

# On the Model Reference Control for Max-Plus Linear Systems

C. A. Maia and L. Hardouin R. Santos-Mendes B. Cottenceau

**Abstract**—This paper deals with the model-reference control of max-plus linear systems. The main contribution of this work is a general control structure that encompasses the previous one found in the literature. It is based on the RST-structure for conventional differentiable system. In addition it provides a comparison among some sub-structures and simulation results are presented.

**Keywords** : Discrete event dynamic systems, Timed Petri nets, Max-plus algebra, dioid, Control, Just-in-Time.

## I. INTRODUCTION

Several applications in manufacturing, computer and communication systems can be modelled by the so called Discrete Event Systems (DES) theory. These of systems are often described by the Petri Net formalism [15]. Timed Event Graphs (TEG) are Timed Petri Nets in which all places have single upstream and single downstream transitions and are used to model DES characterized by delay and synchronization phenomena. TEG can be described by linear equations in the dioid algebra formulation ([1],[7]) and this fact has allowed many important achievements on the modelling and control of DES, as detailed in [5], [14], [6], [12], [9], [10], [8]. TEG control problems are usually stated in a Just-in-Time context, where the design goal is to minimize stocks while guaranteeing performance (e.g. throughput). One possible approach for the control of TEG is the model-reference technique in which a given model describes the desired performance and the design goal is achieved through the calculation of a precompensator or of a feedback controller [6], [12]. The techniques based on feedback control, although favoring stability, are limited in the sense that the reference model must satisfy certain restrictive conditions. Those based on precompensation can guarantee performance for any reference model, but not stability (for the concept of stability in TEG, see [3]). In this context, the present paper proposes a control structure based on well known RST for continuous systems which is a generalization of the work proposed by [6] and [13]. The main advantage of the approach is that it achieves optimality regarding stocks while guaranteeing optimal compliance with any prescribed reference model.

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The paper is organized as follows. Section II introduces some algebraic tools concerning the Dioid and Residuation theories and their applications to max-plus linear system. Section III introduces some control results and develops the proposed control structure. Simulation Results are shown in section IV. A conclusion is given in section V.

## II. MAX-PLUS LINEAR SYSTEM THEORY

An important framework to deal with max-plus systems is the dioid algebra. A *dioid*  $\mathcal{D}$  is an idempotent semiring. The two internal operations are denoted  $\oplus$  and  $\otimes$ . The operation  $\oplus$  is associative, commutative and idempotent, that is,  $a \oplus a = a, \forall a \in \mathcal{D}$ . The operation  $\otimes$  is associative (but not necessarily commutative) and distributive at left and at right with respect to  $\oplus$ . The neutral elements of  $\oplus$  and  $\otimes$  are represented by  $\varepsilon$  and  $e$  respectively, and  $\varepsilon$  is absorbing for  $\otimes$  ( $\forall a \in \mathcal{D}, \varepsilon \otimes a = a \otimes \varepsilon = \varepsilon$ ). In a dioid, a partial order relation is defined by  $a \succeq b$  iff  $a = a \oplus b$  and  $x \wedge y$  denotes the greatest lower bound between  $x$  and  $y$ . A dioid  $\mathcal{D}$  is said to be complete if it is closed for infinite  $\oplus$ -sums and if  $\otimes$  distributes over infinite  $\oplus$ -sums. Most of the time the symbol  $\otimes$  will be omitted as in conventional algebra, moreover,  $a^i = a \otimes a^{i-1}$  and  $a^0 = e$ .

The least solution of the equation  $x = ax \oplus b$  is a relevant issue in many max-plus linear systems problems and an important result is presented in theorem 1.

*Theorem 1* ([1], th. 4.75): The implicit equation  $x = ax \oplus b$  defined over a complete dioid  $\mathcal{D}$ , admits  $x = a^*b$  as least solution, where  $a^* = \bigoplus_{i \in \mathbb{N}} a^i$  (Kleene star operator).

Automatic control synthesis involves the inversion of mappings. Mappings defined over dioid does not always admit inverse, but the residuation theory allows to characterize solution set of inequalities such that  $f(x) \preceq y$ . The reader may consult [2] to obtain an exhaustive presentation of this theory.

*Definition 1* (Residual and residuated mapping): An isotone mapping  $f : \mathcal{D} \rightarrow \mathcal{E}$ , where  $\mathcal{D}$  and  $\mathcal{E}$  are partially ordered sets, is a *residuated mapping* if for all  $y \in \mathcal{E}$  there exists a greatest solution for the inequality  $f(x) \preceq y$  (hereafter denoted  $f^\sharp(y)$ ). The mapping  $f^\sharp$  is called the *residual* of  $f$ .

The mappings  $L_a : x \mapsto a \otimes x$  and  $R_a : x \mapsto x \otimes a$  defined over a complete dioid  $\mathcal{D}$  are both residuated ([1], p. 181). Their residuals are isotone mappings<sup>1</sup> denoted respectively by  $L_a^\sharp(x) = a \setminus x$  and  $R_a^\sharp(x) = x / a$ . Some useful dioid

<sup>1</sup> $f$  is an isotone mapping if it preserves order, that is,  $a \preceq b \implies f(a) \preceq f(b)$ . It is an antitone mapping if it reverses the order, that is,  $a \preceq b \implies f(b) \preceq f(a)$ .

formulæ involving these residuals are given below.

$$a(a \backslash x) \preceq x \quad (1)$$

$$a \backslash (ax) \succeq x \quad (2)$$

$$a(a \backslash (ax)) = ax \quad (3)$$

$$a \backslash a = (a \backslash a)^* \quad (4)$$

$$a(ba)^* = (ab)^*a \quad (5)$$

$$s^* \preceq a^* \Leftrightarrow s \preceq a^* \quad (6)$$

$$(ab) \backslash x = b \backslash (a \backslash x) \quad (7)$$

$$a \backslash (a \backslash x) = a \backslash x \quad (8)$$

A trajectory of a TEG transition  $x$  is a firing date sequence  $\{x(k)\} \in \mathbb{Z}$ . For each increasing sequence  $\{x(k)\}$ , it is possible to define the transformation  $x(\gamma) = \bigoplus_{k \in \mathbb{Z}} x(k) \gamma^k$  where  $\gamma$  is a backward shift operator in event domain (that is  $y(\gamma) = \gamma x(\gamma) \Leftrightarrow \{y(k)\} = \{x(k-1)\}$ , see [1], p. 228). This transformation is analogous to the  $Z$ -transform used in discrete-time classical control theory and the formal series  $x(\gamma)$  is a synthetic representation of the trajectory  $x(k)$ . The set of the formal series in  $\gamma$  is denoted by  $\overline{\mathbb{Z}}_{\max}[\gamma]$  and constitutes a dioid. In general, MIMO max-plus linear System can be represented by

$$x = Ax \oplus u \quad (9)$$

$$y = Cx \quad (10)$$

where vectors  $x \in (\overline{\mathbb{Z}}_{\max}[\gamma])^n$ ,  $u \in (\overline{\mathbb{Z}}_{\max}[\gamma])^p$  and  $y \in (\overline{\mathbb{Z}}_{\max}[\gamma])^m$  represent respectively, the state, the input and output trajectories of the system and  $A, B, C$  are the system matrices of appropriate dimensions. These trajectories can be related ([1], p. 243) by the equation  $y = Hu$ , where  $H = CA^*B \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times p}$  is called the transfer matrix of the TEG. Entries of matrix  $H$  are periodic series ([1], p. 260) in the dioid  $\overline{\mathbb{Z}}_{\max}[\gamma]$ , usually represented by<sup>2</sup>  $p(\gamma) \oplus q(\gamma)(\tau\gamma^\nu)^*$ . The asymptotic slope of a periodic series  $s = p(\gamma) \oplus q(\gamma)(\tau\gamma^\nu)^*$  denoted  $\sigma_\infty(s)$  is defined as the ratio<sup>3</sup>  $\sigma_\infty(s) = \frac{\nu}{\tau}$ .

### III. CONTROL METHOD

The control method proposed herein is based on the Just-in-Time strategy and on the model reference approach [6] and is summarized as follows. First let  $H \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times p}$  be the transfer matrix of the plant and  $G_{ref} \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times p}$  be the reference model, *i.e.*, the desired transfer matrix for the controlled system. In the model reference context, the aim is to delay as much as possible the firing dates of the input transition by assuring that the controlled system transfer function be less than or equal to the reference model (which represents in a manufacturing context the restriction imposed by the

<sup>2</sup> $p(\gamma) = \bigoplus_{i=0}^{n-1} p_i \gamma^i$ ,  $p_i \in \mathbb{N}$ , is a polynomial that represents the transient and  $q(\gamma) = \bigoplus_{j=0}^{\nu-1} q_j \gamma^j$ ,  $q_i \in \mathbb{N}$ , is a polynomial that represents a pattern which is repeated each  $\tau$  time units and each  $\nu$  firings of the transition.

<sup>3</sup>Asymptotic slope in a manufacturing context can be viewed as the production rate of the system. The ratio  $\sigma_\infty(s)$  is calculated in the conventional algebra.

customers demand). This idea is illustrated in the figure 1. In this figure,  $u$ ,  $y$  and  $H$  represents respectively the input, the output and the system transfer function; the reference model is given by  $G_{ref}$  and the external output<sup>4</sup> is given by  $v$ . The control problem is presented in the following. Let  $H \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times p}$  be the matrix transfer function of the system,  $G_{yv} \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times p}$  be the controlled system transfer function and  $G_{ref} \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times p}$  be a reference model *i.e.*, the desired transfer function for the controlled system. So the problem can be stated as follows.

Given a TEG, which controller will assure the greatest transfer function between  $u$  and  $v$  ( $G_{uv}$ ), such that  $G_{yv} \preceq G_{ref}$ ? Considering the just-in-time context, the goal is to obtain a controller satisfying the reference specification ( $G_{yv} \preceq G_{ref}$ ) while delaying as much as possible the product admissions into the system.

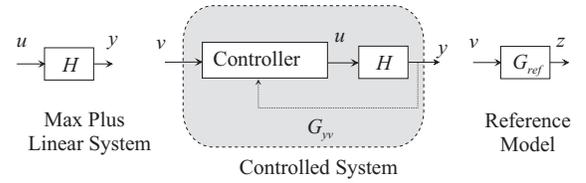


Fig. 1. Model Reference Control

#### A. RST-structure control

It is known that the existence of the greatest feedback controller depends on the model reference choice [6]. A control structure based on a precompensator and a feedback controller improves the controlled system performance, as shown in [13]. This approach always lead to an optimal behavior of the closed-loop system and assures better robustness properties than the simple feedback [11].

In this section, a more general structure is proposed based on the well known RST-structure for continuous systems. This control approach is based on three control matrices as illustrated in the figure 2. By using figure 2, one has the

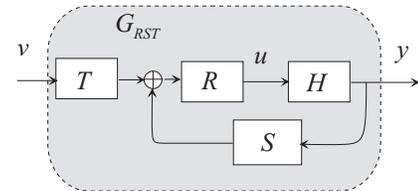


Fig. 2. RST-structure Control

following control law

$$u = R(Tv \oplus Sy). \quad (11)$$

Therefore  $y = Hu = HRTv \oplus HRSy$ , then theorem 1

<sup>4</sup>In a Just-in-Time context,  $v$  represents the available supply of raw material and  $u$  represents the allowance of the raw material into the system.

and formulae 5 yield to the following equations.

$$y = (HRS)^*HRTv = H(RSH)^*RTv \quad (12)$$

$$= G_{RST}v \quad (13)$$

$$u = (RSH)^*RTv. \quad (14)$$

So the objective is to obtain matrices  $R, S, T$  that maximize  $u$  with the restriction that  $G_{RST} \preceq G_{ref}$ , where  $G_{ref}$  is the model reference. As in [13], one can show that:

$$\begin{aligned} & H(RSH)^*RT \preceq G_{ref} \\ \Leftrightarrow & (RSH)^*RT \preceq H \backslash G_{ref} \\ \Leftrightarrow & H(RSH)^*RT \preceq H(H \backslash G_{ref}). \end{aligned} \quad (15)$$

From these inequalities, one can solve the control problem by observing that matrices  $R$  and  $T$  must satisfy:

$$RT \preceq H \backslash G_{ref}. \quad (16)$$

Actually the upper bound  $H \backslash G_{ref}$  is always reachable; for instance take  $R = I^{p \times p}$ ,  $S = [\varepsilon]_{p \times m}$  and  $T = H \backslash G_{ref}$ , which correspond to the precompensator case. In this case, by taking equation 14 and expression 15 into account, the control action is given by  $(H \backslash G_{ref})v$ , and it is maximized since  $u \preceq (H \backslash G_{ref})v$ . Therefore it is important to observe that one possible solution for the proposed control problem is achieved if one takes,  $RT = H \backslash G_{ref}$ . By considering this particular solution, the expression 15 assures the following developments.

$$\begin{aligned} & H(RSH)^*(H \backslash G_{ref}) \preceq H(H \backslash G_{ref}) \\ \Leftrightarrow & (HRS)^*H(H \backslash G_{ref}) \preceq H(H \backslash G_{ref}) \end{aligned} \quad (17)$$

$$\Leftrightarrow (HRS)^* \preceq (H(H \backslash G_{ref})) \backslash (H(H \backslash G_{ref})) \quad (18)$$

$$\Leftrightarrow HRS \preceq (H(H \backslash G_{ref})) \backslash (H(H \backslash G_{ref})) \quad (19)$$

The expression 17 is a consequence of equation 5; 18 is a result of the residuation definition and 19 comes from 4 and 6.

As a result, the RST control problem can be solved if  $RT = H \backslash G_{ref}$  with the restriction given by the inequality 19. Synthetically, an optimal control is obtained if

$$RT = H \backslash G_{ref} \quad (20)$$

$$HRS \preceq (H(H \backslash G_{ref})) \backslash (H(H \backslash G_{ref})). \quad (21)$$

In general, the calculation of matrices  $R, S$  and  $T$  is, in some extent, equivalent to solve the diophantine equation for conventional linear continuous systems. This task can be easily done by using some particular control structures of the proposed RST-structure. These cases are presented and discussed below.

#### R-Structure

This case is known in literature as precompensation approach [4]. In this structure, the controlled system is simplified by considering that  $T = I^{p \times p}$  and  $S = [\varepsilon]_{p \times m}$ . So the problem is solved by finding the greatest precompensator  $R$  such that  $HR \preceq G_{ref}$ . The optimal solution, denoted by  $R_{opt}$ , is obtained directly from equation 20:

$$R_{opt} = H \backslash G_{ref}. \quad (22)$$

This result means that, for a given external input  $v \in (\overline{\mathbb{Z}}_{\max}[\gamma])^p$ , the input vector, given by  $u = Rv$ , will be maximal. In fact, for any  $R$  such that  $HR \preceq G_{ref}$ ,  $R \preceq R_{opt}$ , therefore the isotony property assures that  $u = Rv \preceq R_{opt}v$ . However, since it is an open-loop approach, it does not reinforce the stability<sup>5</sup> of the system and is not efficient when disturbances act on the system( see [13] for details).

#### S-Structure

In this case one sets matrices  $R$  and  $T$  equal to identity and it is known as feedback approach. By using equation 13, the closed-loop transfer matrix between  $y$  and  $v$ , for a given feedback controller  $S$ , is given by  $H(SH)^*$ . Therefore the problem is solved by finding the greatest  $S$  such that  $H(SH)^* \preceq G_{ref}$ . This problem can be solved via residuation theory if some restrictions are imposed on the reference model. The following result is due to [6].

*Proposition 1:* Let  $H \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times p}$  be the transfer function of a TEG. For every reference model  $G_{ref}$  such that  $G_{ref} = E^*H$  or  $G_{ref} = HF^*$  (where  $E \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{m \times m}$  and  $F \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{p \times p}$ ) there exists a greatest feedback controller  $S \in (\overline{\mathbb{Z}}_{\max}[\gamma])^{p \times m}$  such that the transfer function of the closed-loop system is less than or equal to  $G_{ref}$ . The greatest feedback controller is:

$$S_{opt} = H \backslash G_{ref} \backslash H.$$

One should remark from equation 14 that in this strategy the transfer function between  $u$  and  $v$  is  $(SH)^*$ .

*Property 1:* If  $H(SH)^* \preceq G_{ref}$ , for given  $H$  and  $G_{ref}$ , then  $(SH)^* \preceq R_{opt} = H \backslash G_{ref}$ .

The proof comes from the fact that  $ax \preceq b \Leftrightarrow x \preceq a \backslash b$ . This property means that the transfer function between  $u$  and  $v$  for the optimal open-loop strategy is always greater than or equal to the one obtained for any feasible feedback strategy.

It is important to remark that under some restriction on the reference model the simple feedback (S-structure) is better than (R-structure) in the stabilization context [13], [6]. However the precompensation always leads to an optimal control action which is equal to  $R_{opt}v$  and there is no restriction concerning the reference model choice. The next structure to be presented exploits the main advantages of these two approaches.

#### RS-structure

In [13], it was considered a particular case where the precompensator  $T$  is equal to the identity matrix. This structure is shown in figure 3. In this case the direct application of residuation theory to the system given by 20 and 21 leads to optimal controller matrices given by:

$$R_{opt} = H \backslash G_{ref}, \quad (23)$$

$$S_{opt} = (HR_{opt}) \backslash (HR_{opt}) \backslash (HR_{opt}). \quad (24)$$

<sup>5</sup>Stability here means that the number of tokens of system is bounded.

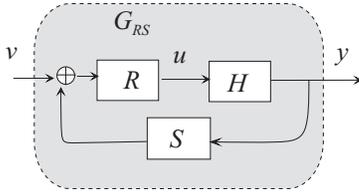


Fig. 3. RS-structure Control

### ST-structure: output feedback

This is a new control structure. It is obtained from the RST one by simply making matrix  $R$  equal to the identity. It is illustrated in the figure 4.

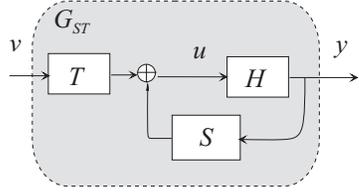


Fig. 4. ST-structure Control

This approach always leads to an optimal control structure. Actually, since  $R = I^{p \times p}$ , by using system given by 20 and 20, one can easily show that the optimal matrices are

$$T_{opt} = H \backslash G_{ref} \quad (25)$$

$$S_{opt} = H \backslash (HT_{opt}) \phi (HT_{opt}) \quad (26)$$

At this point, it is known that both RS-structure and ST-structure lead to an optimal control structure. Therefore, one question arises: what are the differences between them? One difference is pointed out in the property below.

*Property 2:* For the ST structure, the feedback from the output  $y$  to the input  $u$  is greater than or equal the one obtained for the RS-structure. *Proof:* By inspection of figures 4 and 3 one can see that it suffices to show that the feedback given by equation 26 is greater than or equal the product of equation 23 by equation 24. In fact, by using equation 7, equation 24 can be rewritten as:  $(HR_{opt}) \backslash (HR_{opt}) \phi (HR_{opt}) = R_{opt} \backslash (H \backslash (HR_{opt})) \phi (HR_{opt})$ . By equation 23,  $R_{opt} = H \backslash G_{ref}$ , so equation 8 leads to:  $(HR_{opt}) \backslash (HR_{opt}) \phi (HR_{opt}) = R_{opt} \backslash R_{opt} \phi (HR_{opt})$ . Therefore, for the RS-structure, one uses equation 1 to obtain  $R_{opt} S_{opt} \preceq R_{opt} \phi (HR_{opt})$ .

In contrast, for the ST-structure, the feedback is given by  $H \backslash (HT_{opt}) \phi (HT_{opt})$ , where  $T_{opt} = H \backslash G_{ref}$ . So expression 2 assures that  $H \backslash (HT_{opt}) \succeq T_{opt}$ . By isotony,  $H \backslash (HT_{opt}) \phi (HT_{opt}) \succeq T_{opt} \phi (HT_{opt})$ . One concludes the proof by remembering that  $T_{opt} = R_{opt}$ . ■

*Remark 1:* This results may suggest that the ST-structure is more favorable to stability than the RS one since its greatest feedback leads, in the worst case, to the same

number of arcs (from output to input) of the RS one. Concept of stability here means that the number of tokens of the closed-loop system remains bounded and one must remember that a strongly connected graph is always stable. For more detail regarding stability see [1] and [3].

## IV. SIMULATION RESULTS

The objective here is to illustrate the application of model reference approach to control a max-plus linear system represented by a Timed Event Graph.

Consider the TEG depicted in gray box of the Fig. 5 as an example. It models a workshop with 3 machines ( $M_1$  to  $M_3$ ). Machines  $M_1$  and  $M_2$  can process parts which are assembled by machine  $M_3$ . Inputs  $u_1$  and  $u_2$  represent the admission dates of parts into the system; the transportation time is 1 for both parts; machine  $M_1$  can process one part in 2 time units; machine  $M_2$  can process one part in 3 time units. The transportation time between machines  $M_1$  and  $M_3$  is 4 time units and between machines  $M_2$  and  $M_3$  is 5 time units. Machine  $M_3$  can assemble one part in 4 time units, the transportation time until the output is 6 time units and  $y$  represents the output date of the final product. As discussed in section II, one must remind that this system can be modelled by equations 9 and 10 and the input-output relationship is given by the equation  $y = Hu$ , where  $H$  is the transfer function. Therefore appropriated computations<sup>6</sup> lead to:

$$y = [17(4\gamma)^* \quad 19(4\gamma)^*]u. \quad (27)$$

The obtained relationship means that the transfer functions between  $y$  and  $u_1$  and  $u_2$  have a maximum production rate of  $1/4$ . This is a direct consequence of the workshop configuration since the machine  $M_3$  has the slowest production rate (actually, it is the bottleneck for the system).

To compare RS and ST control strategies, the reference model is chosen as  $G_{ref} = [17(4\gamma)^* \quad 19(5\gamma)^*]$ . That is, the objective is to keep the production rate between  $y$  and  $u_1$  unchanged but to allow the one between  $y$  and  $u_2$  to reduce until  $1/5$ . The obtained results are given below.

### A. RS-structure

By using equations 23 and 24, one obtains the solution:  $R_{opt} = \begin{pmatrix} (4\gamma)^* & 2(5\gamma)^* \\ 2\gamma(4\gamma)^* & (5\gamma)^* \end{pmatrix}$  and  $S_{opt} = (3\gamma^5(4\gamma)^* \quad \varepsilon)^t$ . Figure 5 shows one realization of the controlled workshop system.

<sup>6</sup>Softwares to handle dioid algebra using Scilab can be downloaded from the site [16].

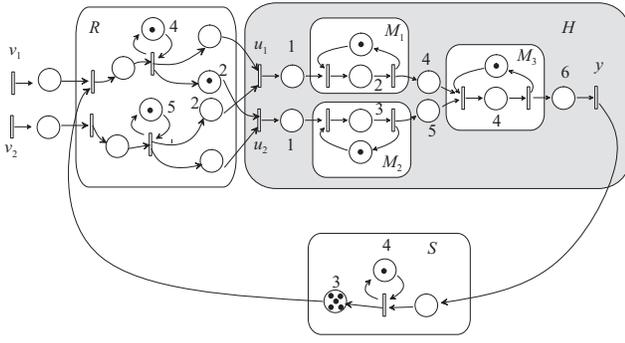


Fig. 5. Controlled Workshop System with the RS-strategy.

### B. ST-structure

By using the equations 25 and 26, one obtains the following solution:  $T_{opt} = \begin{pmatrix} (4\gamma)^* & 2(5\gamma)^* \\ 2\gamma(4\gamma)^* & (5\gamma)^* \end{pmatrix}$  and  $S_{opt} = (3\gamma^5(4\gamma)^* \quad 1\gamma^5(4\gamma)^*)^t$ . Figure 6 shows one realization of the controlled workshop system.

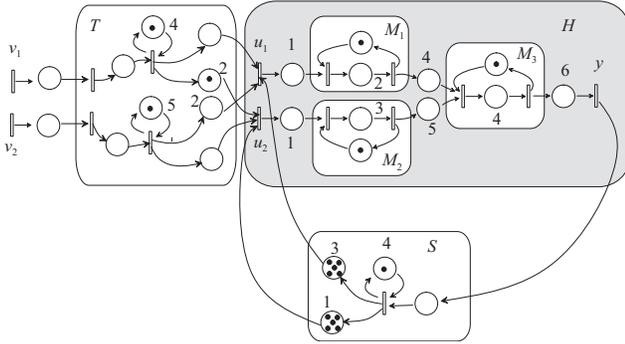


Fig. 6. Controlled Workshop System with the ST-strategy.

**Remark 2:** Both RS and ST approaches lead to an optimal closed-loop system in a just-in-time sense, that is, the control action is maximized. However in this example calculation results in  $R_{opt}S_{opt} = (3\gamma^5(4\gamma)^* \quad 5\gamma^6(4\gamma)^*)^t$  for the RS-strategy. The obtained feedback matrix for the ST-strategy is  $(3\gamma^5(4\gamma)^* \quad 1\gamma^5(4\gamma)^*)^t$ . By remembering that  $1\gamma^5(4\gamma)^* = 1\gamma^5 \oplus 5\gamma^6(4\gamma)^*$ , the feedback from the output  $y$  to the input  $u$  is greater than the one obtained for the RS-structure. This is a consequence of property 2.

## V. CONCLUSIONS AND FUTURE WORKS

This paper presents a general structure for the model-reference control of DES described by the max-plus algebra. The approach is based on the well known RST-structure control for conventional differentiable system. The presented results encompasses the ones found in the literature and some comparisons concerning performance and stability between particular cases are made. Investigations concerning robustness of the proposed structure, as initiated by [11], is an important issue. Another step is the control design in the uncertain environment as proposed by [10].

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