Discrete Event System Control in Max-Plus Algebra : Application to Manufacturing Systems

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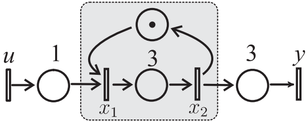
November 11th 2020

Overview

Max-plus algebra is suitable to deal with control and observation of Timed Event Graphs.



Different control strategies can be considered :

- Output/State Feedback controller
- State Estimation
- Observer-based controller

This talk presents a survey of these different strategies and a software tool yielding an efficient way for implementing in order to control an automated system.

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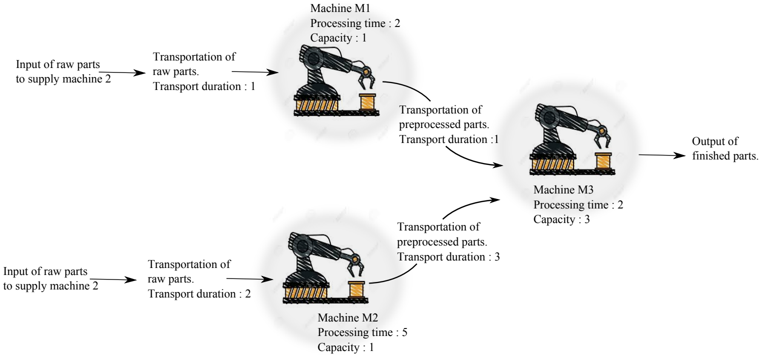
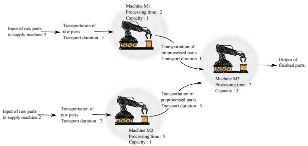
Methodology overview :

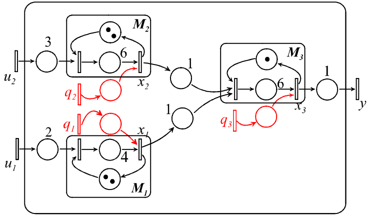
A manufacturing systems to control



Methodology (first step) :

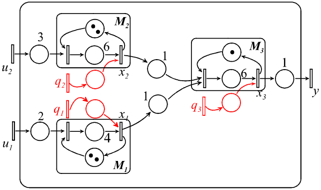
1. The manufacturing system is depicted as a Petri Net





Methodology (second step) :

2. Petri net is transformed in State model in dioid $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$



The system in dioid $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$:

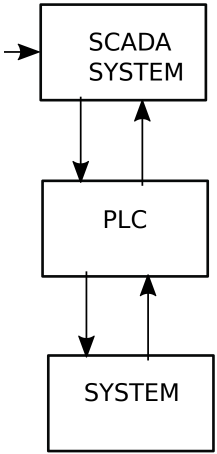
$$
\begin{aligned}
x &= Ax \oplus Bu \\
y &= Cx
\end{aligned}
$$

Methodology (Third step) :

3. The matrices of the model are put in a Supervisory Control And Data Acquisition (SCADA) system and implemented in a Programmable Logical Controller (PLC) in order to control the system.

The system in $\overline{\mathbb{Z}}_{max}[\![\gamma]\!]$

$$\begin{aligned} x &= Ax \oplus Bu \\ y &= Cx \end{aligned}$$



Dioid Theory in a few words [Baccelli et al. 1992]

A Dioid is a set $\mathcal{D}$ with two binary operators addition $\oplus$ and multiplication $\otimes$.

- Addition $\oplus$ is associative, commutative, zero element denoted ε
- Multiplication $\otimes$ is associative, unit element denoted e
- Distributivity property : $(a \oplus b) \otimes c = a \otimes c \oplus b \otimes c$.
- Zero element ε is absorbing : $a \otimes \varepsilon = \varepsilon$
- Addition is idempotent : $a \oplus a = a$

Properties

- Induced order $b \preceq a \Leftrightarrow a \oplus b = a$
- A dioid is said to be complete if it is closed for infinite sums and if multiplication distributes over infinite sums.
- On a complete dioid the Kleene star a^*, is defined by $a^* = \bigoplus_{i=0}^{\infty} a^i$ with $a^0 = e$ and $a^{i+1} = a \otimes a^i$.

Theorem On a complete dioid $\mathcal{D}$, $x = a^* b$ is the least solution of the implicit equation $x = ax \oplus b$.

Example Dioid

Max-plus Algebra

Max,+ Algebra is the set $\mathbb{Z}_{max} = \mathbb{Z} \cup \infty$, endowed with :

- max as addition $\oplus$
- $+$ as multiplication $\otimes$

The zero element $\varepsilon = -\infty$ and the unit element $e = 0$.

- For example : $5 \otimes 4 \oplus 7 = \max(5 + 4, 7) = 9$

Residuation Theory

Residuation Theory [Baccelli et al. 1992]

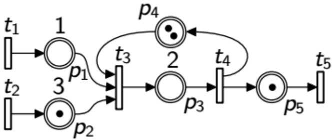
Residuation theory provides, under certain conditions, the greatest solution (in accordance with the considered order) to the inequality $f(x) \preceq b$ where f is an order-preserving mapping defined over ordered sets.

Example : Inequality $a \otimes x \preceq b$ in a complete Dioid

Example left multiplication : the inequality $a \otimes x \preceq b$ admits a greatest solution, denoted, $x = a \diamond\!\!\!\backslash b$ (left division by a).
Right multiplication by a : In analogy the inequality $x \otimes a \preceq b$ has a greatest solution, denoted $x = b \diamond\!\!\!/ a$ (right division by a).

Timed Event Graph (TEG)

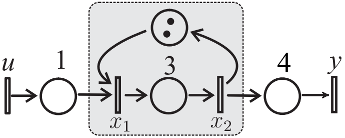
- Subclass of Timed Discrete Event Systems
- Graphical model for manufacturing systems, transportation systems, computer networks, ...



Definition TEG

- $P = \{p_1, \cdots, p_n\}$ is the set of places. $T = \{t_1, \cdots, t_m\}$ is the set of transitions.
- Every place p_i has exactly one upstream and one downstream transition and is attached with holding time $\phi_i \in \mathbb{N}_0$.
- $A \subseteq (T \times P) \cup (P \times T)$ is the set of arcs and all arcs (t_j, p_l) and (p_l, t_o) are equipped with weights 1.

TEG Model in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$



Firing Date [Cohen et al., 85]

$x_i(k)$: date of the firing numbered k for the transition labelled i.

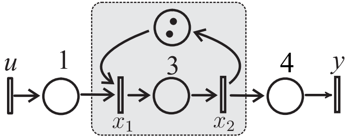
γ transform [Cohen et al,89]

▸ Mo

γ transform of $x(k)$ is a formal series $x(\gamma) = \bigoplus_{k \in \mathbb{Z}} \gamma^k x(k)$.

γ-operator is like a backward shift operator in the event domain, $y(\gamma) = \gamma x(\gamma) \Leftrightarrow y(k) = x(k-1) \forall k$.

TEG Model in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$



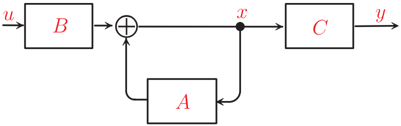
The system in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$:

$$\begin{aligned} x(\gamma) &= Ax(\gamma) \oplus Bu(\gamma) &= \begin{pmatrix} \varepsilon & \gamma^2 \\ 3 & \varepsilon \end{pmatrix} x(\gamma) \oplus \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix} u(\gamma) \\ y(\gamma) &= Cx(\gamma) &= \begin{pmatrix} \varepsilon & 4 \end{pmatrix} x(\gamma) \end{aligned}$$

The system in $\overline{\mathbb{Z}}_{\max}[\![\gamma]\!]$: ▸ Mo

$$\begin{aligned} x(\gamma) &= A^*B &= \begin{pmatrix} (3\gamma^2)^* & \gamma^2(3\gamma^2)^* \\ 3(3\gamma^2)^* & (3\gamma^2)^* \end{pmatrix} \begin{pmatrix} 1 \\ \varepsilon \end{pmatrix} u(\gamma) \\ y(\gamma) &= CA^*B &= \begin{pmatrix} 8(3\gamma^2)^* \end{pmatrix} u(\gamma) \end{aligned}$$

Optimal control : IEEE TAC, Cohen et al. 1989



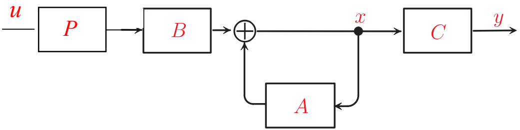
Problem Formulation :

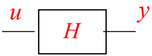
Let z be a desired output. Is it possible to compute a control input in order to get an output y as close as possible to z while respecting the constraint : $y \preceq z$. The optimal input is given by :

$$u_{opt} = (CA^*B) \backslash\!\!\circ z$$

It is the greatest input which achieves the inequality : $y = (CA^*B)u_{opt} \preceq z$. In manufacturing setting z corresponds to the customer demands, u the input of raw parts in the system, y the output of processed parts. Optimal control u_{opt} is the one which minimizes the internal stock while ensuring the customer demand is honored.

Optimal Filtering : JESA, Hardouin et al. 1995





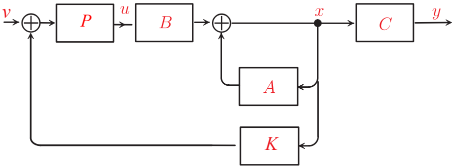
Problem Formulation :

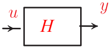
Let H be a matrix describing a desired behavior. Let CA^*B be the transfer of the system to be controlled, and P a filter such that $y = CA^*B \otimes P \otimes u$. The optimal filter such that $CA^*BPu \preceq Hu, \ \forall u$ is given by :

$$P_{opt} = (CA^*B) \circ\!\!\!\!\backslash H$$

In manufacturing setting it is the one which delays as much as possible the input while ensuring that the output $y \preceq Hu \ \forall u$.

State Feedback controller synthesis [Cottenceau 1999, Maia 2003, Lhommeau 2003, Gonçalves 2015]





Equations :

$$\begin{cases} x = Ax \oplus Bu \\ y = Cx \\ u = P(v \oplus Kx) \end{cases}$$

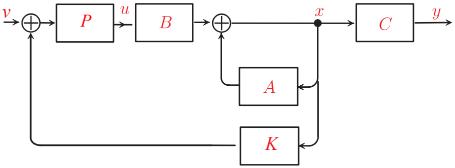
Transfer

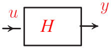
$$\begin{aligned} y &= CA^*Bu \\ &= CA^*BP(KA^*BP)v \end{aligned}$$

Objective

$$y = CA^*BP(KA^*BP)v \preceq Hv$$

State Feedback controller synthesis [Cottenceau 1999, Maia 2003, Lhommeau 2003, Gonçalves 2015]





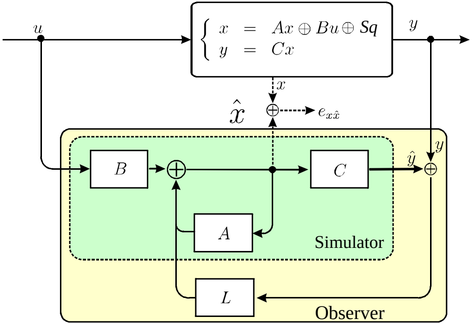
Objective :

Compute the greatest controllers P,K such that :

$$CA^*BP(KA^*BP) \preceq H$$

Optimal Solution :

$$
\begin{aligned}
P_{opt} &= (CA^*B)\backslash\!\!\!\circ H \\
K_{opt} &= P_{opt}\backslash\!\!\!\circ P_{opt}/\!\!\!\circ(A^*BP_{opt}
\end{aligned}
$$

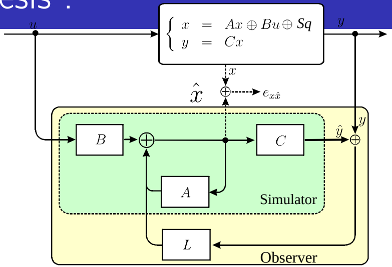


Objective :

Compute an observer matrix L such that the estimated state be as close as possible to the real state under the constraint,

$$\hat{x} \preceq x.$$

Sub Observer Synthesis :



System Equations :

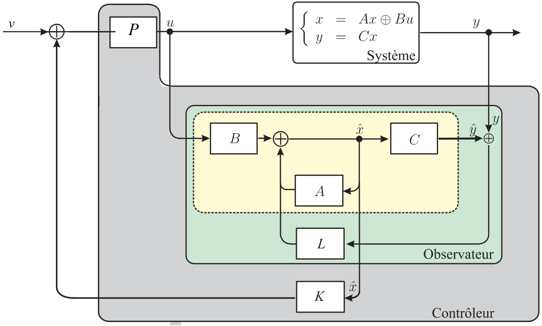
$$\begin{aligned} x &= Ax \oplus Bu \oplus Sq = A^*Bu \oplus A^*Sq \\ y &= Cx = CA^*Bu \oplus CA^*Sq. \end{aligned}$$

Estimated State Equations

$$\begin{aligned} \hat{x} &= A\hat{x} \oplus Bu \oplus L(\hat{y} \oplus y) \\ \hat{y} &= C\hat{x}. \end{aligned}$$

Optimal Matrix :

$$L_{opt} = ((A^*B) \phi (CA^*B)) \wedge ((A^*S) \phi (CA^*S))$$



Optimal Matrices :

$$
\begin{aligned}
L_{opt} &= ((A^*B)\circ\!\!\!/(CA^*B)) \wedge ((A^*S)\circ\!\!\!/(CA^*S)) \\
P_{opt} &= (CA^*B)\circ\!\!\!\backslash H \\
K_{opt} &= P_{opt}\circ\!\!\!\backslash P_{opt}\circ\!\!\!/(A^*BP_{opt})
\end{aligned}
$$

About Implementation :

Realization (Contribution) :

Each entry ($\neq \varepsilon$) of these matrices (L_{opt}, P_{opt}, K_{opt}) are periodic series, e.g., $K_{ij} = p_{ij} \oplus q_{ij} r_{ij}^*$ where p_{ij},q_{ij} are polynomials and r_{ij} a monomial, to realize these series, the following vector is introduced $\zeta_{ij} = \begin{pmatrix} \zeta_{ij_1} \\ \zeta_{ij_2} \end{pmatrix}$

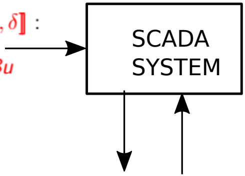
$$
\begin{aligned}
\zeta_{ij} &= \begin{pmatrix} r_{ij} & \varepsilon \\ e & \varepsilon \end{pmatrix} \zeta_{ij} \oplus \begin{pmatrix} p_{ij} \\ q_{ij} \end{pmatrix} \hat{x}_{ij} \\
K_{ij}\hat{x}_{ij} &= \begin{pmatrix} \varepsilon & e \end{pmatrix} \zeta_{ij}
\end{aligned}
$$

Platform of Control

http ://lisabiblio.univ-angers.fr/MASTERSiteWebFreitasOliveiraGabriel/Home.htm

The system in $\mathcal{M}_{in}^{ax}[\![\gamma,\delta]\!]$:

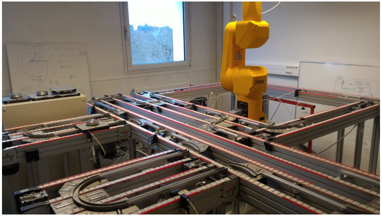
$$x = Ax \oplus Bu$$
$$y = Cx$$

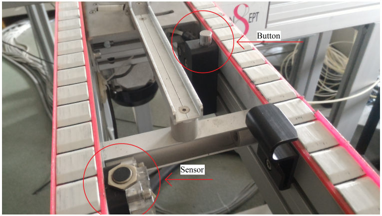


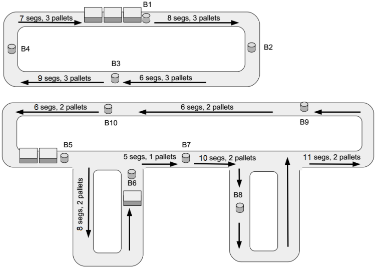
Menu :

- Enter matrices A, B, C, S
- Optimal tracking (a trajectory must be first defined)
- State Estimation
- State Feedback Control (All states are supposed measured)
- Feedback Control
- Observer-based Control

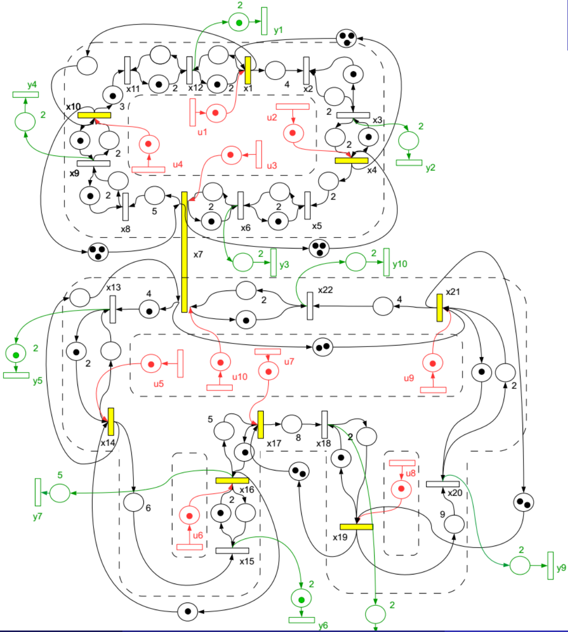
Illustration : Conveyor Belt System





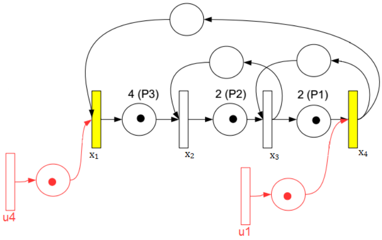


Petri Net Model with 24 states, 10 control inputs, 10 measured outputs (Mariano Gonçalves 2015)



Results and Benefits

On this kind of systems, pallets wait in the front of Button which regulate the flow :



(experiment 6 hours) :

	Mean waiting time	Percent of gain
As soon as possible CA^*B	$8.3s$	0%
Feedback control	$6.6s$	20.5%
Observer-based control	$6.4s$	22.5%

Conclusion

With this software platform it is easy to implement these control strategies

- Improvement of state estimation (G. Winck)
- Control of systems with sharing resources (G. Schafaschek, Friday)
- State estimation of weighted TEG (J. Trunk, Friday)

Thanks to

Literature

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