Analytical solution of the blind source separation Problem using derivatives

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Problem description :

Blind Sources Separation (BSS) deals with the problem of separating sources only from their observed mixture while both the mixing process and the sources are unknown.

This poster presents a new approach for separating sources in linear instantaneous mixtures, based on second order statistics of signals and their first derivatives. Since it only requires second order statistics, the method is then able to separate Gaussian sources.

For two mixtures of two sources, an analytical solution can be derived. For larger mixtures, the method requires the joint diagonalization of the variance-covariance matrix of signals and of signal derivatives.

Theorem :

Let $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$, be an unknown regular mixture of the sources $\mathbf{s}(t)$ whose components are ergodic, stationary, derivable and mutually independent signals.

The separating matrix $\mathbf{B}$, such that $\mathbf{y}(t) = \mathbf{B}\mathbf{x}(t)$ has mutually independent components, is the solution of the equation set :

$$\left.\begin{array}{ll} \mathbf{B}E[\mathbf{x}\mathbf{x}^T]\mathbf{B}^T & = E[\mathbf{y}\mathbf{y}^T] \\ \mathbf{B}E[\dot{\mathbf{x}}\dot{\mathbf{x}}^T]\mathbf{B}^T & = E[\dot{\mathbf{y}}\dot{\mathbf{y}}^T] \end{array}\right\} (1)$$

where $E[\mathbf{y}\mathbf{y}^T]$ and $E[\dot{\mathbf{y}}\dot{\mathbf{y}}^T]$ are diagonal matrices.

Statistical Independence Results :

Notation: Two independent random signals $s_1(t)$ and $s_2(t)$ will be denoted $s_1(t)\mathbb{I}s_2(t)$.

Properties:

$$x_1(t)\mathbb{I}s(t) \Rightarrow s(t)\mathbb{I}x_1(t)$$

$$x_1(t)\mathbb{I}s(t),\dots,x_n(t)\mathbb{I}s(t) \Rightarrow (x_1(t)+\cdots+x_n(t))\mathbb{I}s(t)$$

$$x_1(t)\mathbb{I}s(t) \Rightarrow \lambda x_1(t)\mathbb{I}s(t)$$

Lemma: Let $s_1(t)$ and $s_2(t)$ be differentiable signals, then

$$s_1(t)\mathbb{I}s_2(t) \Rightarrow \left\{\begin{array}{l} s_1(t)\mathbb{I}\dot{s}_2(t) \\ \dot{s}_1(t)\mathbb{I}s_2(t) \\ \dot{s}_1(t)\mathbb{I}\dot{s}_2(t) \end{array}\right.$$

Sketch of proof: $s_1(t)\mathbb{I}s_2(t)$ and $s_1(t)\mathbb{I}s_2(t+\tau)$

Hence $s_1(t)\mathbb{I}\frac{s_2(t+\tau)-s_2(t)}{\tau}$.

Since $s_2(t)$ is differentiable, $s_1(t)\mathbb{I}\dot{s}_2(t)$.

The other relations are proved similary.

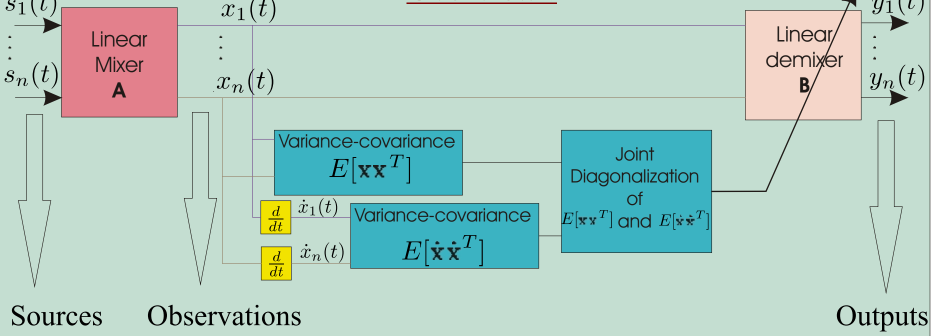
Algorithm :

For two mixtures of two sources, an analytical solution of (1) can be easily derived (see paper for details).

For larger mixtures, algorithm requires two steps :

- computation of variance-covariance matrices of signals and their first derivatives,
- joint diagonalization (e.g. with JADE) of these two matrices.

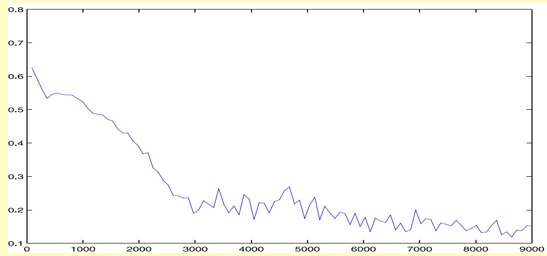
Overview :



Experimental Result :

The Figure presents the separation performance for Gaussian mixture (using index $E[norm(s-\hat{s})]$) versus the sample number.

Over 2600 samples, the analytical solution provides good performance.



Futher works

Conditions on signals for insuring joint diagonalization of $E[\dot{\mathbf{y}}\dot{\mathbf{y}}^T]$ And $E[\dot{\mathbf{y}}\dot{\mathbf{y}}^T]$ which lead to separation.

Extentions to higher order derivatives for improving algorithm robustness.

Extensions to more complex signals mixtures, e.g. based on state variable models.

Demonstration

A solver can be downloaded on my web page
http://www.istia.univ-angers.fr/~lagrange