COINC Library

A Scilab toolbox for Network Calculus

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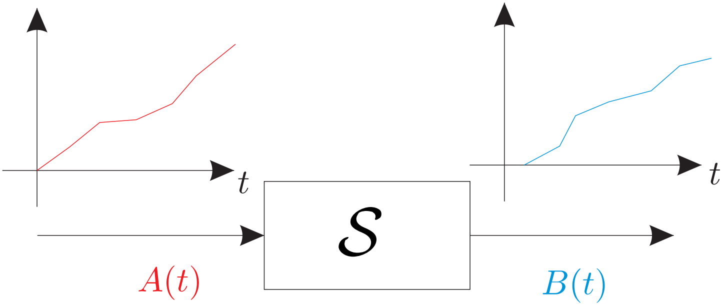
Overview

- Network Calculus
  - Arrival curves
  - Service curves
  - Backlog and Delay bounds
- COINC library
  - Data structure
  - Operations
- Conclusion

Network Calculus

Network Calculus is a theory based on the $(min, +)$ algebra that enables the computation of deterministic performance bounds in communication networks.



Network Calculus

Arrival Curve

The flow $A(t)$ is constrained by the arrival curve $\alpha(t)$ if

$$\forall s, 0 \leq s \leq t, A(t) \leq \alpha(t-s) + A(s)$$

$$A(t) \leq A(t) \otimes \alpha(t)$$

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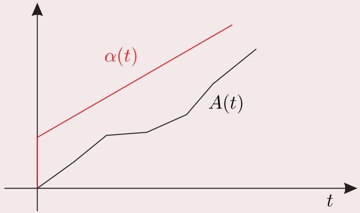
Example : $\alpha(t) = \sigma + \rho t$



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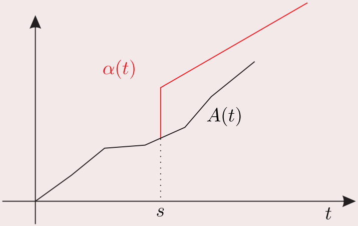
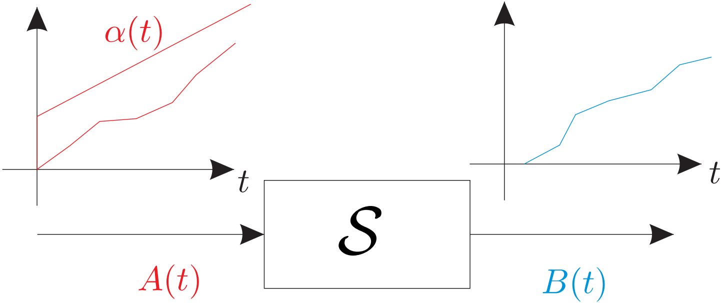
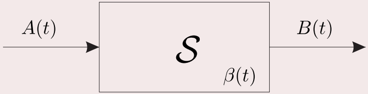


Network Calculus



Network Calculus

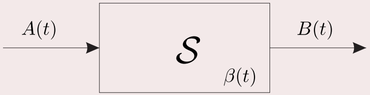
Service curve



Consider a system $\mathcal{S}$ with input function $A(t)$ and output function $B(t)$. $\beta(t)$ is a service curve if $B(t) \geq A(t) \otimes \beta(t)$

Network Calculus

Service curve



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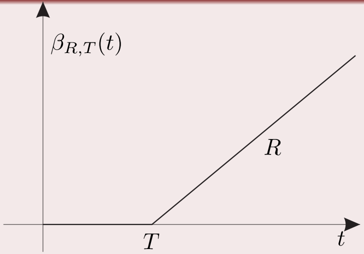
Example : rate-latency service curve :$\beta_{R,T}(t) = R(t - T)_+$



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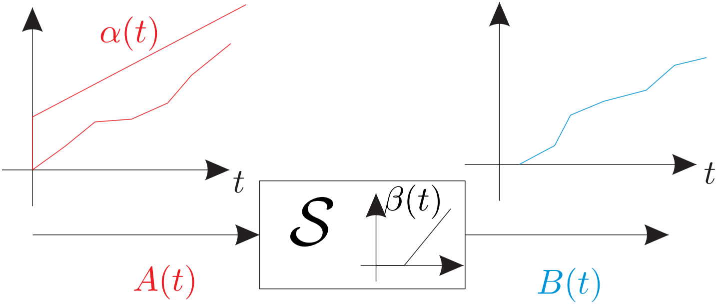
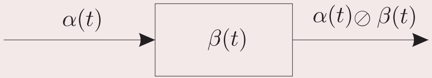


Network Calculus

Output arrival curve

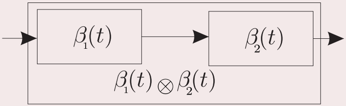
Assume a flow, constrained by arrival curve α, traverses a system that offers a service curve of β. The output flow is constrained by the arrival curve $\alpha \oslash \beta$.



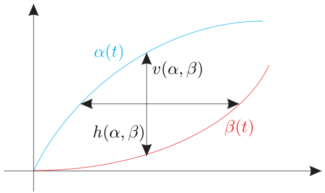
Network Calculus

Concatenation of systems

Assume a flow traverses systems $\mathcal{S}_1$ and $\mathcal{S}_2$ in sequence that offers a service curve $\beta_i, i = 1, 2$. Then the concatenation of $\mathcal{S}_1$ and $\mathcal{S}_2$ offers a service curve of $\beta_1 \otimes \beta_2$.



Network Calculus : Bounds



Virtual Backlog :
$V(t) = A(t) - B(t)$
Delay :
$D(t) = \inf\{d | B(t+d) \geq A(t)\}$

Performance bound

Assume a flow, constrained by arrival curve α, traverses a system that offers a service curve β. One has

$$V_{max} \leq (\alpha \oslash \beta)(0)$$

$$D_{max} \leq \inf\{d : (\alpha \oslash \beta)(-d) \leq 0\}$$

Network Calculus : Elementary Operations

Definitions

Let f and g be two functions from $\mathbb{R}$ to $\mathbb{R} \cup \{+\infty, -\infty\}$,

- $\min(f,g)(t) = \min(f(t), g(t))$.
- $\max(f,g)(t) = \max(f(t), g(t))$.
- $(f+g)(t) = f(t) + g(t)$.
- $(f-g)(t) = f(t) - g(t)$. Example
- $(f \otimes g)(t) = \inf_{0 \le s \le t}(f(s) + g(t-s))$ (Convolution).
- $(f \oslash g)(t) = \sup_{s \ge 0}(f(t+s) - g(s))$ (Deconvolution).
- $f^\star(t) = \inf(f^0, f^1, f^2, f^3, \dots)$ (sub-additive closure).

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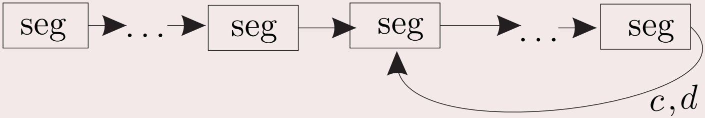
Data Structure

Ultimately Pseudo Periodic Function



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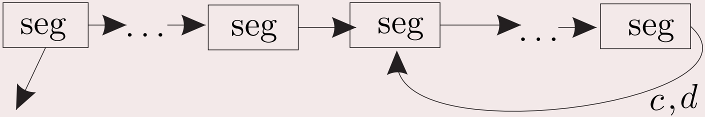
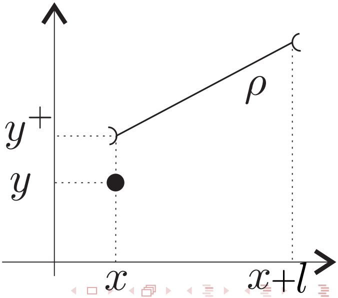


Segment s

s consist of 6 elements

$s = (x, y, y^+, \rho, l)$ where

- (x, y) is the initial point
- y^+ is the right image of x
- ρ is the slope
- l is the length



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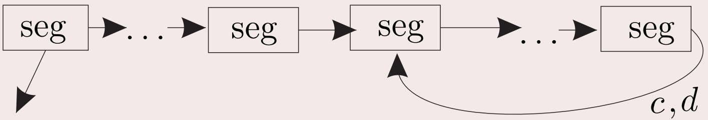

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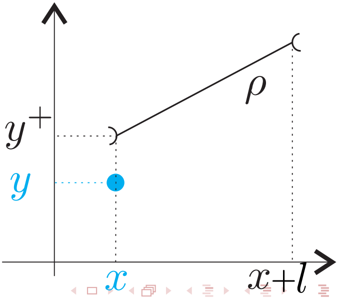

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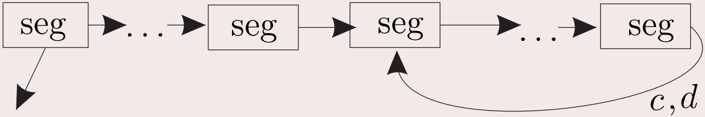

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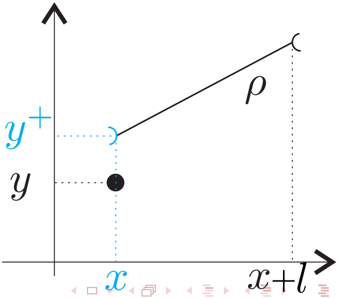

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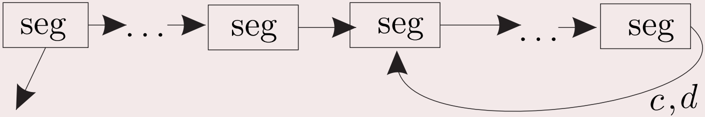

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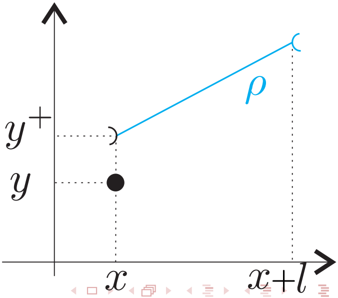

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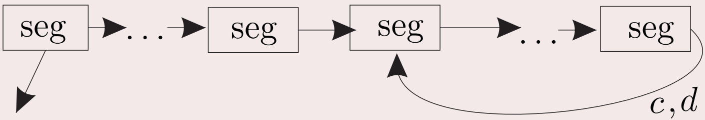

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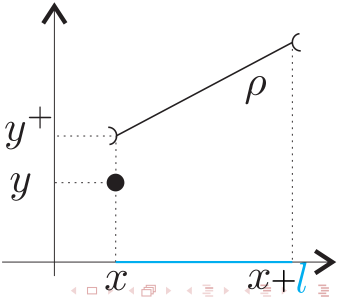

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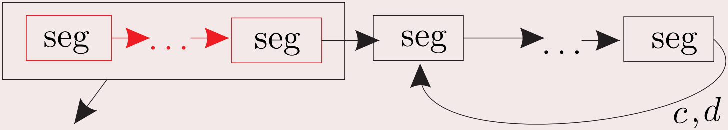

Transient Part

A list of segments

$f = s_1 \oplus s_2 \oplus s_3 \oplus s_4$

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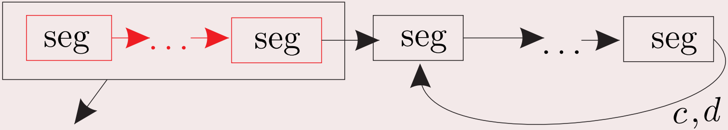

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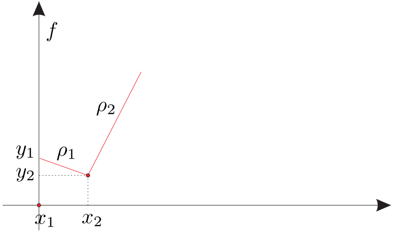

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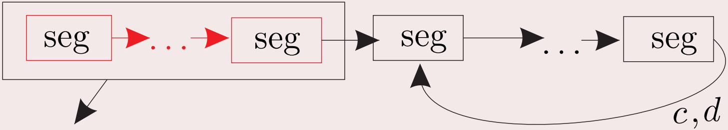

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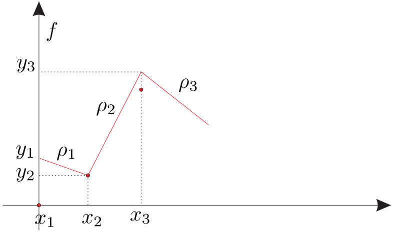

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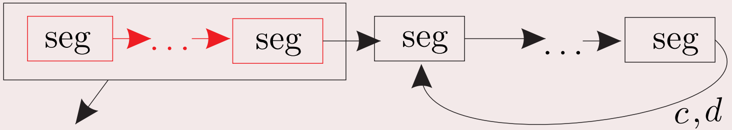

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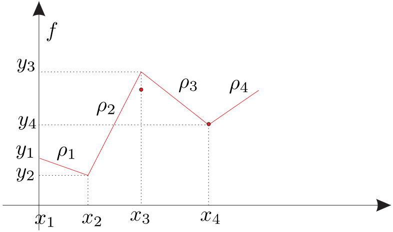

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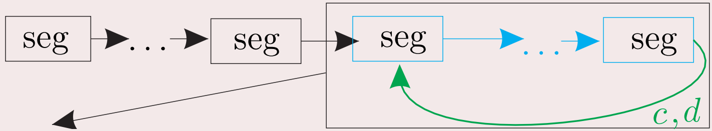
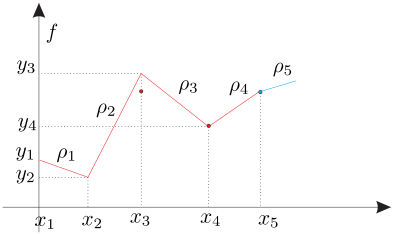


Periodic Part

A list of segments $f = s_1 \oplus s_2 \oplus s_3 \oplus s_4 \oplus (s_5 \oplus s_6)(c, d)^{\star}$



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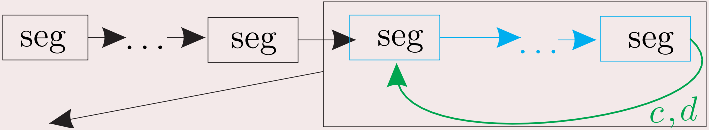

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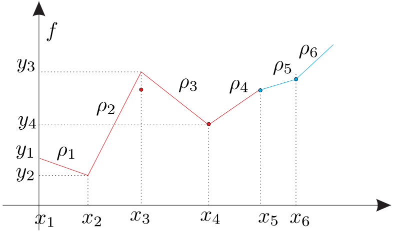

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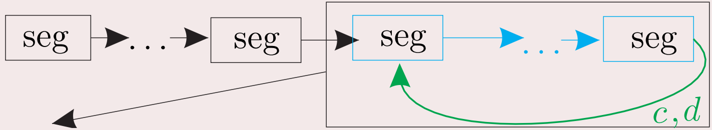

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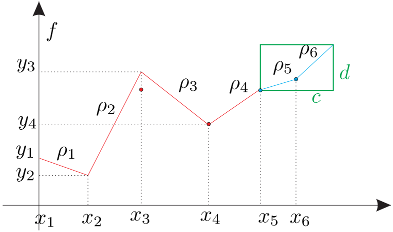

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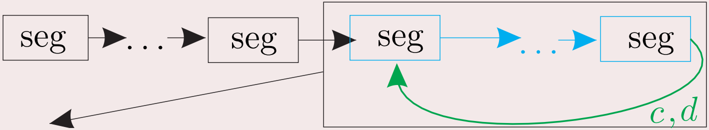

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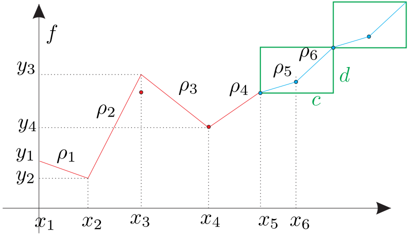
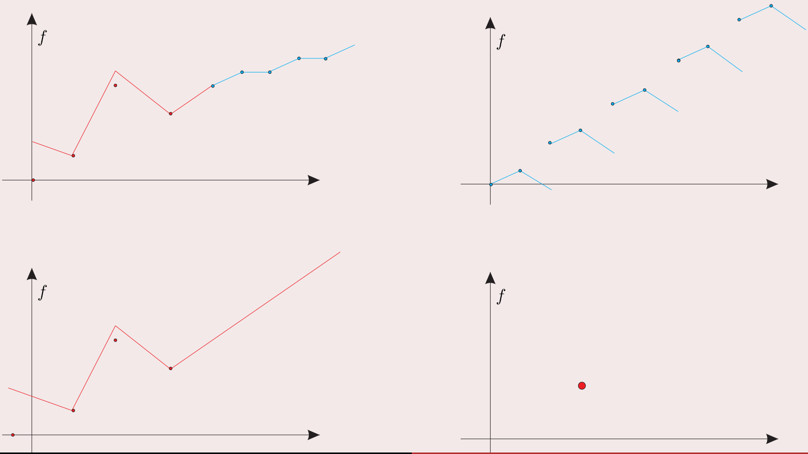


Data Structure

Example of Ultimately Pseudo Periodic Functions



Min of ultimately pseudo periodic functions

$\min(f_1(t), f_2(t))$

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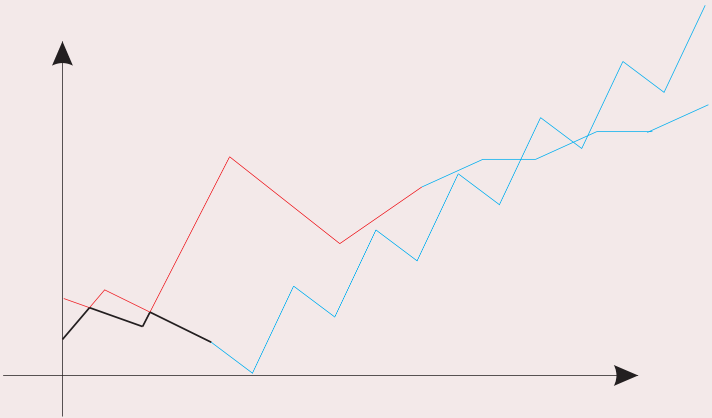

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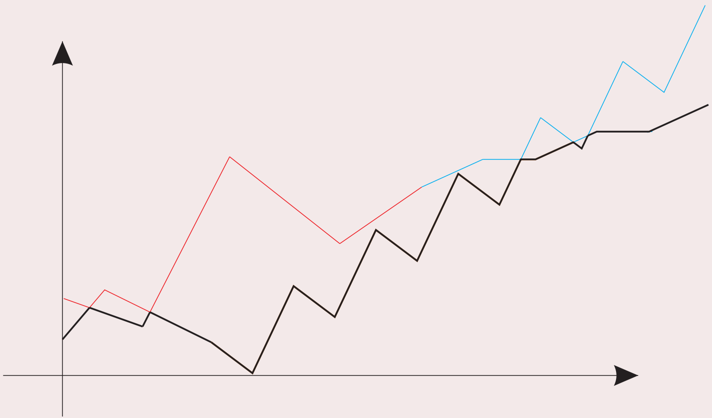
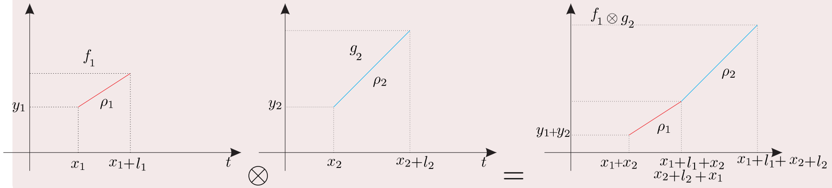


Convolution of ultimately pseudo periodic functions

$f(t) \otimes g(t) = \inf_{0 \leq s \leq t} f(t) + g(t-s)$

$f \otimes g = (\min_i(f_i)) \otimes (\min_j(g_j)) = \min_{ij}(f_i \otimes g_j)$ (Distributivity).

convolution of segments



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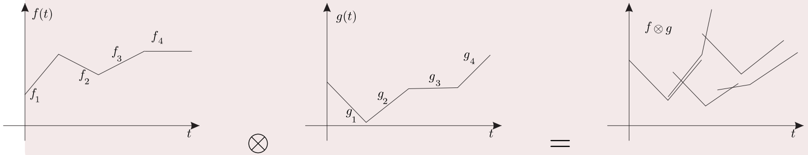


Convolution of ultimately pseudo periodic functions

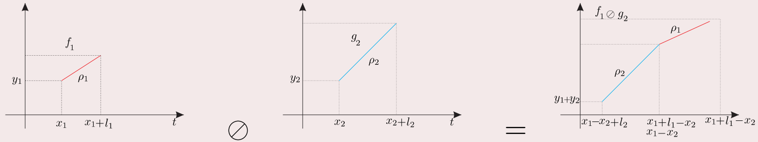
geometrical representation of convolution

Deconvolution of ultimately pseudo periodic functions

$$f(t) \oslash g(t) = \sup_{0 \leq s \leq t} f(t+s) - g(t)$$

$f \oslash g = (\max_i(f_i)) \oslash (\min_j(g_j)) = \max_{ij}(f_i \oslash g_j)$ (Distributivity).

Deconvolution of segments



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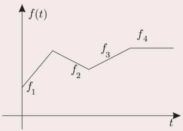
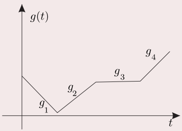


$\oslash$



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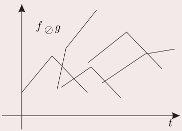

Sub-additive closure of ultimately pseudo periodic functions

Sub-additive closure of ultim. pseud. period. functions

Let $f = p \oplus q \otimes r^\star$. One has

$$f^\star = p^\star \otimes (e \oplus q \otimes (q \oplus r)^\star)$$

- $p^\star$ and $(q \oplus r)^\star$: Sub additive closure of a finite set of segments.
- Minimum and Convolution operations.

Algo

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How to use the Scilab Toolbox

Conclusion

software

- Formal series $(\max, +)$ (C++/Scilab) [Gaubert, Hardouin et al, 1992].
- CyNC - Cyclic Network Calculus (Matlab/Simulink) [Schioler et al, 2005].
- DISCO Network Calculator (Java) [Schmitt et al, 2006].
- RTC - Real Time Calculus Toolbox (Java/Matlab) [Thiele et al, 2006].
- COINC - COmputational Issues in Network Calculus (C++/Scilab).

http://perso.bretagne.ens-cachan.fr/~bouillar/coinc/

Steck of Sub additive closure algorithm

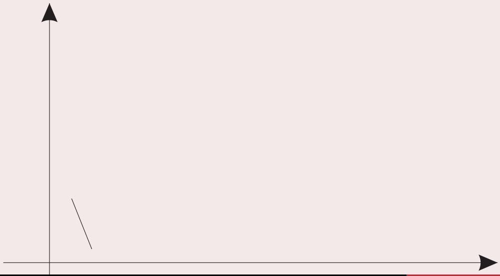
Sub additive closure of a finite set of segments

$$p^\star = (p_1 \oplus p_2 \oplus \cdots \oplus p_n)^\star$$

$$p^\star = p_1^\star \otimes p_2^\star \otimes \cdots \otimes p_n^\star$$

Back

Sub additive closure of segments : $p_1^\star = e \oplus p_1 \oplus p_1^2 \oplus p_1^3 \oplus \ldots$



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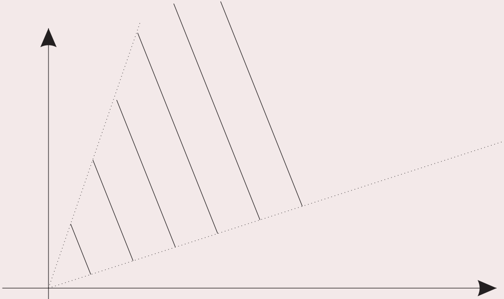
Back

Sub additive closure of segments : $p_1^\star = e \oplus p_1 \oplus p_1^2 \oplus p_1^3 \oplus \ldots$



Steck of Sub additive closure algorithm

Sub additive closure of a finite set of segments

$$p^\star = (p_1 \oplus p_2 \oplus \cdots \oplus p_n)^\star$$

$$p^\star = p_1^\star \otimes p_2^\star \otimes \cdots \otimes p_n^\star$$

Back

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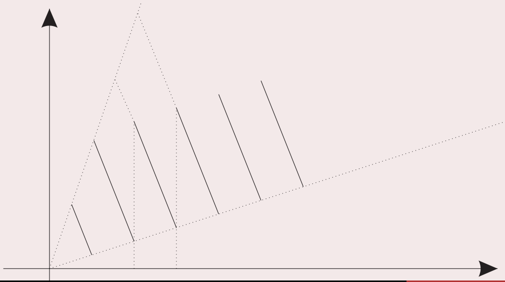
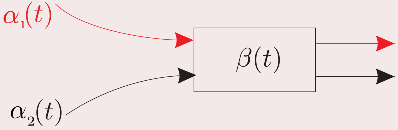


Residual service curve

Consider a node offering a strict service curve β and two flows entering that server, with respective arrival curves α_1 and α_2. Then a service curve for flow 1 is $\beta_1 = (\beta - \alpha_2)_+$.



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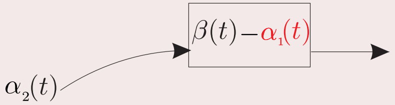

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