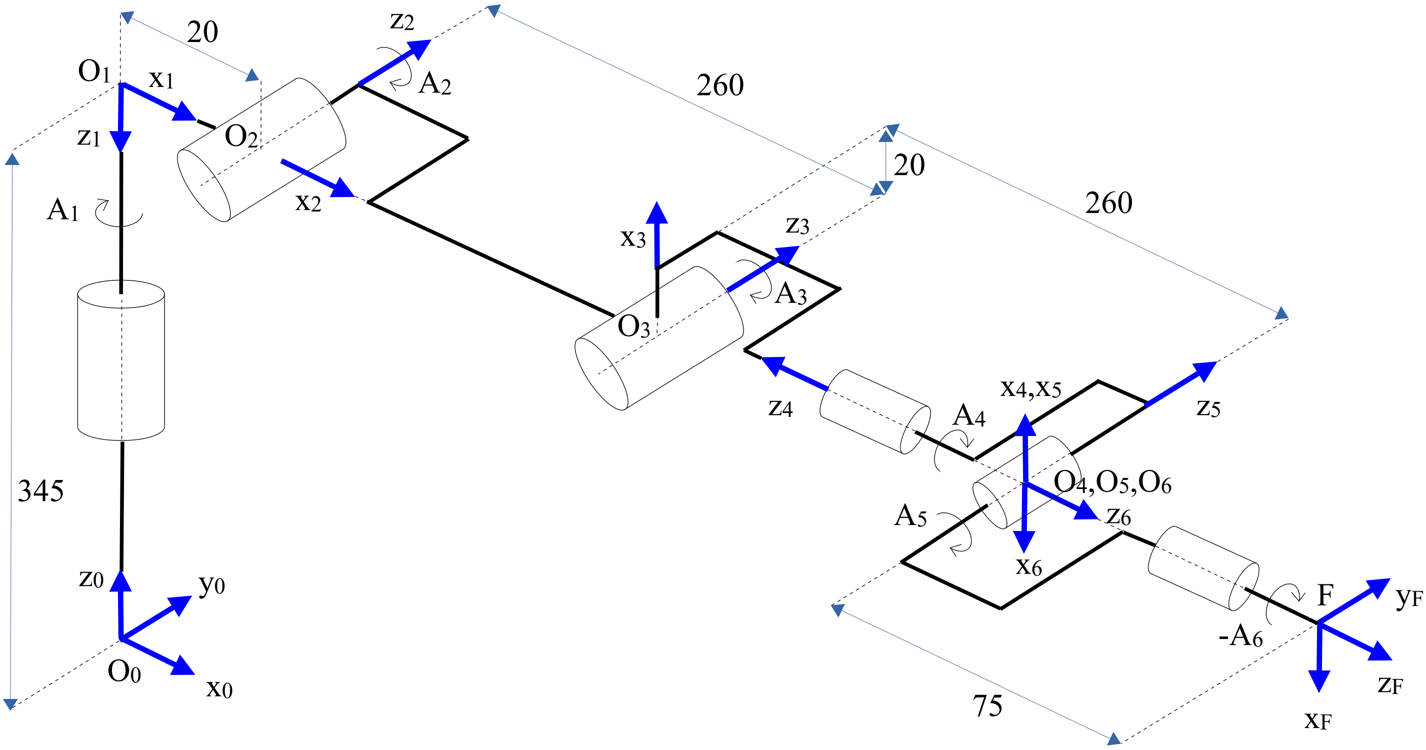
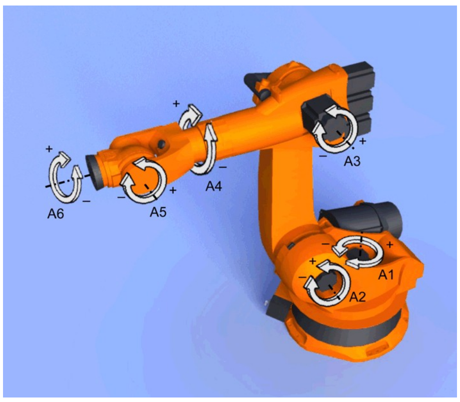
Robotics Tutorial 2: Inverse geometric model of KUKA KR3 R540

We aim to apply the method of Paul to compute the inverse geometric model of KUKA KR3. Let us consider the posture of the robot described below.



R_0 is the fixed reference frame (attached to the robot base), R_F is the frame attached to the robot flange (the coordinates of *F* are situated at the centre of the flange).
The direction of vectors z_1, …, z_6 is chosen according to the sign value of the joints given by the robot manufacturer, see the following figure.



The modified Denavit-Hartenberg parameters of the robot are given below (angles are in *rad*, distances are in *mm*):

j	α_j	d_j	θ_j	r_j
1	π	0	A1	-345
2	$\pi/2$	20	A2	0
3	0	260	A3-$\pi/2$	0
4	$\pi/2$	20	A4	-260
5	$-\pi/2$	0	A5	0

6	-π/2	0	π-A6	0
7 (F)	0	0	0	75

Let *P* be the point equal to points O_4, O_5, O_6 (located at the intersection of the last 3 concurrent axes of joints A4, A5, A6 corresponding to a spherical wrist). Since the robot has 6 *d.o.f.* with a spherical wrist, the inverse geometric model can be computed in 2 steps:

- the computation of joints A1, A2, A3 for a given planned point *P* through the *position equation* solving,
- the computation of joints A4, A5, A6 for a given planned orientation of frame R_7 through the *orientation equation* solving.

A) Position equation solving $\overrightarrow{O_0 P}_{|0} = T_{04} \overrightarrow{O_4 P}_{|4}$, *i.e.*, $\begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = T_{04} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

1) Compute the 2 possible values of joint A1 from the equation:

$$T_{10} \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = T_{14} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} .$$

2) From the equation $T_{20} \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix} = T_{24} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, deduce the following equation system:

$$\begin{cases} b_1 \cos(A2) + b_2 \sin(A2) = 13\cos(A3) + \sin(A3) + 13 \\ b_2 \cos(A2) - b_1 \sin(A2) = -\cos(A3) + 13\sin(A3) \end{cases} ,$$

by expressing the values of b_1 and b_2 as a function of A1 and constant values.

3) From the previous equation system, deduce the 2 possible values of joint A3 as a function of b_1, b_2 and constant values.

4) From the same equation system, deduce the 2 possible values of joint A2 as a function of A3, b_1, b_2 and constant values.

B) Orientation equation solving $A_{30}[S\,N\,A] = A_{36}(A4, A5, A6)$

5) Let $[F\,G\,H] = A_{30}[S\,N\,A]$, compute the vectors:

$$F = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}, G = \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix}, H = \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} .$$

6) From the equation $A_{43}[\,F\,G\,H\,] = A_{46}(A4, A5, A6)$, deduce the 2 possible values of joint A4 as a function of constant values, then the values of joints A5 et A6 as a function of A4 and constant values.

C) Solutions programming

7) Propose a script computing the 8 possible sets of joint parameters from the matrix $\begin{bmatrix} S_x & N_x & A_x & P_x \\ S_y & N_y & A_y & P_y \\ S_z & N_z & A_z & P_z \end{bmatrix}$ (singular configurations of the robot are not considered), this matrix defines the planned location (*i.e.* the position and the orientation) of frame R_F (also noted R_7) with respect to the reference frame R_0.

8) Test your script from the information (read on the SmartPAD of the robot) on the 3 following postures:

first posture:
Fx = 318.3; Fy = -258.43; Fz = 624.05; A = -108.62; B = 16.24; C = -72.32;
A1 = 42.27; A2 = -76.52; A3 = 79.96; A4 = 41.42; A5 = -27.40; A6 = 29.35;

second posture:
Fx = -76.53; Fy = 344.74; Fz = 733.23; A = -148.17; B = 13.80; C = 39.06;
A1 = 82.73; A2 = -109.95; A3 = -46.10; A4 = 41.42; A5 = 40.57; A6 = 44.28;

third posture (the one represented in the first figure):
Fx = 615; Fy = 0; Fz = 365; A = 0; B = 90; C = 0;
A1 = 0; A2 = 0; A3 = 0; A4 = 0; A5 = 0; A6 = 0.

To use the script of question 7, you have to deduce the coordinates of point *P* (with respect to R_0) from the ones (with respect to R_0) of point $F=[F_x \;\; F_y \;\; F_z]^t$ described in the first figure.