Robotics Tutorial 1: Basic Concepts

Exercise 1:

Consider the following matrix: $R_{A,B}(\rho,\beta) = \begin{pmatrix} \cos\rho & -\sin\rho & 0 \\ \sin\rho\cos\beta & \cos\rho\cos\beta & -\sin\beta \\ \sin\rho\sin\beta & \cos\rho\sin\beta & \cos\beta \end{pmatrix}$.

1) Prove that $R_{A,B}(\rho,\beta)$ is a rotation matrix (representing thus the orientation of a frame B with respect to a fixed frame A) for any value of angles (ρ,β).
2) Express the matrix $R_{A,B}(\rho,\beta)$ as a product of 2 elementary rotation matrices[1] $R(\beta)R(\rho)$ (specify the axes of rotation for each of the 2 matrices).
3) Let $\rho = 90°$ and $\beta = -90°$, give the coordinates with respect to frame A of vectors x_B, y_B, z_B (of frame B). Give also the coordinates of a point P knowing that its coordinates with respect to frame B is $(0{,}5 \quad 0{,}5 \quad 0)$.

Exercise 2:

A frame $B = \{O_B, \ \ x_B, \ \ y_B, \ \ z_B\}$ is displaced and rotated with respect to a fixed reference frame $A = \{O_A, \ \ x_A, \ \ y_A, \ \ z_A\}$. The displacement is represented by the vector $\overrightarrow{O_A O_B} = (3 \quad 7 \quad -1)^T$, while the orientation of B with respect to A is represented by the following sequence of three Euler angles:

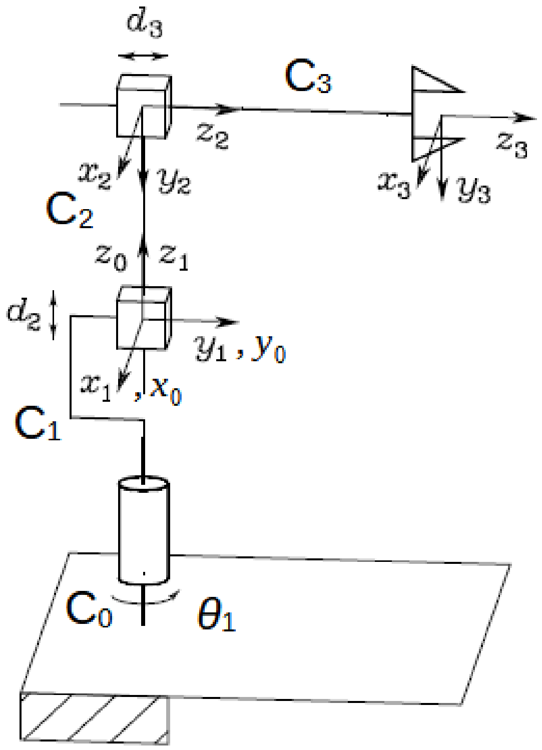
$$\psi = {}^{\pi}/_{4}, \theta = -{}^{\pi}/_{2}, \varphi = 0,$$

according to the convention (Z, Y', X'').

Provide the coordinates of a point P with respect to frame A knowing that its position with respect to frame B is given by $(1 \quad 1 \quad 0)^T$.

Exercise 3:

Consider the cylindrical arm represented in the following figure where a frame is attached at each link of the robot.



[1] The rotation is about an elementary axis, that is, x, y or z.

1) Provide the homogeneous transformation matrices $T_{0,1}, T_{1,2}, T_{2,3}$. Calculate $T_{0,3}$ to deduce the direct geometric model of the robot (that is $R_{0,3}$ and $\overrightarrow{O_0O_3}|_0$).
2) From $T_{0,3}$, give the location of the end-effector ($R_{0,3}$, $\overrightarrow{O_0O_3}|_0$) when the configuration/posture of the arm is initial. Verify on the figure the location of the frame R_3 with respect to the frame R_0.
3) From $T_{0,3}$, give the location of the end-effector when $\theta_1 = \frac{\pi}{4}, d_2 = 5\ cm, d_3 = 10\ cm.$
4)
   i) Deduce from $T_{0,3}$:
   - the joint variables that set the orientation of the frame R_3 with respect to the frame R_0,
   - the ones that set the coordinates *x* and *y* of $O_3|_0$,
   - the ones that set the coordinate *z* of $O_3|_0$.

   ii) Calculate the inverse geometric model.