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# Robust stabilization of the current profile in Tokamak plasmas using sliding mode approach in infinite dimension

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## Abstract

This paper deals with the robust stabilization of the spatial distribution of tokamak plasmas current profile using a sliding mode feedback control approach. The control design is based on the 1D resistive diffusion equation of the magnetic flux that governs the plasma current profile evolution. The feedback control law is derived in the infinite dimensional setting without spatial discretisation. Numerical simulations are provided and the tuning of the controller parameters that would reject uncertain perturbations is discussed. Closed loop simulations performed on realistic test cases using a physics based tokamak integrated simulator confirm the relevance of the proposed control algorithm in view of practical implementation.

*Keywords:* Tokamak plasmas control, Parabolic partial differential equations, Lyapunov stabilization, sliding mode control, Robust control

## 1. Introduction

Fossil fuels (oil, gas, coal) account for approximately 85% of the worldwide sources of primary energy today. But they should run out with in some tens of years and they are responsible for a climate change via the contribution in the greenhouse effect of the $CO_2$ generated by their combustion.

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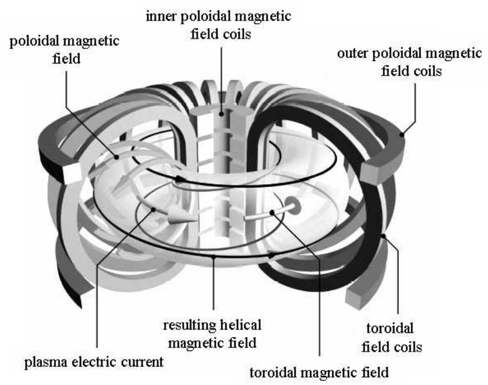


Figure 1: Tokamak magnetic configuration

The controlled thermonuclear fusion is one of the options being studied in order to eventually provide an answer to these issues. Its main assets are to be a potential inexhaustible and safe source of energy because the reserves in nuclear fuel are plenty (Deuterium can be extracted from sea water, and Lithium, that has to be used to breed Tritium, can be found in continental crust) and because there is no risk of runaway reaction nor long lasting radioactive waste.

The key world project in the domain, ITER, is led by seven partners (Europe, United States of America, Japan, China, India, South Korea, Russia) accounting for one half of the world population. The main objective of the ITER project is to demonstrate the scientific feasibility of thermonuclear fusion.

Several conditions have to be met to produce fusion reactions (Wesson (2004)): the fuels have to be heated up to very high temperature (around 100 millions degrees) in order to overcome the electrostatic potential barrier between positively charged nucleus. To reach such a temperature, the ionized gas or plasma must be confined, for example by magnetic confinement which seems to be the most promising way. This magnetic confinement is obtained in a tokamak (Wesson (2004) and Saoutic et al. (2009)) by superimposing different electric currents in a torus-like configuration device, including a high current, of the order of the MA, within the plasma itself (see Fig. 1).

This plasma current can be produced by inductive means, in particular at the beginning of the plasma pulses, and by non-inductive means through the injection of fast particles and /or waves (Wesson (2004) and Fisch (1987)). The 1D radial profile of this plasma current, via the so-called safety factor profile, is known to be a key parameter for tokamak plasma performance. It indeed plays a crucial role in the global magnetohydrodynamic (MHD) (Wesson (2004) and Freidberg (2007)) stability of plasma experiments. Moreover, it has been observed that some specific profiles may generate some enhanced confinement of the plasma energy (see for example the work of Baranov et al. (2004), Challis et al. (2002), Taylor (1997) and Wolf (2003)). It is obvious that such profiles are

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very attractive and may at the end reduce the size and cost of future fusion reactors. Several approaches have been developed regarding the control of tokamak plasmas current profile. The control of one single shape parameter, based on Single Input - Single Output (SISO) semi-empirical approaches, has been performed experimentally (e.g plasma internal inductance by Wijnands et al. (1997) or non-inductive current drive profile width by Mazon et al. (2002)). But this is clearly not enough to match the main requirements of MHD stability and/or internal transport barrier issues in advanced tokamak scenarios.

The control of the safety factor profile in a few number of points, based on a Multi Input Multi Output (MIMO) approach in finite dimensional setting was developed using linear models identified from experimental data (Moreau et al. (2008)). It was experimentally tested but showed severe limitations in terms of robustness, given also the lack of power from the actuators as shown by Ariola et al. (2008). Some other recent approaches have been developed on simulations (see the work of Ou & Schuster (2009a), Xu et al. (2010), Argomedo et al. (2010) and Ou et al. (2011)).

Some nonlinear control techniques for tokamak reactors are presented in Ou et al. (2007), Ou & Schuster (2009b) and Ou et al. (2010). In Ou et al. (2007), extremum seeking approach is implemented for optimal control of a nonlinear distributed parameter system. In Ou & Schuster (2009b) and Ou et al. (2010), the evolution of the poloidal magnetic flux profile (modeled by a nonlinear partial differential equation), is controlled using a proper orthogonal decomposition (POD) and Galerkin projection.

The approach proposed in this paper is to design the control law in the infinite dimensional setting (without spatial discretisation) and without linearisation. This allows to keep as long as possible the physical phenomena at stake, and in particular the meaning of the terms manipulated, and thus to better integrate the knowledge of model uncertainties and disturbances during the design control process (whereas for linear finite dimensional models obtained for instance from identification, the physics parameters are mixed up in the resulting transfert functions of $A$, $B$, $C$, $D$ matrices coefficients, thus making the physical uncertainties very difficult to handle). The model reduction is performed at the latest stage, that is to say when the real engineering inputs have to be set using a model based optimisation process (a realistic estimation of the infinite dimensional control inputs is then obtained).

In order to investigate the practical revelance of the proposed control approach, closed loop simulations using one of the internationally recognised physics based tokamak integrated simulator have been performed.

The paper is organized as follows. The overall description of the control problem is given in section 2. In section 3 a mathematical analysis of the distributed control model is proposed. Details are given on the transformation of the PDE that are required in order to prepare the control design. Section 4 is devoted to the construction of a control law. Simulation results, using the METIS (see the work of Artaud (2008)) code dedicated to plasma scenario studies, are provided in section 5. A preliminary version of the proposed results without simulation METIS has been published by Gaye et al. (2011). Finally,

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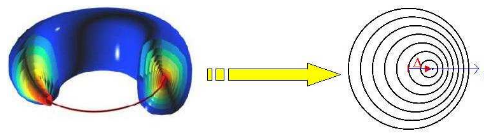


Figure 2: 1D geometry of the simplified formulation of the problem

conclusion remarks are collected in section 6.

## 2. Control problem description

The quantity to be controlled is the so called $q$ safety factor profile. Indeed, this physics quantity which characterises the twisting of the magnetic field lines on the magnetic equilibrium surfaces ($q$ measures the number of turns around the tokamak main axis that a magnetic field lines has to make before going back to the same position in a meridian plane) is extremely important in tokamak research. In particular, different energy confinement regimes were observed depending on the $q$ profile being either monotonous or reversed. Rational values of $q$ play also a very important role for what concerns MHD stability.

An equivalent output quantity is the magnetic flux profile relative to its edge value (at $x = 1$)

$$\psi_r(t,x) = \psi(t,x) - \psi(t,1) \tag{1}$$

as

$$\psi_r(t,x) = a^2 B_0 \int_x^1 \frac{r}{q(t,r)} dr.$$

Provided usual assumptions (axisymmetry, MHD equilibrium, averaging over the magnetic surfaces, cylindrical approximation, etc. (see the work of Blum (1989) and Witrant et al. (2007))), the evolution of the plasma safety factor $q$ can be obtained by solving the following 1D PDE

$$\begin{cases} \frac{\partial \psi}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x}\left( x \frac{\partial \psi}{\partial x}\right) + \eta_{||} R_0 j_{ni}; \\ \left.\frac{\partial \psi}{\partial x}\right|_{x=0} = 0, \quad \frac{\partial \psi}{\partial t}(t,1) = -V_0(t). \end{cases} \tag{2}$$

$$q = -\frac{a^2 x B_0}{\frac{\partial \psi}{\partial x}}$$

and

$$j_T = -\frac{1}{\mu_0 R_0 a^2 x} \frac{\partial}{\partial x}\left( x \frac{\partial \psi}{\partial x}\right)$$

where $x \in (0,1)$ is the $1D$ (radial) profile coordinate, $\psi(t,x)$ the magnetic flux, $R_0$ and $a$ are respectively the major and minor radius of the plasma boundary

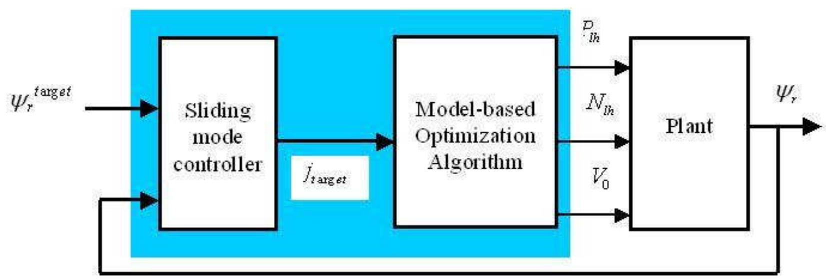


Figure 3: Control scheme of the system

(assumed to be fixed), $\mu_0$ the permeability of vacuum, $\eta_{||}(t,x)$ the parallel resistivity of the plasma, $B_0$ the toroidal magnetic field at $R_0$ and $j_T(t,x)$ is the total current density. $q(t,x)$ is the safety factor to be controlled (plant output).

The control variables in the infinite dimensional setting are the plasma loop voltage $V_0(t)$ and the non-inductive current density $j_{ni}(t,x)$. $V_0$ can basically be directly set using the inner poloidal magnetic field coils voltage (see Fig. 1). The non-indutive current density is composed of the bootstrap current density which is self-generated by the plasma (Hirshman (1998)) and of additional source terms provided by different actuators, namely the Lower Hybrid, Electron Cyclotron or Ion Cyclotron wave systems and/or the Neutral Beam Injection system. The most efficient is the Lower Hybrid Current Drive (LHCD) system that is routinely used on the Tore Supra tokamak (Saoutic et al. (2009)), which now has a capability to inject up to around 7 MW in steady state that shall allow to sustain plasma currents in the megaAmpere range on very long pulse (see the work of Ekedahl et al. (2010) and Becoulet et al. (2011)).

The control law design that will be derived in section 4 is generic to all kinds of current drive system. However, in this paper, the numerical simulations will be performed on Tore Supra typical conditions, using mainly Lower Hybrid waves as current drive means, so that in this particular case :

$$j_{ni} = j_{bs} + j_{lh}$$

where $j_{bs}$ and $j_{lh}$ are respectively the bootstrap current density and the current density provided by the LHCD system. $j_{bs}$ can be modelled by a nonlinear function of the flux (Hirshman (1998)). $j_{lh}$ is modelled by Gaussian fonctions controlled by two engineering parameters: the LH power $P_{lh}$ and the wave refractive index $N_{lh}$, (see the work of Witrant et al. (2007) for further details). $V_0$, $P_{lh}$ and $N_{lh}$ are the considered control variables (plant inputs) for the simulations of section 5.

In the present investigation, the resistivity $\eta_{||}(t,x)$ is assumed to be lower and upper bounded by some positive constants $\eta_1$ and $\eta_2$. Moreover, let us consider that $\eta_{||}(t,x)$ is available for feedback purposes through some on-line estimation, basically from electronic temperature measurements (see the work of Witrant et al. (2007) for more details).

The feedback control designed in this paper is composed of two steps (see Fig. 3).

The first step provides the current density $j_{target}$ to be applied to the system. Here, a sliding mode controller has been designed in infinite dimensional setting whose derivation will be detailed in the following. It takes as inputs the target flux profile $\psi_r^{target}$ and the current flux profile provided by the plant. The fact that the output $j_{target}$ has to be reachable by the actuators means that the reference has to be carefully set. The difference between the target current density $j_{target}$ and the one that the actuators can provide is handled by the robustness of the controller as an unknown bounded disturbance on the input ($|j_T - j_{target}| < \Delta_j, \forall x \in [0,1]$).

The second step is an optimization process which finds the best set of parameters $P_{lh}, N_{lh}$ and $V_0$ to fit the current density target $j_{target}$ provided by the sliding mode control law in the infinite dimensional setting. The optimisation algorithm miminizes the quadrature (Ouarit et al. (2011))

$$\varepsilon\left(P_{lh}, N_{lh}, V_0\right) = \int_0^1 \left(j_{target} - \left(j_\Omega + j_{lh} + j_{bs}\right)\right)^2 dx$$

under the constraints

$$V_{\min} < V_0 < V_{\max},\ P_{\min} < P_{lh} < P_{\max},\ N_{\min} < N_{lh} < N_{\max}$$

where $j_\Omega = j_T - j_{ni}$ is the steady state ohmic part of the current density, equal to $\frac{-V_0}{\eta_{||} R_0}$.

In the simulations, the minimum / maximum value of the engineering inputs will be taken from the Tore Supra tokamak constraints, i.e.

$$-5V < V_0 < 5V,\ 0MW < P_{lh} < 7MW,\ 1.43 < N_{lh} < 2.37$$

At each time step, the optimization algorithm looks for the set of control variables that minimize the quadratic criterion $\varepsilon\left(P_{lh}, N_{lh}, V_0\right)$ considering the previous constraints on $P_{lh}$, $N_{lh}$ and $V_0$. Then the total current $j_T = j_\Omega + j_{lh} + j_{bs}$ is the real current applied at the following time step. In practice, in-line minimization is performed using a numerical Matlab optimization toolbox (gradient-based algorithm *fmincon*).

## 3. Control model mathematical analysis

The state equation (2), rewritten in terms of $\psi_r$, reduces to

$$\begin{cases} \frac{\partial \psi_r}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x}\left(x \frac{\partial \psi_r}{\partial x}\right) + \eta_{||} R_0 j_{ni} + V_0(t) \\ \left.\frac{\partial \psi_r}{\partial x}\right|_{x=0} = 0, \quad \psi_r(t,1) = 0 \end{cases} \tag{3}$$

Then, let us introduce the error variable

$$\phi(t,x) = \psi_r(t,x) - \psi_r^{target}(x) \tag{4}$$

with respect to the steady state target $\psi_r^{target}(x)$ which has to be reached in the Sobolev space

$$W^{1,2}(0,1) = \left\{\psi \in L^2(0,1) : \frac{\partial \psi}{\partial x} \in L^2(0,1)\right\}$$

of differentiable functions, whose spatial derivative is square integrable on the interval $(0,1)$. Then the error variable is governed by

$$\begin{cases} \frac{\partial \phi}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2}\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \phi}{\partial x}\right) + \frac{\eta_{||}}{\mu_0 a^2}\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \psi_r^{target}}{\partial x}\right) + \eta_{||}R_0 j_{ni} + V_0(t); \\ \left.\frac{\partial \phi}{\partial x}\right|_{x=0} = -\left.\frac{\partial \psi_r^{target}}{\partial x}\right|_{x=0}, \quad \phi(t,1) = -\psi_r^{target}(1). \end{cases} \tag{5}$$

In order to deal with the regular term $\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \psi_r^{target}}{\partial x}\right)$ in (5) and homogeneous boundary conditions let us assume that

$$\lim_{x\to 0}\left|\frac{1}{x}\frac{\partial \psi_r^{target}}{\partial x}\right| < +\infty; \tag{6}$$

$$\left.\frac{\partial \psi_r^{target}}{\partial x}\right|_{x=0} = 0; \qquad \psi_r^{target}(1) = 0. \tag{7}$$

These assumptions lead to the system with homogeneous boundary conditions

$$\begin{cases} \frac{\partial \phi}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2}\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \phi}{\partial x}\right) + \frac{\eta_{||}}{\mu_0 a^2}\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \psi_r^{target}}{\partial x}\right) + \eta_{||}R_0 j_{ni} + V_0(t); \\ \left.\frac{\partial \phi}{\partial x}\right|_{x=0} = 0, \quad \phi(t,1) = 0. \end{cases} \tag{8}$$

Since

$$\frac{\eta_{||}}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \phi}{\partial x}\right) = \frac{\partial}{\partial x}\left(\eta_{||}\frac{\partial \phi}{\partial x}\right) + \left(\frac{\eta_{||}}{x} - \frac{\partial \eta_{||}}{\partial x}\right)\frac{\partial \phi}{\partial x} \tag{9}$$

the equation (8) with the singular term $\frac{1}{x}$ can be brought into the regular form without singularities by applying the following feedback transformation

$$\eta_{||}R_0 j_{target} = \eta_{||}R_0 j_{ni} + V_0 = \eta_{||}u + v \tag{10}$$

with a virtual control input $u$ and the relation

$$v = \frac{1}{\mu_0 a^2}\left(-\frac{\eta_{||}}{x} + \frac{\partial \eta_{||}}{\partial x}\right)\frac{\partial \phi}{\partial x} - \frac{\eta_{||}}{\mu_0 a^2}\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \psi_r^{target}}{\partial x}\right) \tag{11}$$

is deduced from the state transformations (1) and (4). Then substituting (10) subject to (11) into (8) yields

$$\begin{cases} \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\eta_{||}}{\mu_0 a^2}\frac{\partial \phi}{\partial x}\right) + \eta_{||}u; \\ \left.\frac{\partial \phi}{\partial x}\right|_{x=0} = \phi(t,1) = 0. \end{cases} \tag{12}$$

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The resulting equation is a standard parabolic equation in the Sobolev space

$$H = \left\{ \phi \in W^{1,2}(0,1) : \left. \frac{\partial \phi}{\partial x} \right|_{x=0} = \phi|_{x=1} = 0 \right\}$$

of the square integrable functions subject to the boundary conditions corresponding to the boundary value problem (12) and equipped with the norm $\|\phi\| = \sqrt{\int_0^1 \left(\frac{\partial \phi}{\partial x}\right)^2 dx}$. The operator

$$A\phi = \frac{\partial}{\partial x}\left(\frac{\eta_{||}}{\mu_0 a^2}\frac{\partial \phi}{\partial x}\right), \tag{13}$$

that appears in the right-hand side of the PDE (12), is defined on the domain $D(A) = \left\{ \phi \in H : \frac{\partial^2 \phi}{\partial x^2} \in L^2(0,1) \right\} \subset L^2(0,1)$. This operator is recognized as a Sturm-Liouville operator (see Naylor & Sell (1982)) and its spectrum consists of the discrete values

$$\lambda_k = -\frac{1}{\mu_0 a^2}\left(\frac{\left(k-\frac{1}{2}\right)\pi}{\int_0^1 \frac{ds}{\eta_{||}(s)}}\right)^2, \; k = 1, 2, \ldots$$

that correspond to the following eigenfunctions

$$\phi_k^0(x) = \cos\left(\left(k - \frac{1}{2}\right)\pi \frac{\int_0^x \frac{ds}{\eta_{||}(s)}}{\int_0^1 \frac{ds}{\eta_{||}(s)}}\right) \tag{14}$$

for the scalar product $< f|g >_{\frac{1}{\eta_{||}}} = \int_0^1 f(x)g(x)\frac{dx}{\eta_{||}(x)}$ and for all $k$

$$I_1(k) = \| \phi_k^0(x) \| = \sqrt{\int_0^1 \phi_k^0(x)^2 \frac{dx}{\eta_{||}(x)}}.$$

The Sturm-Liouville operator $A$ generates an exponentially stable semigroup. Moreover, the following theorem, being applied to (12), shows that the system (12) has continuous solutions (see the work of Cannarsa et al. (2004) and Rosier (2007) for more details).

**Theorem 1.** *For given $z_0 \in \boldsymbol{H}$ where $\boldsymbol{H}$ is a Hilbert space, let us consider the Cauchy problem*

$$\left\{ \begin{array}{l} \dot{z} = Az + Bu; \\ z(0) = z_0. \end{array} \right. \tag{15}$$

*where $A$ is the infinitesimal generator of a strongly continuous semigroup $(S(t))_{t \geq 0}$ on $L^2(0,1)$. Let $u$ belongs to $L^2((0,T) \times (0,1))$ for all $T > 0$. Then for all $z_0 \in D(A)$ where $D(A)$ is the domain of the operator, there exists a unique* *strong solution $z \in \mathcal{C}([0,+\infty); D(A))$ of (15); and for all $z_0 \in L^2(0,1)$ there exists a unique weak solution $z \in \mathcal{C}\left([0,+\infty); L^2(0,1)\right)$ of (15). The strong solution of (15) is given by the Duhamel formula*

$$z(t) = S(t)z_0 + \int_0^t S(t-s)Bu(s)ds \quad \forall t \in [0,T].$$

*The above formula is still meaningful and defines the weak solution of (15).*

The system (12) is null controllable if $u$ belongs to $L^2$ (see the work of Cannarsa et al. (2004, 2006) for details). It implies that the system (12) is exponentially stabilizable ((Rosier, 2007, Theorem 4.12, p. 407)). Consequently, the sliding mode strategy developed by Orlov (2000) can be applied.

## 4. Sliding mode controller

The problem of the control of partial differential equations (PDEs) is an active area of research (see the work of Coron (2007) and Smyshlyaev & Krstic (2010) for example), but very few constructive methods are available. For robust stabilization, a sliding mode strategy has nevertheless recently been developed by Orlov (2000).

The sliding mode control approach, developed by Orlov (2000) for infinite dimensional systems, is further adapted to be applied to our control problem. The virtual control input $u$ is designed as follows:

$$u(\phi) = -\left(\mathcal{N} + L\sqrt{\sum_{k=0}^{k=N_d} \left(P^k(\phi)\right)^2}\right) sign\left(\sum_{k=0}^{k=N_d} c_k P^k(\phi)\right) \tag{16}$$

where $C = (c_k)_{k=0}^{N_d}$ is the matrix which defines the sliding surface, $\mathcal{N}$ is determined to reject the disturbances, $L$ is determined to ensure the Lyapunov stability and $N_d$ is the number of projections $P^k$ on the eigenfunctions of the operator $A$ defined by $P^k(\phi) = \frac{1}{I_1(k)^2}\int_0^1 \phi(t,x)\phi_k^0(x)\frac{dx}{\eta_{||}}$, $k \in \{0,\ 1, \ldots\}$. The following relation describes the sliding motion along the sliding surface (see the work of Orlov (2000))

$$\dot{x}_1 = [A_1 - B_1\left(CB_1\right)^{-1} CA_1]x_1 \tag{17}$$

where $x_1 = \left(P^0\phi(t,.), ..., P^{N_d}\phi(t,.)\right)^T$ is the projection of $\phi$ on the first $N_d + 1$ eigenfunctions of operator $A$ and where

$$A_1 = diag\left\{\lambda_k\right\}_{k=0}^{N_d} \in \mathbb{R}^{N_d+1 \times N_d+1}$$

and

$$B_1 = \left(P^0\eta_{||}, P^1\eta_{||}, ..., P^{N_d}\eta_{||}\right)^T.$$

Let us consider

$$A_c = A_1 - B_1\left(CB_1\right)^{-1} CA_1 \tag{18}$$

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then

$$CA_c = CA_1 - CB_1 \left(CB_1\right)^{-1} CA_1 = 0 \Leftrightarrow A_c^T C^T = 0. \tag{19}$$

Consequently $C^T$ is an eigenvector of $A_c^T$ associated to the eigenvalue $\lambda = 0$. Now let us introduce $K = \left(CB_1\right)^{-1} CA_1$. From (17),

$$\dot{x}_1 = [A_1 - B_1 K]x_1. \tag{20}$$

The matrix $K$ is chosen in order to define an appropriate system dynamics on the sliding surface. The choice of $K$ does not affect the stability but determine the characteristics of sliding mode. $K$ is chosen such that $A_c^T$ has a first eigenvalue equal to zero while other eigenvalues are strictly negative. In order to determine $L$, the system in $x_1$ without disturbance ($\mathcal{N} = 0$) is considered

$$\dot{x}_1 = A_1 x_1 + B_1 u \tag{21}$$

and the Lyapunov function

$$V = \frac{1}{2} S^2 > 0, \quad with\ S = Cx_1.$$

It follows that

$$\begin{array}{rcl} \frac{dV}{dt} & = & S\dot{S} = Cx_1 C\left(A_1 x_1 + B_1 u\right) \\ & = & Cx_1 CA_1 x_1 - Cx_1 CB_1 L \parallel x_1 \parallel sign(S). \end{array}$$

Then $\dot{V} < 0$ if and only if

$$Cx_1 CA_1 x_1 < CB_1 L \parallel x_1 \parallel \left|Cx_1\right|. \tag{22}$$

Knowing that

$$Cx_1 CA_1 x_1 \leq \left|Cx_1\right| \parallel CA_1 \parallel \parallel x_1 \parallel \tag{23}$$

and

$$CB_1 L \parallel x_1 \parallel \left|Cx_1\right| \leq \left|Cx_1\right| \parallel CB_1 \parallel L \parallel x_1 \parallel. \tag{24}$$

In order to have $\dot{V} < 0$ it is sufficient to have

$$\left|Cx_1\right| \parallel CA_1 \parallel \parallel x_1 \parallel < CB_1 L \parallel x_1 \parallel \left|Cx_1\right|.$$

Then from (23) and (24),

$$L > \frac{\parallel CA_1 \parallel}{\parallel CB_1 \parallel}. \tag{25}$$

The constant $L$ is lower bounded. Moreover, the proposed control law (16), specified with (12), rejects any additive external disturbances satisfying a matching condition

$$\alpha(t,x) = \eta_{||} h(t,x) \tag{26}$$

$$\left\{ \begin{array}{l} \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\eta_{||}}{\mu_0 a^2}\frac{\partial \phi}{\partial x}\right) + \eta_{||} u + \alpha(t,x); \\ \left.\frac{\partial \phi}{\partial x}\right|_{x=0} = \phi(t,1) = 0. \end{array} \right. \tag{27}$$

O. Gaye, E. Moulay, S. Brémond, L. Autrique, R. Nouailletas, J.F. Artaud, Y. Orlov,
Robust stabilization of the current profile in Tokamak plasmas using sliding mode approach in infinite dimension,
Control Engineering Practice, vol. 21, pp. 1350-1358, 2013.

with a priori known upper bounds $\mathcal{H} > 0$ provided that

$$\| h \| \leq \mathcal{H} < \mathcal{N}. \tag{28}$$

The difference between the target current density $j_{target}$ and the one that the actuators can provide is $\delta(t,x) = j_{target} - j_T$. It appears in our EDP in the form $\eta_{||} R_0 \delta = \eta_{||} \widetilde{h}$, which is the matching condition. Then the different $\delta(t,x)$ is handled by the robustness of the controller if $\| \widetilde{h} \| < \mathcal{N}$.

Summarizing the following result is obtained.

**Theorem 2.** *Considering the error system (12) and the sliding mode controller (16), if* $L > \frac{\|CA_1\|}{\|CB_1\|}$*, the closed-loop system is globally exponentially stable in the state space* $H$.

*Moreover, if system (12) is additively affected by an external disturbance* $\alpha(t,x)$ *(see system (27)) then the stability remains in force if* $\| h \| \leq \mathcal{H} < \mathcal{N}$.

The system (12) is approximated by the finite dimensional system (21) which is indeed finite time stabilized by the proposed controller (16). However, the infinite-dimensional rest of the system possesses the spectrum with negative real parts so that it is globally exponentially stable in the open-loop and it remains so while driven by the sliding mode controller, vanishing in finite time.

## 5. Simulation results

In this section, numerical simulations are performed in order to illustrate the sliding mode approach relevance in the context investigated (robust stabilisation of the current profile in Tokamak plasmas in infinite dimension). Evolution of the current spatial profile $q$ from an initial state to a desired target profile is considered (see Fig. 4). We then also checked the effect of the disturbance

$$h(t,x) = \begin{cases} 0, & 0s \leq t < 10s \\ -\frac{5}{100\mu_0 R_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi_r^{target}}{\partial x} \right), & 10s \leq t \leq 20s \end{cases}$$

added at the time instant $t = 10s$ and representing 5% of the target total current density $j_T^{target}$. First of all, without engineering parameters research optimization process, parameters of the sliding mode controller are tuned. In order to illustrate closed loop system behavior, time evolution of $\psi_r$ is drawn at point $x = 0.4$ (behaviors are similar for other points). In Fig. 5a, Fig. 5b and 6 we have the result of simulations without METIS code. In Fig. 5a, effect of parameter $L$ is shown (while other control parameters are fixed). In Fig. 5b, effect of parameter $\mathcal{N}$ is shown for disturbances rejection purpose. As expected, (i) the closed loop system is stable, (ii) the larger the parameter $L$, the smaller the steady state error resulting from a disturbance; moreover time response can be speeded up using large parameter $L$ at the expense of possibly overshoots, (iii) parameter $\mathcal{N}$ allows to reject the steady state disturbance ($\mathcal{N}$ in section 4 aims to reject the disturbances $\alpha(t,x) = \eta_{||} h(t,x)$ if $\mathcal{N} > \| h \|$), (iv) large values of parameter $\mathcal{N}$ can speed up the rejection of steady state disturbance (but can induce large overshoots). In Fig. 6 we have the evolution of the variable $\psi_r$ at some instants.

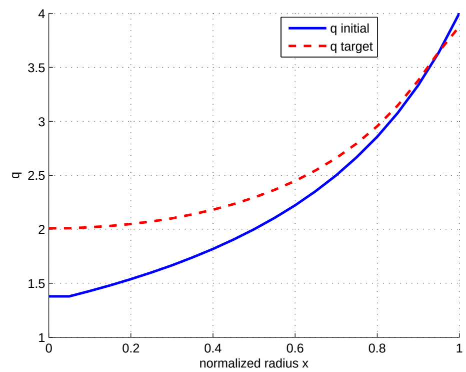
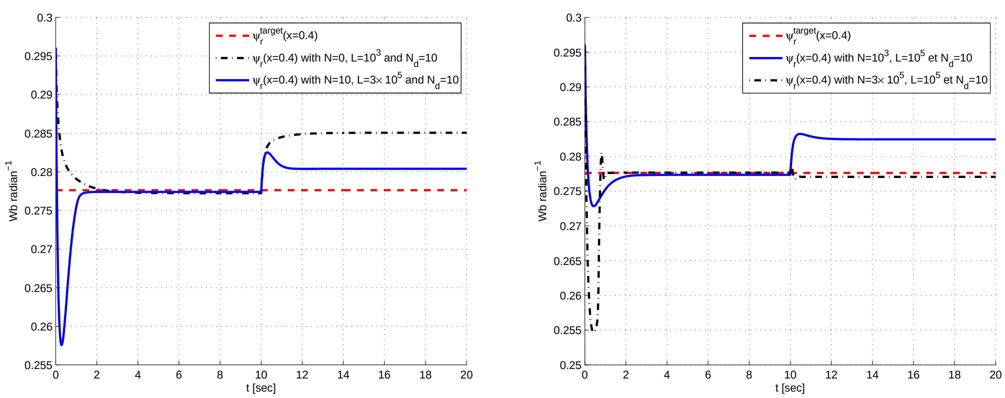



Figure 4: Initial and target profile of safety factor $q$




(a) Time evolution of $\psi_r$ profile at $x = 0.4$

(b) Time evolution of $\psi_r$ profile at $x = 0.4$

Figure 5: Time evolution of magnetic flux $\psi_r$

Then, in order to evaluate if the proposed control approach is realistic from the experimental point of view, simulations using METIS (Minute Embedded Tokamak Integrated Simulator) (Artaud (2008)) tokamak plant simulator are performed. METIS is a simplified version of the CRONOS suite of codes developed by Artaud et al. (2010) adapted to the final simulation test of tokamak control algorithms on physics relevant model. The METIS code includes a full

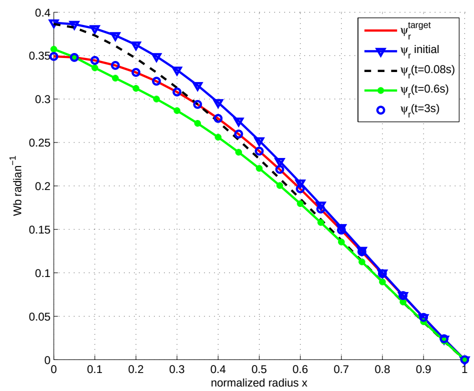



Figure 6: Evolution of $\psi_r$ at some instants with $\mathcal{N} = 310^5$, $L = 10^5$ and $N_d = 10$

fast current diffusion solver and takes into account various nonlinear couplings between physical quantities. Such numerical simulator is quite time consuming (simulation for a $20s$ long Tore Supra tokamak plasma discharge takes about 3 hours). In the following, METIS is used jointly with the $Matlab/Simulink^{TM}$ toolbox to simulate Tore Supra plasma discharges. As an initial step, considering real pulse engineering inputs ($V_0$, $P_{lh}$ and $N_{lh}$) METIS open loop simulations were performed in order to determine a reachable target profile $\psi_r$.

The sliding mode controller feedback is then implemented with $\mathcal{N} = 3\ 10^5$, $L = 10^5$ and $N_d = 10$ (parameters previously adjusted in infinite dimension see 5a and Fig. 5b). The global feedback control scheme described in section 2 (see Fig. 3) is taken into account. In order to illustrate the method robustness, a disturbance

$$h(t,x) = \begin{cases} 0, & 0s \leq t < 9s \\ -\frac{5}{100\mu_0 R_0 a^2}\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial \psi_r^{target}}{\partial x}\right), & 9s \leq t \leq 20s \end{cases}$$

was added at $t = 9s$.

An example of simulation results is given at Fig. 7, 8a, 8b, 9a and 9b. At $t = 1s$, the controller is activated in order to force the magnetic flux profile to reach the desired target. The time evolution of the magnetic flux $\psi_r$ at point $x = 0.2$, $x = 0.6$ and $x = 0.8$ are shown on Fig. 8b, 9a and 9b. Time evolution of the engineering plant inputs is plotted in Fig. 8a. The $P_{lh}$, $N_{lh}$ and $V_0$ parameters behave as expected and experimental constraints for the engineering inputs are satisfied.

## 6. Conclusion and future work

In this paper, a sliding mode control in the infinite dimensional setting has been designed for the control of tokamak plasma current profiles using 1D re-

O. Gaye, E. Moulay, S. Brémond, L. Autrique, R. Nouailletas, J.F. Artaud, Y. Orlov,
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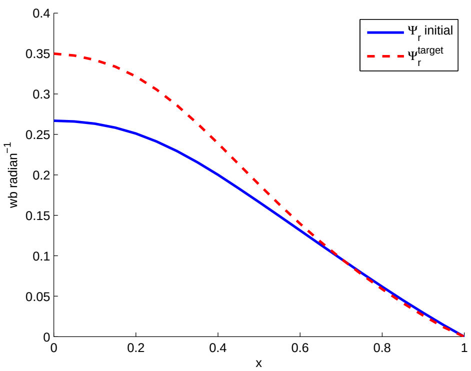
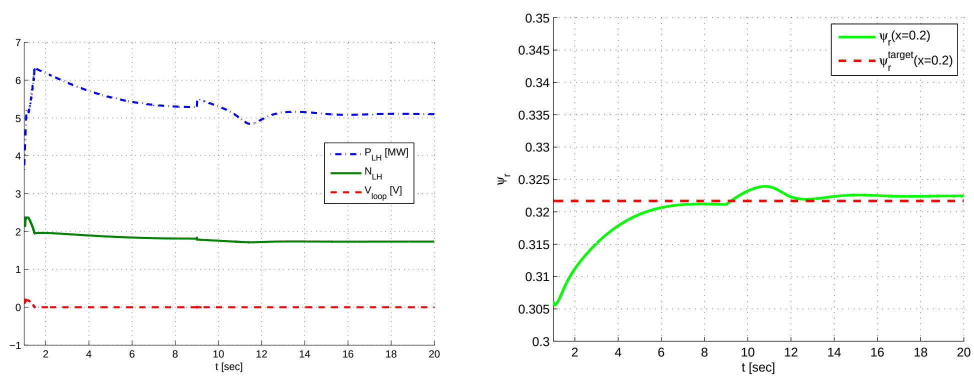



Figure 7: Initial and target profile of $\psi_r$ with METIS code




(a) Time evolution of engineering inputs

(b) Time evolution of $\psi_r$ at $x = 0.2$

Figure 8: Time evolution of engineering inputs and $\psi_r$ at $x = 0.2$

sistive diffusion equation of the magnetic flux. The investigated PDE system was reformulated so as to exhibit a Sturm-Liouville operator. Recent results on sliding mode control in infinite dimension were further developed to this control problem.

Then a model-based optimisation process was used to derive the engineering plant inputs related to both inductive and non-inductive current drive means. Numerical simulations were performed using typical Tore Supra values that provide quite positive results with promissing robustness properties. The practical interest of the new robust controler based on sliding mode approach in infinite dimension is demonstrated using METIS (Minute Embedded Tokamak Integrated Simulator). Future work will consist in preparing implementation on

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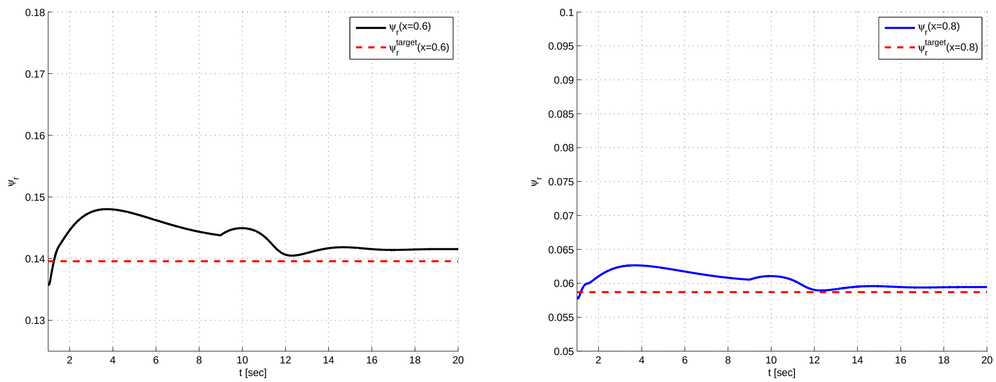


(a) Time evolution of $\psi_r$ at $x = 0.6$ (b) Time evolution of $\psi_r$ at $x = 0.8$

Figure 9: Time evolution of $\psi_r$ at $x = 0.6$ and $x = 0.8$

real Tore Supra experiments.

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