ or $L_o = 0$, i.e. when the valve is positioned at the duct end. In this case, equation (37) gives the head-loss Λ_0 caused by the valve:

$$\Lambda_0 \simeq \frac{1}{M}. \quad (39)$$

Note that this is simply a function of the inverse of the Mach number so the incidence of the valve leading to such head-loss is easily obtained by considering the nonlinear relation $\Lambda = F(\theta)$ shown in Figure 3. Equation (39) immediately provides the expression of the impedance associated with the valve: $Z_v = Z_c$, i.e. the impedance of the valve is equal to the characteristic impedance of the duct.

Figure 7a provides an illustration of the complete equation (33), i.e. an example of the changes for different frequencies in the reflection coefficient module as a function of the head-loss coefficient Λ_0 (according to a logarithmic scale). Calculation of Z_2 requires knowledge of the open-end impedance Z_0 determined by using the empirical formula of (Ingard and Singhal, 1974; Ingard and Singhal, 1975). Figure 7b shows the changes in the module of the reflection coefficient when $L_o = 0$. This module tends toward zero ($|R| \simeq 0.02$) for $\Lambda_0 \simeq 20$ ($M = 0.05$), i.e. $\theta_0 \simeq 45°$.

To summarize, on respect of the assumptions used to establish the fixed valve model (18), i.e. $M_2 \ll 1$ and Λ_0 relatively low, the results above indicate that it is only necessary to position the valve in incidence θ_0 at the duct end in order to obtain the desired adaptation of impedance. However, as the head-loss coefficient is practically the same as the inverse of the Mach number, the load thus added to the circuit becomes very large for low flow velocities ($M < 0.1$). This could disturb the functioning of the primary source or even modify the characteristics of the circuit to be studied. In fact, experience relative to studies concerning the use of an oscillating valve in active control systems has shown that the head-loss coefficient Λ_0 must be below 10, i.e. at about a 40° incidence. This major drawback led us to propose an active termination susceptible of producing identical results with less head-loss.

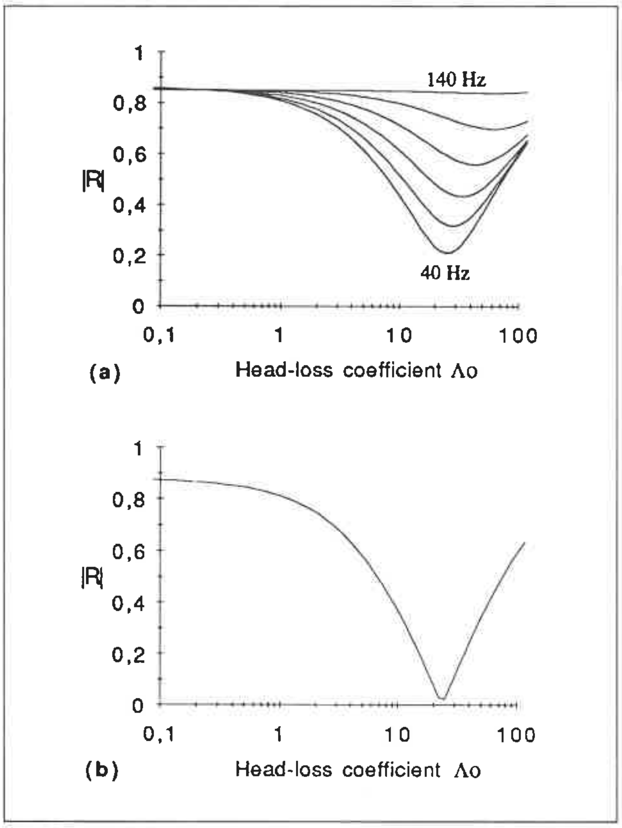

Figure 7. Module of the reflection coefficient R. $M = 0.05$, $L_i = 0.7$ m; frequency: 40, 60, 80, 100, 120, 140 Hz.
(a): $L_o = 0.6$ m, (b): $L_o = 0$ m.

3.2. Active minimization of the reflection coefficient

In consideration of the comments above, the purpose of this section was to study the valve in an active manner so as to control (and thus reduce) its passive influence on the circuit. Accordingly, we shall now consider the active control system described and modeled in section 2.4 (Figure 4). Studies concerning the use of this system for the attenuation of periodic fluctuations in acoustic circuits, such as for exhaust from reciprocating engines, have made it possible to determine the transfer function H of the control filter which abolishes pressure fluctuations downstream from the valve. When this control filter is perfectly tuned, it has been shown that the wave from the primary source is totally reflected by the valve ($|R| = 1$).

In our case, we determine the transfer function H allowing the reflection coefficient to be cancelled out at the level of the valve. Accordingly, we consider the pressure at point "i" (30) and introduce again the load impedance Z_i downstream from this point, i.e.:

$$p_i = \frac{G_i}{1-HF_i}\, p_{gi} = \frac{Z_i}{Z_i+Z_{gi}}\, p_{gi}. \quad (40)$$

By replacing transfer functions G_i and F_i with equations (27) and (29), a new expression of Z_i (41) and the reflection coefficient (42) can be deduced:

$$Z_i = \frac{(Z_2 + Z_v)\cos k_c L_i + \mathrm{j} Z_c \sin k_c L_i}{\cos k_c L_i + \mathrm{j}\frac{Z_2+Z_v}{Z_c}\sin k_c L_i - H\mathrm{e}^{-\mathrm{j}k_c M L_i}} \quad (41)$$

and

$$R = \frac{\frac{Z_2+Z_v-Z_c}{Z_c}\,\mathrm{e}^{\mathrm{j}k_c L_i(M-1)} + H}{\frac{Z_2+Z_v+Z_c}{Z_c}\,\mathrm{e}^{\mathrm{j}k_c L_i(M-1)} - H} \quad (42)$$

In this case as well, the reflection coefficient R does not depend on the characteristics of the primary source (p_{gi}, Z_{gi}). When $H = 0$, we find the expressions (24) and (32) for respectively Z_i and R coresponding to the motionless valve.

Note that $H_{\text{anec.}}$ is the optimal transfer function for filter H, enabling reflection coefficient R to be cancelled out. Equation (42) immediately gives the expression of this filter:

$$H_{\text{anec.}} = \frac{Z_c - Z_v - Z_2}{Z_c}\,\mathrm{e}^{\mathrm{j}k_c L_i(M-1)} \quad (43)$$

i.e.,

$$H_{\text{anec.}} = (1 - \Lambda_0 M - Z_2/Z_c)\,\mathrm{e}^{\mathrm{j}k_c L_i(M-1)}. \quad (44)$$

When this expression is introduced into equation (41), it is clearly demonstrated that $Z_i = Z_c$, i.e. that the impedance downstream from point "i" is equal to the characteristic impedance Z_c. Equation (44) shows that $H_{\text{anec.}}$ depends uniquely on conditions downstream from point "i". If we consider the same simplifying hypotheses as in section 3.1, i.e. $k_c \simeq k_{co}$ and $Z_2/Z_c \simeq \mathrm{j}\tan k_{co}L_o$, equation (44) is simplified in the following manner:

$$H_{\text{anec.}} = (1 - \Lambda_0 M - \mathrm{j}\tan k_{co}L_o)\,\mathrm{e}^{\mathrm{j}k_{co}L_i(M-1)}. \quad (45)$$

This transfer function possesses an infinity of poles for $k_{co}L_o \simeq (2n+1)\pi/2$. The module $|H_{\text{anec.}}|$ tends toward infinity when $\Lambda_0 \to \infty$, which corresponds to the obstructed duct. It presents a minimum for $\Lambda_0 = 1/M$ and tends asymptotically toward $\sqrt{1 - (\tan k_{co}L_o)^2}$ when Λ_0 tends toward zero. These results are illustrated by Figure 8a which represents, for a given configuration and several frequencies, a series of changes in the module of $H_{\text{anec.}}$ as a function of Λ_0 (i.e. as a function of Z_v).

We shall now consider the case in which the oscillating valve is positioned at the duct end, i.e. $L_o = 0$, thereby reducing equation (45) to:

$$H_{\text{anec.}} = (1 - \Lambda_0 M)\,\mathrm{e}^{\mathrm{j}k_{co}L_i(M-1)}. \quad (46)$$

The module for this transfer function is of course independent of the frequency and is abolished for $\Lambda_0 = 1/M$, which is the passive condition for cancellation of the reflection coefficient. The comments above also apply to this case. In particular, it appears that $|H_{\text{anec.}}|$ tends asymptotically toward 1 when Λ_0 tends toward zero. An example of the changes in this module as a function of the head-loss coefficient is given in Figure 8b.

Thus, this study shows the existence of a control filter which can regulate the valve so as to cancel the reflection coefficient while controlling its passive influence on the circuit. This passive influence is represented by the head-loss coefficient induced by the presence of the valve.

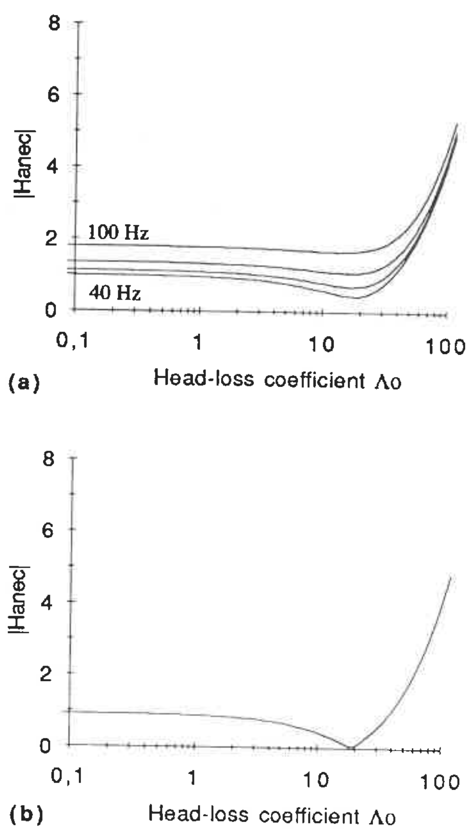

Figure 8. Module of the transfer function $H_{\text{anec.}}$. $M = 0.05$, $L_i = 0.7$ m; frequency: 40, 60, 80, 100 Hz.
(a): $L_o = 0.6$ m, (b): $L_o = 0$ m.

4. Experimental validation

4.1. Description of the experimental set-up

The primary source is a device serving to break up the flow generated by a fan. This scheme, described in (Laumonier, 1990), enables pulsed flow to be obtained in a frequency range of 10 to 100 Hz. This unit is connected to a 45 mm diameter duct.

The auxiliary source is a circular valve made to oscillate sinusoidally by an electric motor. The mean incidence θ_0 is adjustable in a range of 0 to 60 degrees and the instantaneous incidence $\theta(t)$ is measured by a position sensor.

The control circuit of the source is composed of a band-pass filter, an amplifier and a phase shifter. The input signal for the filter come from an optical encoder installed on the primary source and is synchonized with the pulsation generated.

The pressures are measured with piezoresistive sensors, temperature with a thermocouple. The mean flow is obtained by means of a hot-wire anemometer placed at the duct center line at which we applie a profile correction.

Measurement of the reflection coefficient

The experimental validations are based on measurement of the reflection coefficient performed by means of the two-

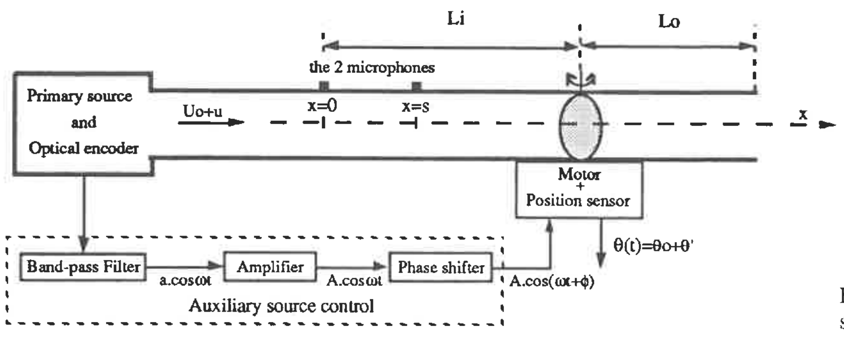

Figure 9. Design of the experimental system.

microphone method (Chung and Blaser, 1980). This method was adapted to measure low frequencies (40 to 100 Hz) with mean flow according to the work (Abom and Boden, 1988) concerning the error analysis. Accordingly, the essential parameter is the distance s between the two microphones which is set at 0.4 m. With this method, the reflection coefficient for the microphone positioned at $x = 0$ (Figure 9) is determined by the following equation:

$$R(x=0) = \frac{e^{-jk^+ s} - H_{12}}{H_{12} - e^{jk^- s}} \qquad (47)$$

where $H_{12} = p(x = s)/p(x = 0)$ is the transfer function measured between the two microphones.

4.2. Experimental results

4.2.1. Valve at a fixed incidence

In this section, we compare the theoretical equation (33) and experimental reflection coefficients obtained upstream from a stationary valve after modification of its incidence and thus its head-loss coefficient. The first series of results concerned the valve positioned at a distance $L_o = 0.6$ m from the duct end. Figure 10 shows the theoretical and experimental changes in the reflection coefficient module for frequencies of 40 and 80 Hz as a function of Λ_0. The experimental results clearly indicate a valve incidence at which the reflection coefficient is minimized. The variations between theoretical and experimental values were due to several factors. In particular, the two-microphone method is difficult to optimize at low frequencies and not very accurate for the extreme values of the reflection coefficient ($|R| = 0$ and $|R| = 1$). Another source of error is the difficulty in accounting perfectly for the influence of flow characterized by the Mach number in the modeling and the measurement of the reflection coefficient.

In accordance with the theory, the distance between the valve and the duct end ($L_o = 0.1$ m) was decreased in order to reduce this minimum. Figure 11 shows a clear reduction in the minimum reflection coefficient for the two frequencies studied. In fact, the results were identical with either frequency, which is in agreement with equation (38). The reflection coefficient was practically cancelled out ($|R| < 0.1$) for $\Lambda_0 \simeq 30$ or $\theta_0 \simeq 50°$.

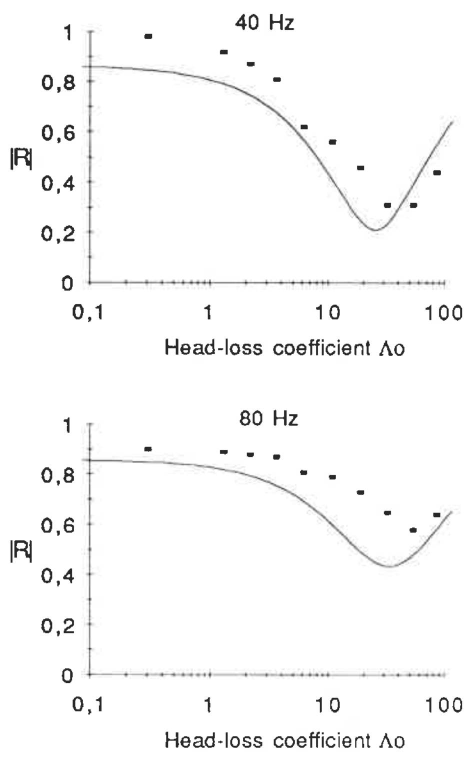

Figure 10. Module of the reflection coefficient R. $M = 0.05$, $L_i = 0.7$ m, $L_o = 0.6$ m, frequency: 40 and 80 Hz. (——) theoretical, (▪) measured.

4.2.2. Oscillating valve

The experimental results presented in this section demonstrate that oscillation of the valve enabled us to reduce the reflection coefficient and validate the expression of the transfer function $H_{\text{anec.}}$ defined by equation (45). For this purpose, we use the experimental configuration described above (Figure 10), i.e. an oscillating valve positioned at a distance $L_o = 0.6$ m from the duct end. The mean incidence of the valve is $\theta_0 = 47°$ ($\Lambda_0 = 25$), that is, a head-loss coefficient characterized by a passive reduction in the reflection coefficient. In this case, the objective is to regulate the movement of the valve (amplitude and phase) so as to cancel the reflection

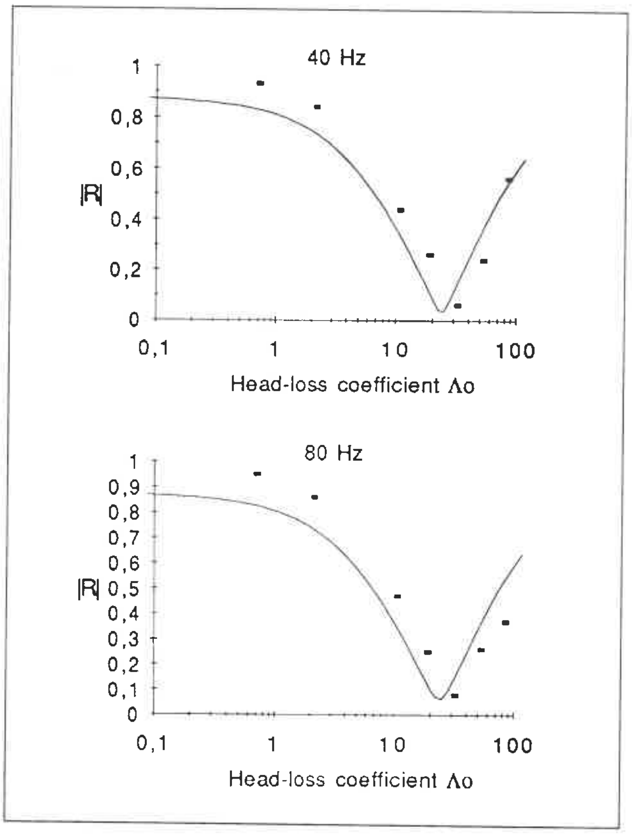

Figure 11. Module of the reflection coefficient R. $M = 0.05$, $L_i = 0.7$ m, $L_o = 0.1$ m, $s = 0.4$ m, frequency: 40 and 80 Hz. (——) theoretical, (▪) measured.

coefficient ($|R| < 0.1$). For each of the excitation frequencies, we measure the displacement of the valve required for the abolition of the reflection coefficient as well as the pressure at the level of the upstream sensor. We thus deduce the experimental transfer function $H = H_{\text{anec.}}$, noting that it is defined by $H = p_v/p_i$. The results for the configuration studied are presented in Figure 12. A reflection coefficient of less than 5% is obtained from 40 to 100 Hz (Figure 12a), whereas it was always above 30% in the passive situation (Figure 11). These results are obtained with valve oscillations of about two degrees around its mean position. The module and the phase of the corresponding transfer function $H_{\text{anec.}}$ (Figure 12b and 12c) coincide with the theoretical estimates. The variations in the module are due to uncertainties about measurements noted in the preceding section.

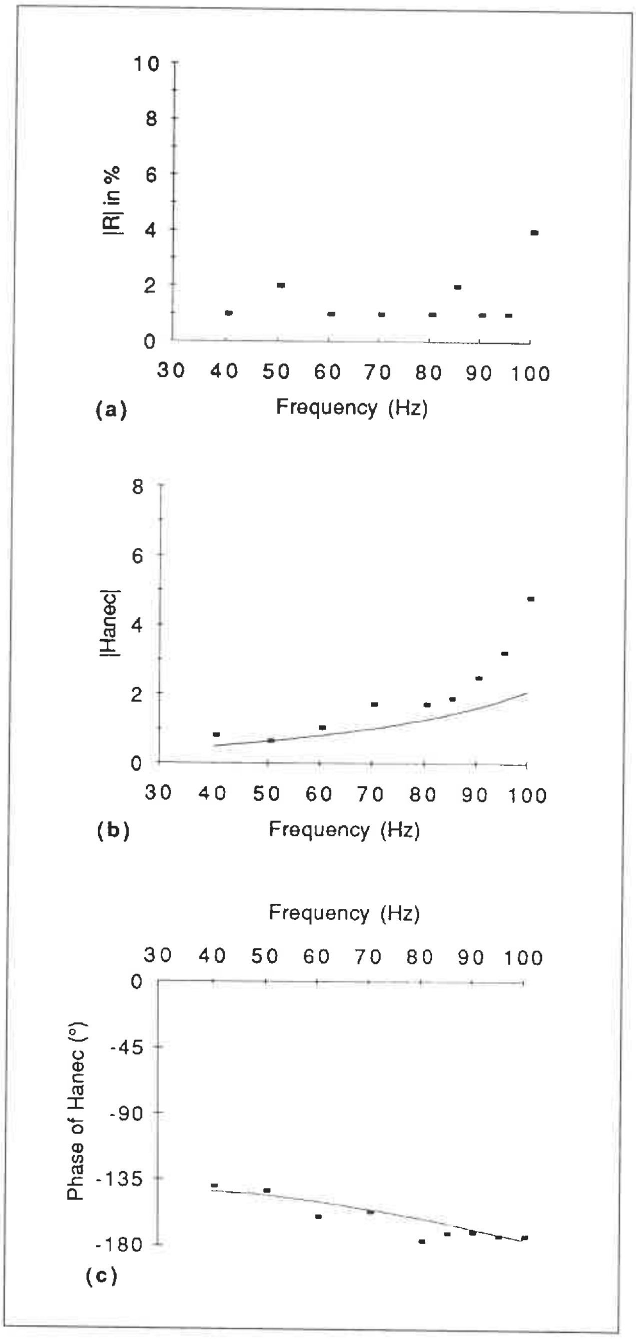

Figure 12. Module (b) and phase (c) of the transfer function minimizing the reflection coefficient (a). $M = 0.05$, $\theta = 47°$, $L_i = 0.74$ m, $L_o = 0.6$ m, $s = 0.4$ m. (——) theoretical, (▪) measured.

5. Conclusion

This paper demonstrates that it is possible to develop a low-frequency anechoic termination in the presence of flow. The principle is to use a valve characterized by an impedance $Z_v = \Lambda_0 M Z_c$ which is a function of the Mach number M and the mean head-loss coefficient Λ_0 (consequently, its incidence).

Two situations were studied by modeling and experimental validation: passive minimization of the reflection coefficient through use of a fixed valve and active minimization through oscillation of the valve.

With the fixed valve positioned at the duct end, the reflection coefficient ($|R| < 10\%$) can be cancelled when the head-loss coefficient Λ_0 of the valve is of the order of magnitude of the inverse of the Mach number of the flow. However, in many applications the Mach number is less than 0.1, which implies that the optimal head-loss coefficient is greater than 10, a value too high to allow the device to be used without disturbing the acoustic circuit in which it is installed.

For an equivalent result, the active system serves to limit the head-loss induced by the valve oscillating around a mean position. In the experiments performed, a reflection coefficient of less than 5% was obtained from 40 to 100 Hz, with a Mach number of around 0.05. In addition to its good performance, this device is robust, takes up little space and is easy to integrate into the acoustic circuit. The next step in the development of this device will be to work out an automatic control strategy for the valve. This device can be used to advantage in measuring the transmission loss of mufflers in the exhaust system of reciprocating engines which are characterized by very low frequencies, a corrosive environment and high temperatures.

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